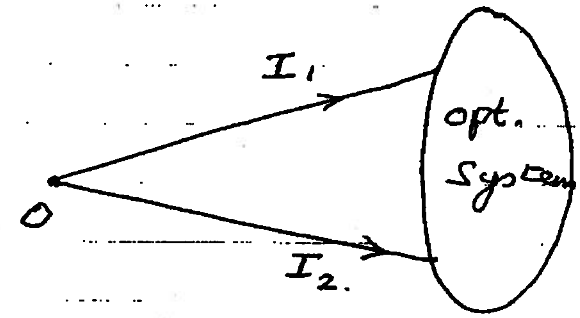
FORMATION OF IMAGES – MIRRORS.

General Construct to locate image of a point Object O using the laws of reflection and refraction to locate the path of light

Step 1 Take two incident rays I_1 and I_2 starting from O.



Step 2 Follow them through the optical system using

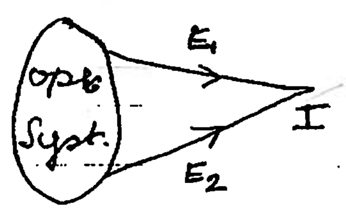
$$r = i \quad \text{for Reflection}$$

and

$$n_2 \sin R = n_1 \sin i \quad \text{for Refraction.}$$

Step 3. Locate the two rays E_1 and E_2 that emerge from the optical system. Two Cases arise:

Case I

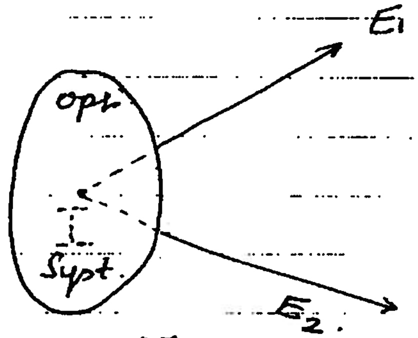


Point of intersection of E_1 and E_2 defines position of image I. Here, light actually goes through the point I so it is called a

REAL IMAGE

Note: A real Image Can be projected on a Screen.

Case II

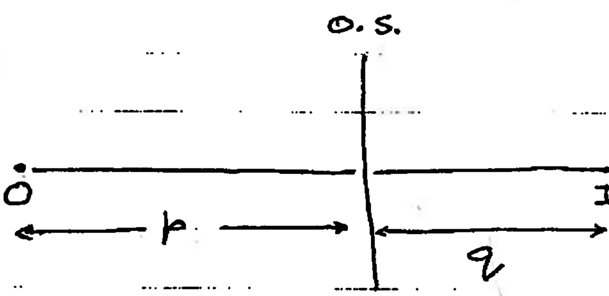


E_1, E_2 are diverging so they will not intersect. You will have to extrapolate the E_1, E_2 lines to locate the image I as the point from which E_1, E_2 "appear" to be Coming. No light actually goes through I so it is Called

VIRTUAL IMAGE

Note A virtual image cannot be projected on a Screen (your eye can see it)!

SOME DEFINITIONS



p = Distance of object from opt. Syst. (O.S.)

q = Distance of image from opt. system.

Magnification $m = \dfrac{\text{Size of image}}{\text{Size of object}} = -\dfrac{q}{p}$.

The minus sign on the right side ensures that for a single element O.S. all real images (q +ive) are Inverted (m -ive) while all virtual images (q -ive) are upright (m +ive).

MIRROR

A mirror is an optical system in which only reflections occur; so there is a light side and a dark side. We use the "sign" convention that distances are "+ive" on light side and "-ive" on dark side.

1. PLANE MIRROR: PLANE, SILVERED Sheet of Glass

Two rays:

