ENERGY CONSERVATION PRINCIPLE - REVISITED.

(~~E-field~~ POTENTIAL ENERGY, POTENTIAL, EQUIPOTENTIALS.)

FIRST, RECALL FROM PHYS 121 WHERE WE TALKED ONLY OF MECHANICAL ENERGY:

MECHANICAL WORK

$$\Delta W = \vec{F} \cdot \Delta\vec{S}$$
$$= F \Delta S \cos(\vec{F}, \Delta\vec{S})$$

where

$\vec{F}$ = force

$\Delta\vec{S}$ = Displacement.

Note: NO work is done if $\vec{F} \perp \Delta\vec{S}$.

KINETIC ENERGY

WORK STORED IN MOTION. AS SHOWN BEFORE IF AN OBJECT OF MASS M IS SITTING AT REST THE WORK REQUIRED TO GIVE IT A VELOCITY V IS EXACTLY

$$K = \frac{1}{2} MV^2$$

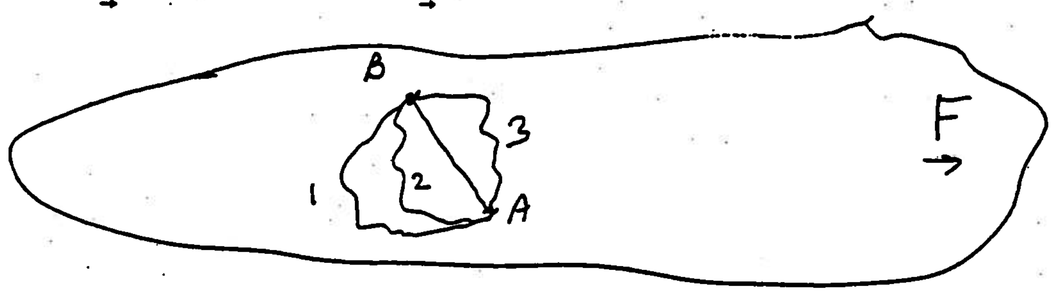
or since momentum is $p = MV$

$$K = \frac{p^2}{2M}$$

Potential Energy (P) presents a greater conceptual challenge.

P is the mechanical work stored in a system when it is prepared (or put together) in the presence of a prevailing conservative force.

Suppose we have a region of space in which there is a prevailing force (weight near Earth's surface comes to mind). That is, at every point in this region an object will experience a force. Let the object be at point B [First, notice that you cant let the object go as $\vec{F}$ will immediately cause $\vec{a}$ and object will move].



To define P at B we have to calculate how much work was needed to put the object at B in the presence of $\vec{F}$. Let us pick some point A, where we can claim that P is known, and calculate the work needed to go from A to B. As soon as we try to do that we realize that the only way we can get a meaningful answer is if the work required to go from A to B is independent of the path taken. So our prevailing force has to be special. Such a force is called a CONSERVATIVE FORCE – WORK DONE DEPENDS ONLY ON END-POINTS AND NOT ON THE PATH TAKEN.

If that is true we have a unique answer

$$\Delta w_1 = \Delta w_2 = \Delta w_3 = \Delta w_{AB}$$

and we can use this fact to calculate the change in P in going from A to B

$$\Delta P_{AB} = -\vec{F} \bullet \Delta \vec{S}_{AB}$$

NOTE THE –SIGN: It comes about because as stated above we cannot let the object go. In fact, the displacement from A to B must be carried out in such a way that the object cannot change its speed (if any). That is, we need to apply a force $-\vec{F}$ to balance the ambient $\vec{F}$ at every point. The net force will become close to zero at all points. ΔP_{AB} is work being done by $-\vec{F}$.

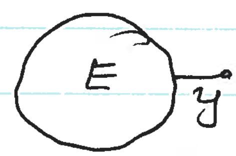
So when $\vec{F}$ is conservative ΔP_{AB} is unique. In the final step we can choose A such that $P_A = 0$. Then $P_B = -\vec{F} \bullet \Delta \vec{S}_{AB}$

Using the above definition for P we derived

(i) P for Earth-Mass system [taking zero when M is on surface of Earth]

$$\vec{F}_g = -Mg\,\hat{y}$$

$$P_g(y) = Mgy$$



as long as y is very small.

