Devices

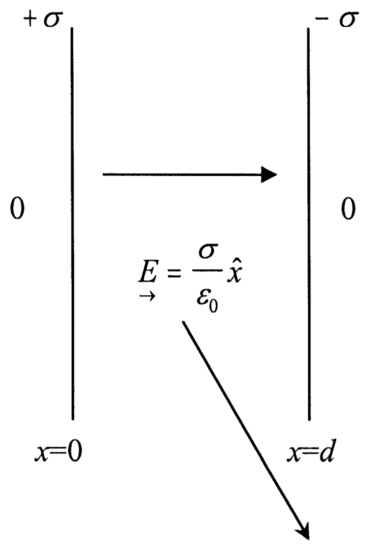
Battery: Generates $\vec{E}$-field using chemical energy.

$\varepsilon = \varepsilon mf$ is actually potential difference between + and – plates.

Capacitor: "Bucket for $\vec{E}$-field Capacitance $C = \frac{Q}{V}$ [For a given V, C tells you how much charge can you store].

Parallel Plates of Area A separated by d, each plate has charge density $\sigma = \frac{Q}{A}$.

$\vec{E}$-field is
$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{x} \quad [0 < x < d]$$
$$\vec{E} = 0 \quad \textit{at all other } x.$$



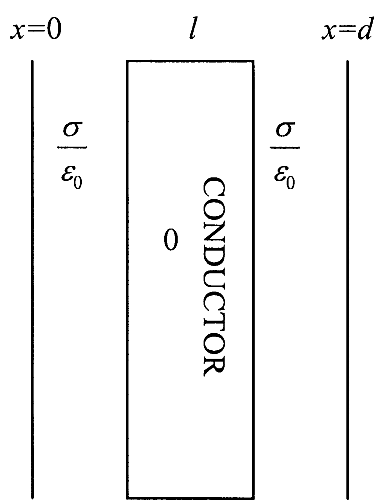
air or vacuum between plates

$$\Delta V = -\vec{E} \bullet \Delta\vec{S} = -\frac{\sigma}{\varepsilon_0}d \quad so,\ V = \left|\frac{\sigma}{\varepsilon_0}d\right|$$

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$

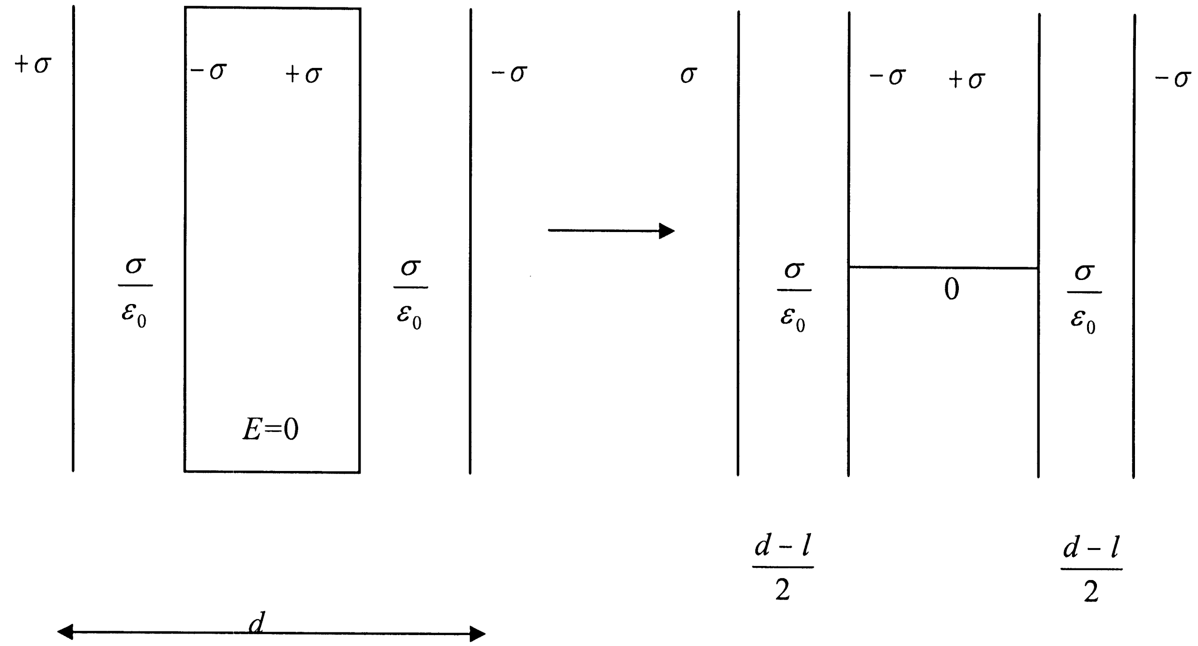
$$V = \frac{\sigma}{\varepsilon_0}(d - l)$$