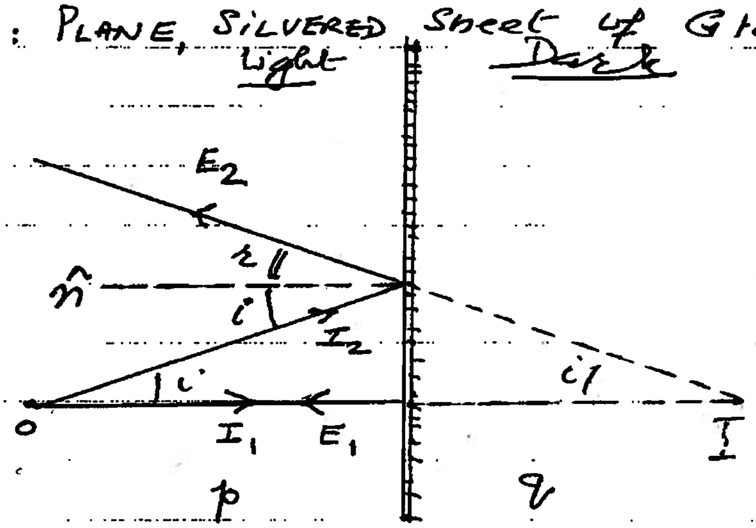
$\vec{I}_1 \parallel \hat{n}$

$i = 0, \; r = 0$

$\vec{E}_1 \parallel \hat{n}$

I_2: $r = i$

Locates E_2.



E_1 and E_2 diverge.

Locate image by extrapolation at I.

q is -ive.

Image is virtual, clearly $|q| = p$.

$$m = -\frac{q}{p} = +1.$$

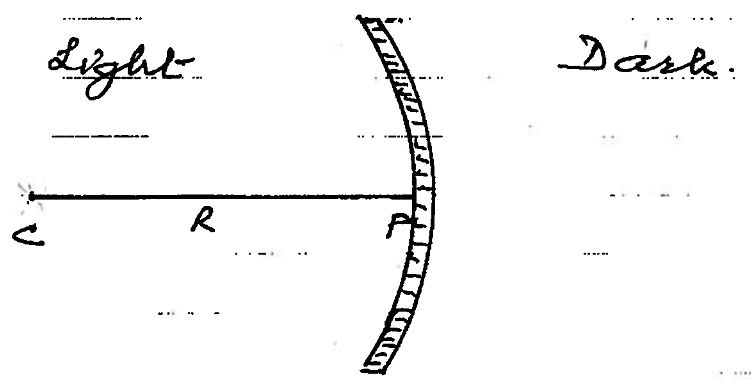
upright, virtual Image is as far behind the mirror as the object is in front of it

2 SPHERICAL MIRRORS: Mirror cut from a spherical shell of radius R.

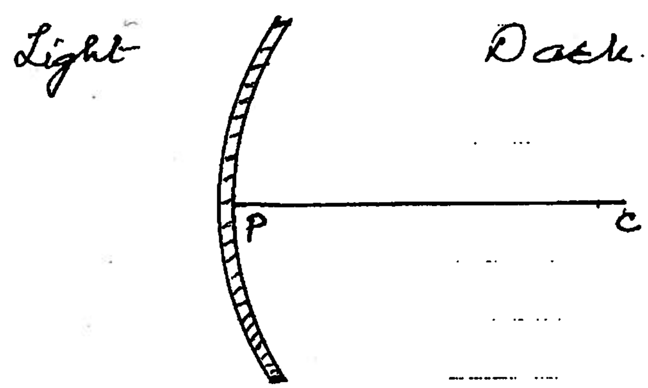
Two Cases arise

A CONCAVE. CENTER OF SPHERE IN FRONT OF MIRROR.

R is +ive



B CONVEX CENTER OF SPHERE BEHIND MIRROR.

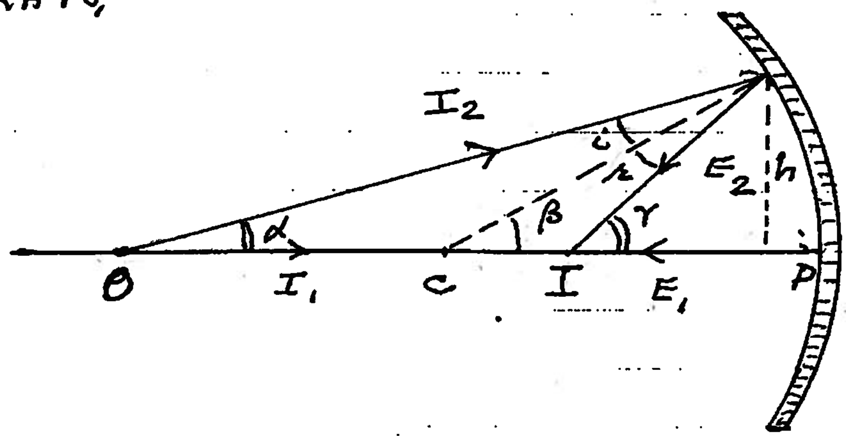


R is -ive

A Images in Concave Mirror

We will consider only paraxial rays, that is all angles are taken to be very small so $\sin\theta \approx \tan\theta \approx \theta$.

