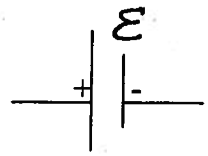
DEVICES – AC CIRCUITS

Battery Source of Coulomb $\vec{E}$ -field

Output is *emf*: ε

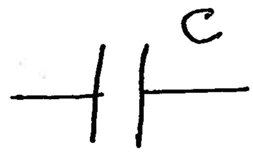


Capacitor: Container for $\vec{E}$ -field $C = \frac{Q}{V}$

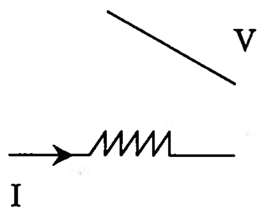
Potential Energy $U_E = Q^2/2C$

η_E = Energy stored per $m^3 = \frac{1}{2}\epsilon_0 E^2$

$\epsilon_0 = 9 \times 10^{-12}\ F/m$



Resistor: Represents that it costs energy to transport charge through a conductor

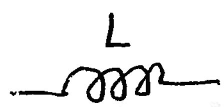


$R = \frac{V}{I}$ $\quad \left[\vec{J} = \sigma \vec{E}\right]$

Power loss $P = I^2 R = \frac{V^2}{R}$

Inductor: A time varying current causes a Non-Coulomb $\vec{E}$ -field, or an induced *emf*, $L = \frac{-\varepsilon}{\left(\Delta i / \Delta t\right)}$

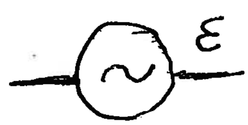
Container for $\vec{B}$ -field, Potential Energy $U_B = \frac{1}{2}Li^2$



η_B = Energy stored per $m^3 = \frac{B^2}{2\mu_0}$

A.C. Generator: Wire loops of area A rotated at ω rad/s in a $\vec{B}$ -field. Generates non-coulomb $\vec{E}$ -field in the loops, produces an *emf*: $\varepsilon = \omega NBA\ Sin(\omega t)$

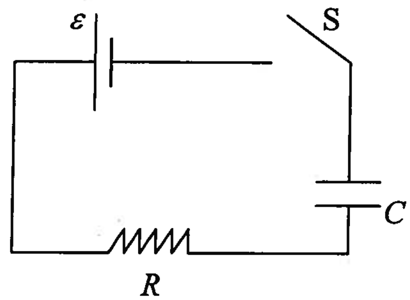
Where N= # of turns in the loop. Hence *ac-generator*

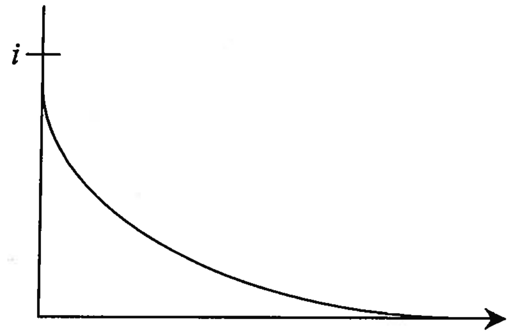


$\Phi_B = NBA\cos(\theta)$
where θ is angle
between $\hat{n}$ and $\vec{B}$
Rottn by ω rad/s makes $\theta = \omega t$, $\Phi_B = NBA\cos\omega t$
so $\frac{\Delta\phi_B}{\Delta t} = -NBA\omega\sin\omega t$, $\varepsilon = -\frac{\Delta\phi_B}{\Delta t} = \omega NBA\sin\omega t$

CIRCUITS

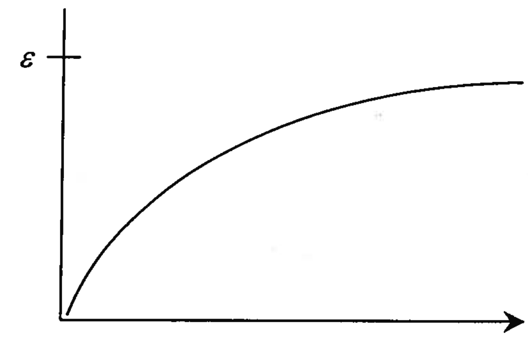
I. RC with battery, close switch at t=0, current flows immediately, potential across C appears later $\varepsilon = {}^{q}\!/_{C} + iR$