$$C = \frac{\varepsilon_0 A}{d - l} \longrightarrow (1)$$



Put conductor of thickness l in middle. Now $\vec{E}$ =0 inside conductor

<u>We have</u>



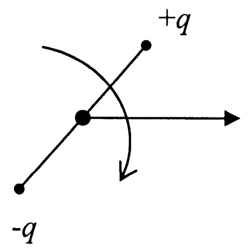
That is, as if two capacitors were in series, Each having $C_1 = C_2 = \dfrac{2\varepsilon_0 A}{d-l}$ (2)

And comparing Eqs. (2) & (1) you see $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

So when capacitors are connected in series, total equivalent capacitance is given by

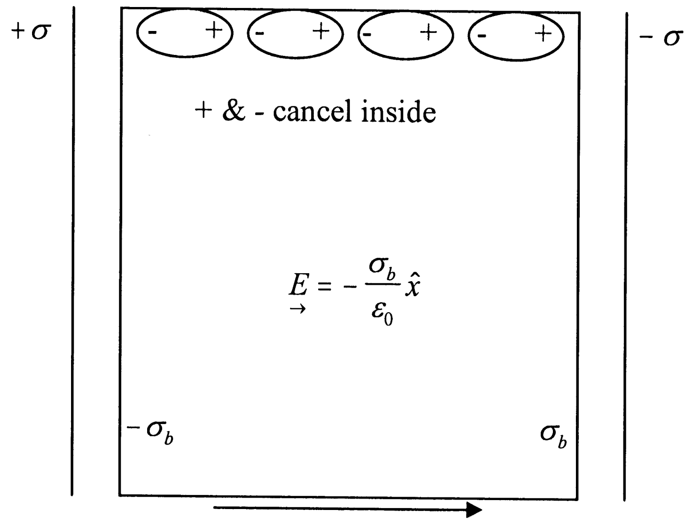
$\dfrac{1}{C_s} = \Sigma \dfrac{1}{C_i}$ ⟵ SERIES CONNECTION

<u>Dielectric between plates</u>. Dielectric consists of Dipoles. Dielectric is an insulator.



$\vec{\tau} = \vec{p} \times \vec{E}$

Every dipole feels a torque which makes it parallel to the $\vec{E}$-field between the plates hence the system looks like what is shown below:



$$\underset{\rightarrow}{E} = +\frac{\sigma}{\varepsilon_0}\hat{x}$$

Dipole in $\underset{\rightarrow}{E}$ -field experiences torque, which causes each Dipole to line up along $\underset{\rightarrow}{E}$.

On surfaces of dielectric charge sheets $+\sigma_b$ and $-\sigma_b$ appear.

$\underset{\rightarrow}{E}$ -field inside dielectric is the sum of field due to charge on plates $\left[+\frac{\sigma}{\varepsilon_0}\hat{x}\right]$ and that due to charge on $[-\frac{\sigma_b}{\varepsilon_0}\hat{x}$ surfaces.

Hence,

$$\underset{\rightarrow}{E_k} = \frac{\sigma - \sigma_b}{\varepsilon_0}\hat{x}$$

$$\frac{E_k}{E} = \frac{\sigma - \sigma_b}{\sigma} = \frac{1}{k}$$

k= Dielectric Const. $[N.B.\ k\ is\ always > 1]$.