ALL OF OUR DISCUSSION ASSUMES "PARAXIAL" RAYS, THAT IS ALL ANGLES, (α, β, γ) are very small.



$\beta \approx \tan\beta$
$\alpha \approx \tan\alpha$
$\gamma \approx \tan\gamma$

Same construct. I_1, I_2 start from O, after reflection we get E_1, E_2, intersection locates I.

Equations

$$\beta = \alpha + i$$
$$\gamma = \beta + r$$
$$i = r$$

Hence $\gamma = \beta + i = 2\beta - \alpha.$

or $\gamma + \alpha = 2\beta$.

Angles are small, hence

$$\tan\gamma + \tan\alpha = 2\tan\beta.$$

$$\frac{h}{IP} + \frac{h}{OP} = \frac{2h}{CP}$$

or

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} \qquad (1)$$

and combined with

$$m = -\frac{q}{p} \qquad (2)$$

these two equations describe all possible images, and their sizes, formed by a concave mirror.

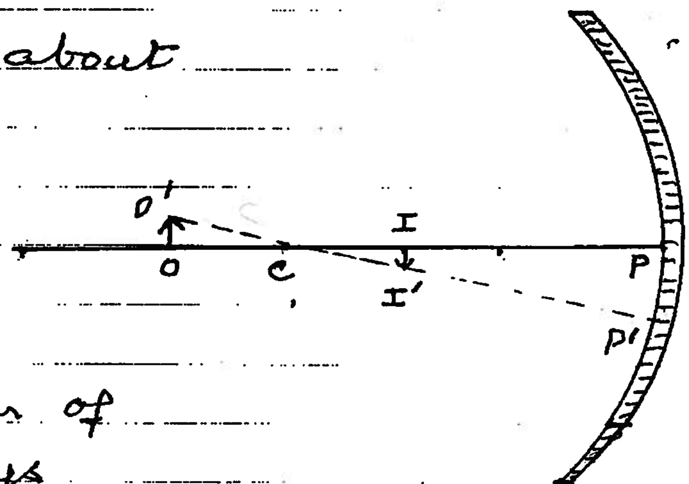
~~SPECIAL CASES~~
OBJECT SIZE.

a) Small object

Rotate picture about the center (of curvature) C

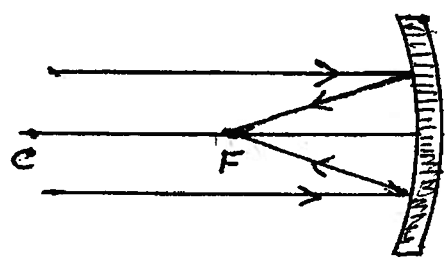
The above calculations apply.

Our assumption of paraxial rays requires that all objects are small.



SPECIAL CASES [Eqs (1) and (2)].

I $p \to \infty, \quad q = \frac{R}{2}$.



$p \to \infty$ Implies that incident ~~rays~~ light is a parallel beam, and we learn that when a parallel beam falls on the mirror, after reflection it converges to a point. This defines the focus and the focal length

$$f = R/2.$$

So $p \to \infty$, $q \to f$ ~~∞~~ $m \to 0$ Real Image

Concave Mirror makes parallel light converge to a point hence
Concave Mirror $\Rightarrow$ CONVERGENT MIRROR.

NOTE NO REAL IMAGE CAN COME CLOSER THAN f!

II $p > R$ object lies beyond C.