$$i = \frac{\varepsilon}{R} e^{-t/RC}$$

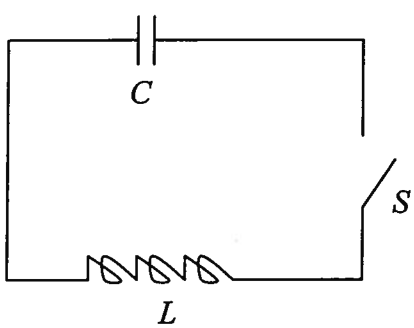
$\boxed{\tau = RC}$

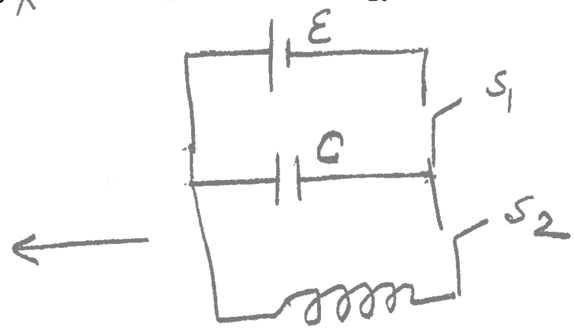


$$v_c = \varepsilon\left[1 - e^{-t/RC}\right]$$

III. LC-Circuit: Undamped Oscillator

Close S_1, First charge C to Q_0. open S_1 Close ~~switch~~ S_2 at t=0. Energy stored in capacitor $U_E = \dfrac{Q_0^2}{2C}$





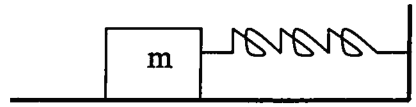
Charge begins to flow. Total Energy = (Energy in $\vec{E}$-field) + (Energy in $\vec{B}$-field)

= (Energy in C) + (Energy in L)

$$\frac{Q_0^2}{2C} = \frac{q^2}{2C} + \frac{1}{2}L\left(\frac{\Delta q}{\Delta t}\right)^2$$

Recognize, similarity to spring-mass oscillator

$$\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}m\left(\frac{\Delta x}{\Delta t}\right)^2$$



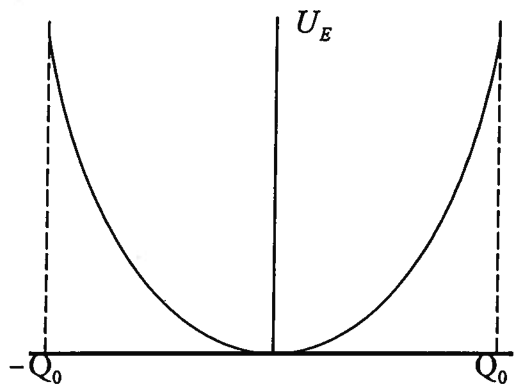
$x \rightarrow q$ $\qquad$ $x = A\ Cos\,\omega t$

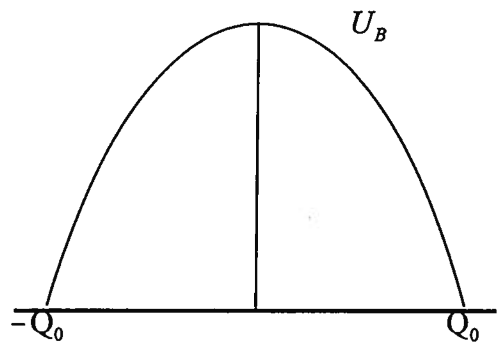
$m \rightarrow L$ $\qquad$ $\omega_0 = \sqrt{\frac{k}{m}}$

$k \rightarrow \frac{1}{C}$

Now $\qquad$ $q = Q_0\ Cos\,\omega_0 t$

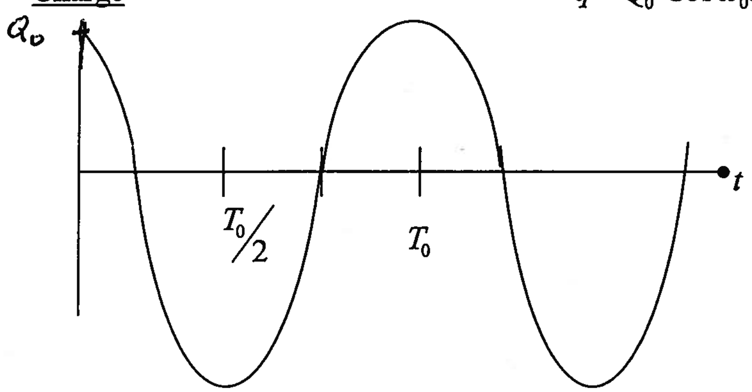
$$\omega_0 = \frac{1}{\sqrt{LC}}$$



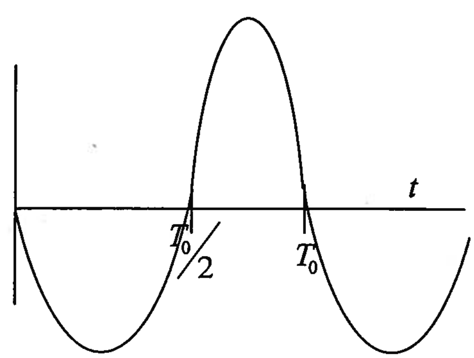


$\vec{E}$ -field collapses giving rise to $\vec{B}$ -field and vice versa.