(ii) P for object attached to spring [taking zero when spring unstretched].

$$\vec{F}_{sp} = -kx\,\hat{x}$$

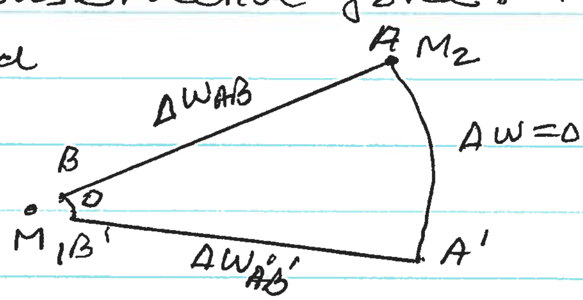
$$P_{sp}(x) = \frac{1}{2}kx^2.$$

(iii) P_G for M_1, M_2 separated by r

$$* \quad \vec{F}_G = -\frac{GM_1M_2}{r^2}\hat{r}.$$

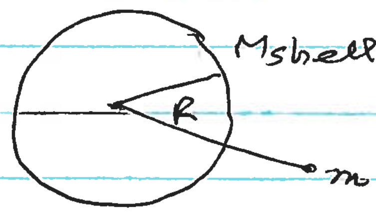
$$P_G = -\frac{GM_1M_2}{r}.$$

Note that we showed explicitly that this is a conservative force. Fixed M_1 at $r=0$, moved M_2 either along radius AB



or on path $A - A' - B' - B$. Since $A'B'$ is also a radius $\Delta W_{A'B'} = \Delta W_{AB}$. On AA' & $B'B$ no work is done because $\vec{F}_G \perp \vec{\Delta S}$.

(iv). P_G for M_{shell} and m.



Now $\vec{F}_G = 0 \qquad r < R$

$$\vec{F}_G = -\frac{GM_{shell}\, m}{r^2} \qquad r > R.$$

hence

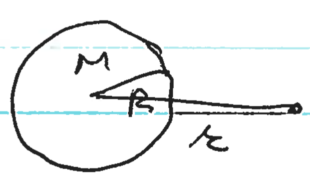
$r > R$

$$P_G = -\frac{GM_{shell}\, m}{r}$$

$r < R$

$$P_G = -\frac{GM_{shell}\, m}{R}.$$

(v) P_G for Solid sphere M and m.



Now $\vec{F}_G = -\frac{4\pi}{3} G\rho m r \hat{r} \qquad r < R$

$$\vec{F}_G = -\frac{GMm}{r^2}\hat{r} \qquad r > R.$$

so $P_G = -\frac{GMm}{r} \qquad r > R$

$$P_G = -\frac{GMm}{R} - \frac{GMm}{2R}\left[1 - \frac{r^2}{R^2}\right] \qquad r < R.$$

CONSERVATION OF ENERGY

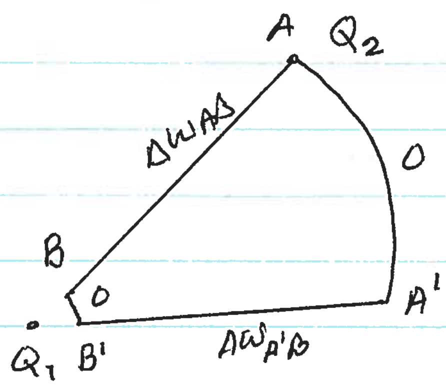
$$K_f + P_f(G) + P_f(SP) = K_i + P_i(G) + P_i(SP) + W_{NCF}.$$

Now that we have charge. We have a new force

$$\underset{\rightarrow}{F_E} = \frac{k_e Q_1 Q_2}{r^2} \hat{r}$$

This force has exactly the same structure as $\underset{\rightarrow}{F_G}$. Hence it is also a <u>CONSERVATIVE</u> FORCE

Fix Q_1



Move Q_2 [Note that you must apply a force $-\underset{\rightarrow}{F_E}$ to Q_2 at all times. Otherwise, it will be pushed away by Q_1]. along AB or on $A \to A' \to B' - B$ path.

$\Delta W_{A'B'} = \Delta W_{AB}$.

$\Delta W = 0$ for AA' & BB'

so again work done independent of path.

Potential Energy for $\underset{\rightarrow}{F_E}$ $\quad \Delta P_E = -\underset{\rightarrow}{F_E} \cdot \underset{\rightarrow}{\Delta S}$ $\quad$ Electrostatic Potential Energy.