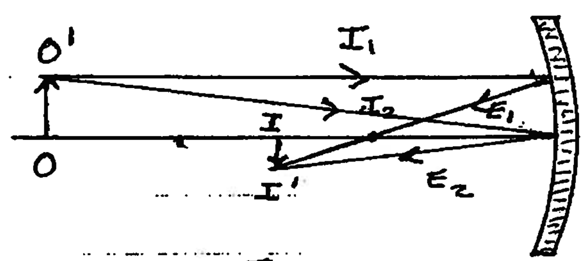
$$\frac{1}{q} = \frac{2}{R} - \frac{1}{p} = \frac{2p-R}{pR}$$

$$\frac{p}{q} = \frac{2p-R}{R} = \frac{>2R-R}{R} = >2-1 = >1$$

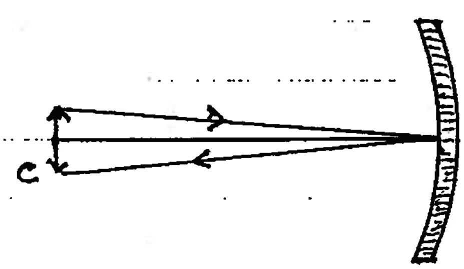
so $q < R$.

$m = -\frac{q}{p}$, $|m| < 1$.

Inverted, Real, Reduced Image.



III $p = R$, $q = R$, $m = -1$. [Lamp Expt.].
Object lies at C.
All light falls on mirror at $i=0$, so it leaves at $r=0$ and goes right back to C.



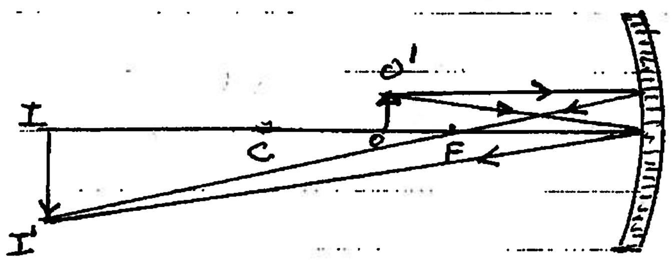
REAL, INVERTED IMAGE SAME SIZE AS OBJECT.

IV $p < R$, : object ~~at~~ lies between C and F

$$\frac{p}{q} = \frac{2p-R}{R} = \frac{<2R-R}{R} = <1$$

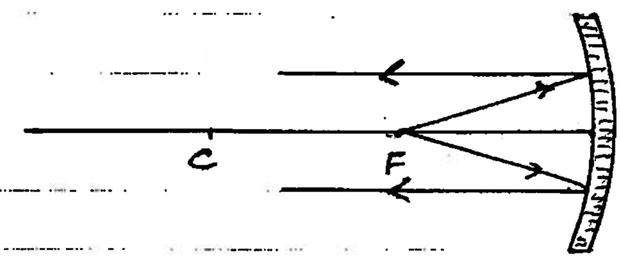
$q > R$, $\quad m = -\frac{q}{p}$, $|m| > 1$

Inverted, Real, Enlarged Image



V $p = f$, object at F, $q \to \infty$, $m \to \infty$

Point Source at F on reflection produces a parallel beam.

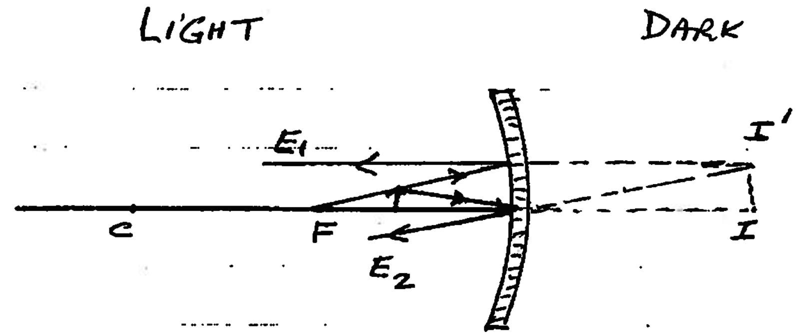


VI: $p < f$, object closer to mirror than F,

Since $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

this equation cannot be satisfied unless q becomes $\underline{-ive}$: IMAGE IS BEHIND MIRROR, ON THE DARK SIDE.