Charge $\qquad$ $q = Q_0\ Cos\,\omega_0 t$



Current

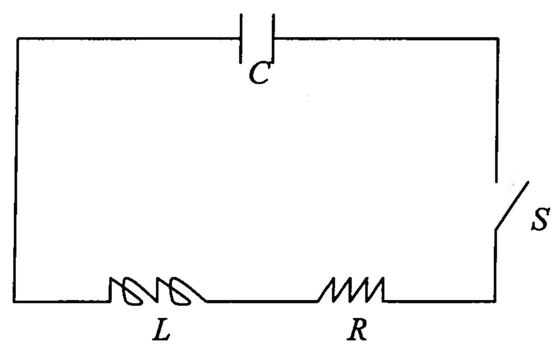


$i = -Q_0\omega_0\ Sin\,\omega_0 t$

Period $T = \frac{2\pi}{\omega_0}$

IV. LCR-CIRCUIT: DAMPED OSCILLATOR.

At t=0, charge C to Q_0 close switch. Now driving i through R costs i^2R per second.



So $\left(\frac{q^2}{2C}+\frac{1}{2}Li^2\right)$ IS NOT CONSTANT

$$\frac{\Delta\left(\frac{q^2}{2C}+\frac{1}{2}Li^2\right)}{\Delta t} = -\, i^2R$$

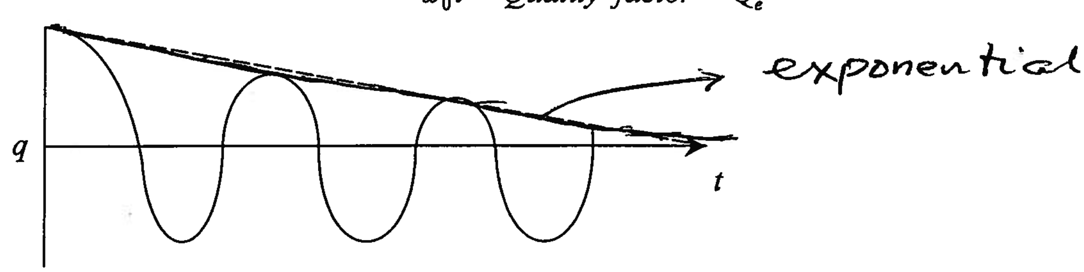
-ive sign on right because energy is being lost (R is getting warmer).

Now $q = Q_0 e^{-Rt/2L}\, Cos\,\omega t$

$$\omega = \omega_0\left[1-\frac{1}{(2\omega_0\tau)^2}\right]^{1/2}$$

$$\tau = \frac{L}{R}$$

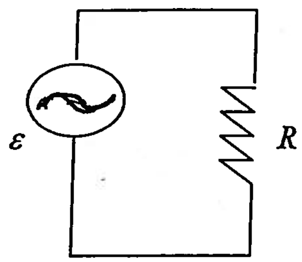
$$\omega_0\tau = \textit{Quality factor} = Q_e$$



Note 1: smaller R, larger the duration for which the oscillations persist.
Note 2: R plays role of friction; as always energy lost goes to raise temperature. Electrical Equivalent of Heat.

CIRCUITS: AC

I. Resistor and Generator



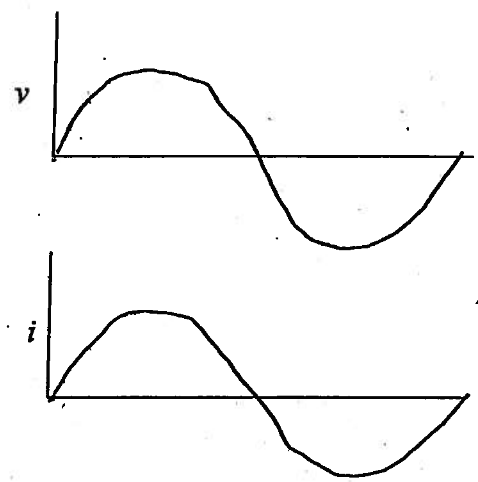
$$V_R = IR$$

so

If $\varepsilon = \varepsilon_0 \, Sin\,\omega t$

$$i = \frac{\varepsilon_0}{R} Sin\,\omega t$$

Current and voltage are in phase.



Power

$$P(t) = iv$$

$$= \frac{\varepsilon_0}{R} Sin^2\ \omega t$$

$$\langle \sin^2 \omega t \rangle = \frac{1}{2}$$

averaged over a cycle. $< P >= \dfrac{\varepsilon_0^{\,2}}{2R} = \dfrac{i_0 \varepsilon_0}{2} = \dfrac{i_0^{\,2} R}{2}$ and the power loss is as if R was connected to a battery ~~who~~ whose $\varepsilon = \dfrac{\varepsilon_0}{\sqrt{2}}$. In this sense one talks of $\dfrac{\varepsilon_0}{\sqrt{2}}$ *and* $\dfrac{i_0}{\sqrt{2}}$ as root-mean-square or r.m.s. voltage and current.

R.m.s. voltage in our house, $V_{rms} \approx 115$ Volts

$f = 60\,Hz$

$\omega = 377$ rad/sec

$V_{max} \simeq 155$ Volts