Now we define a new Quantity Electrostatic Potential :

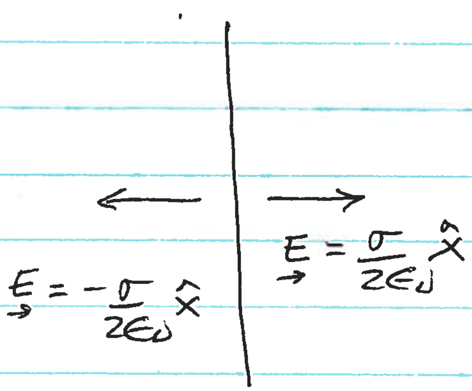
$$\Delta V = \frac{\Delta P_E}{q} = -\frac{\underset{\rightarrow}{F_E}}{q} \cdot \underset{\rightarrow}{\Delta S} = -\underset{\rightarrow}{E} \cdot \underset{\rightarrow}{\Delta S}.$$

The Energy Conservation Equ. now has both mech. Energy and El. Energy

$$K_f + P_f(G) + P_f(SP) + P_f(E) = K_i + P_i(G) + P_i(SP) + P_i(E) + W_{NCF}$$

SOME CALCULATIONS OF POTENTIAL

(i) Single plate with σ C/m²,



$$\Delta V = -\vec{E}\cdot\Delta\vec{s}$$

Since $\vec{E} \parallel \hat{x}$.

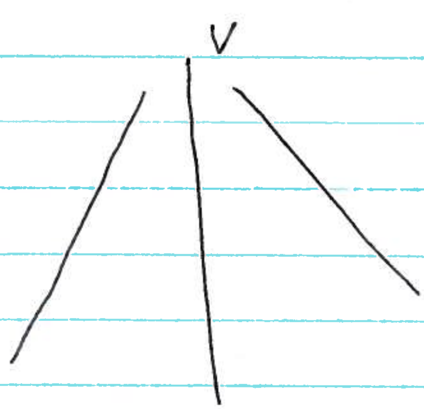
Nonzero ΔV only if $\Delta\vec{s} \parallel \hat{x}$.

$x > 0$ $\quad \Delta\vec{s} = x\,\hat{x}$

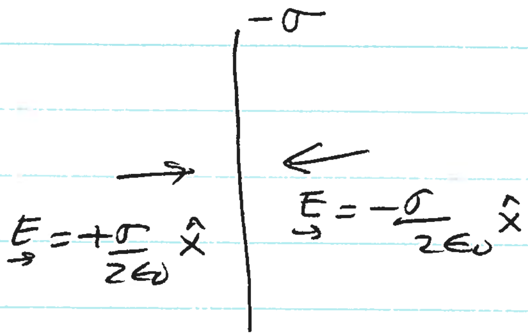
$$\Delta V = -\frac{\sigma}{2\epsilon_0}x$$

$x < 0$ $\quad \Delta\vec{s} = -x\,\hat{x}$.

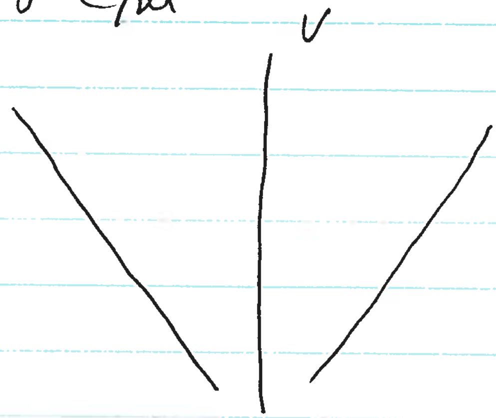
$$\Delta V = +\frac{\sigma}{2\epsilon_0}(-x)$$