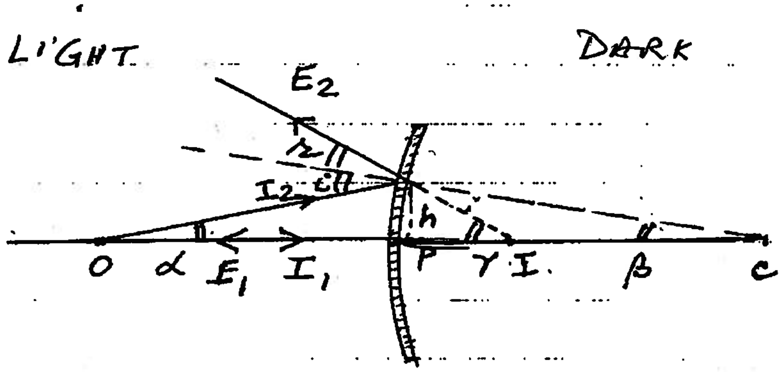
We get a VIRTUAL, ENLARGED IMAGE



To summarize: Start O far away, I at f (closest to mirror), bring O closer I moves away from mirror, gets bigger but remains inverted real, O at C, I at C, $m = -1$, O between C and F, I beyond C, enlarged, inverted, real, O at F, $I \to \infty$, Parallel light; O closer than F, I goes <u>BEHIND</u> mirror becomes virtual and upright.

<u>B</u> Images in Convex Mirror

<u>R is -ive</u>



$OP = p$
$IP = q$ [q is -ive]

<u>Eqns</u>

$$\gamma = \cancel{\varepsilon} + \beta = i + \beta$$
$$i = \alpha + \beta$$
$$\gamma = \alpha + 2\beta$$
$$\alpha - \gamma = -2\beta$$

$$\tan\alpha - \tan\gamma = -2\tan\beta.$$

$$\frac{h}{p} - \frac{h}{q} = -\frac{2h}{R}$$

or

$$\frac{1}{p} - \frac{1}{q} = -\frac{2}{R}$$

However, both q and R are -ive so one should write

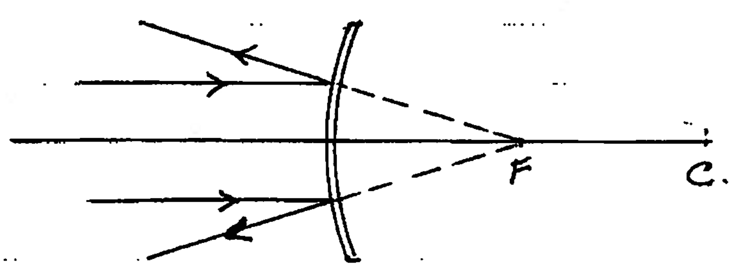
$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R}. \begin{bmatrix} R \text{ -ive} \\ q \text{ -ive} \end{bmatrix} \quad (3)$$

and supplement it with

$$m = -\frac{q}{p} \qquad (4)$$

SPECIAL CASES

I $p \to \infty$, Parallel light falls on mirror $q = R/2 = f$ but f is -ive, f is behind mirror on dark side.



A parallel beam of light incident on a convex mirror, on reflection appears to start from F and becomes divergent.

Hence

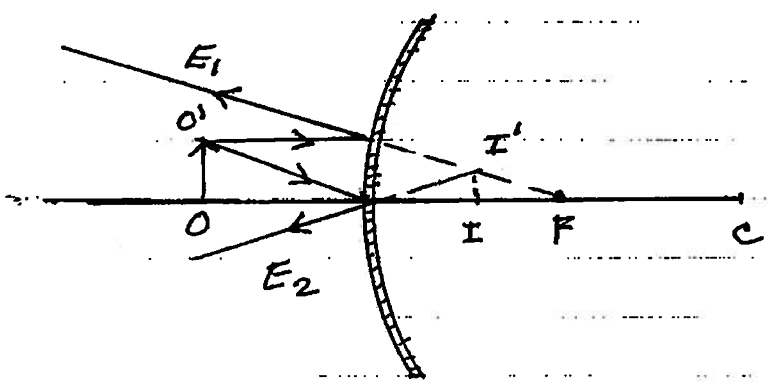
CONVEX MIRROR $\Rightarrow$ DIVERGENT MIRROR.

II Since q is −ive all Images are VIRTUAL and upright.

Also
$$\frac{1}{p} - \frac{1}{q} = -\frac{2}{R}$$
or
$$\frac{1}{q} = \frac{1}{p} + \frac{2}{R} \qquad \left[\frac{1}{q} > \frac{1}{p}\right].$$

So q is always less than p.
and always less than f.

All Images are Virtual, upright and reduced.



Right rear view mirror of automobiles!!
That is why Cars appear small and we have a warning "objects in mirror are closer than they ~~ap~~ appear.