ii) Single plate with $-\sigma\ C/m^2$



So



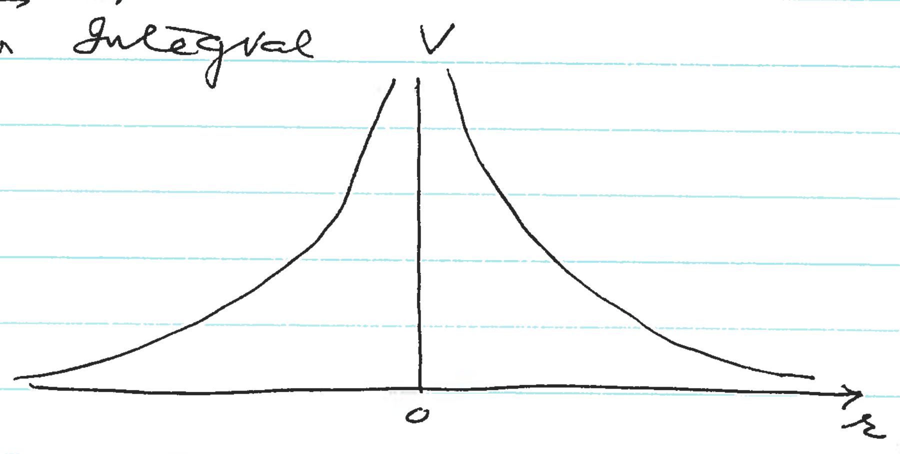
(iii) Single Q at $r=0$.

$$\vec{E} = \frac{k_e Q}{r^2}\hat{r}.$$

Take $V=0$ at large r ($r \to \infty$).

Now $\Delta V = -\vec{E} \cdot \Delta \vec{r}$ needs to be evaluated by an integral

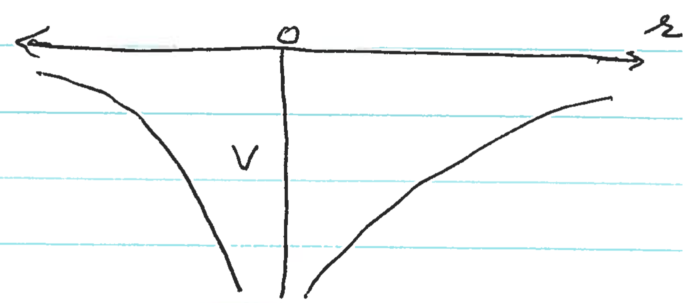
$$V(r) = \frac{k_e Q}{r}$$



(iv) Single $-|Q|$ at $r=0$

$$\vec{E} = -\frac{k_e Q}{r^2}\hat{r}$$

So



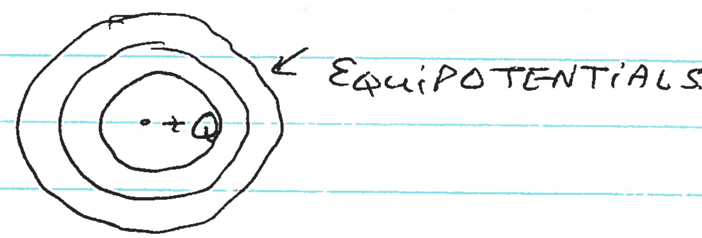
EQUIPOTENTIALS

CURVES (in 2-d) and Surfaces (in 3-d) of Constant Potential (V = Const.).
Two immediate Consequences: i) if a charge moves on an equipotential it costs no Energy, (ii) The $\vec{E}$-field must be perpendicular to an equipotential.

Examples

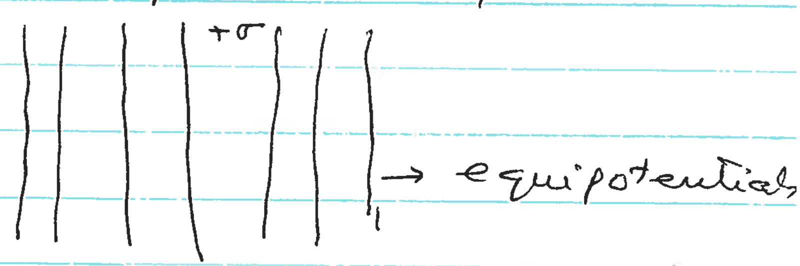
i) Pt. charge Q at $r=0$

$V = \frac{k_e Q}{r}$ so Equipotentials are spheres of radius r whose center is at $r=0$



(ii) Plate Carrying $+\sigma$ C/m^2 $\Delta V = \frac{-\sigma}{2\epsilon_0} x$.

Equipotentials are planes parallel to plate



(iii) Surface of a Conductor. Since $\vec{E} \perp$ surface, Surface of a Conductor is always an Equipotential.