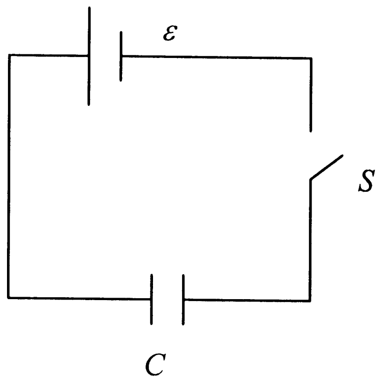
Now $V_k = E_k d = \frac{Ed}{k} = \frac{\sigma d}{\varepsilon_0 k}$

So $C_k = \frac{Q}{V_k} = \frac{\sigma A \varepsilon_0 k}{\sigma d} = \frac{k\varepsilon_0 A}{d}$

Capacitance is increased by factor k.

Note: in both cases conductor of thickness l between plates or Dielectric of thickness d, potential difference is reduced but physics is totally different!

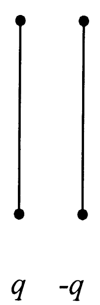
Next, put the two devices in a circuit:



Note: the lines connecting the devices are perfect conductors and so under stationary conditions they must become equipotentials. That is if we close the switch S and wait for a while the Potential difference across C will become ε.

$$V = \varepsilon$$
$$Q = \varepsilon C$$

So the capacitor plates now have $+Q$ (left) and $-Q$ (right). How did this happen? Clearly, the +ive plate of the battery pulled electrons from the left sheet of capacitor while the –ive plate pushed electrons on the right sheet. Effectively, you start with q=0 on either sheet, transfer charge from one sheet to the other, so one becomes $-q$ & the other q [However, no flow of charge between capacitor plates]

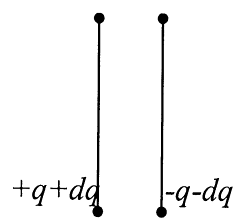


$V = q/c$

Now take Δq from rt. To Left. Work done will be

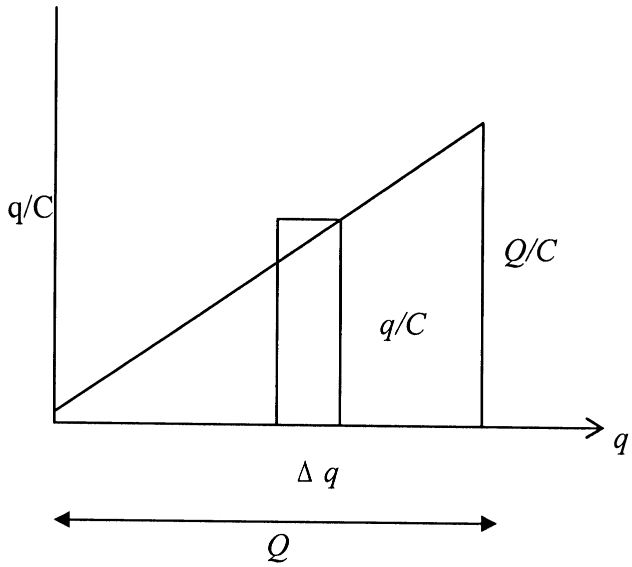
$DW = \Delta q \dfrac{q}{C}$ area of rectangle. (next page picture)

And we get



CONSERVATIVE FORCE: Work done is independent of the path.

To build up charge from 0 to Q you need area of Δ . This work is now $U_E = \frac{1}{2} \cdot Q(Q/C) = \frac{Q^2}{2C}$



Where does this work go? Notice, space between plates is not empty, there is an $\underset{\rightarrow}{E}$ -field in it. This energy is stored as potential energy in that $\underset{\rightarrow}{E}$ -field.

Apply it to ||-plate [air bet plates] $Q = \sigma A, C = \frac{\varepsilon_0 A}{d}$

$$U_E = \frac{\sigma^2 A^2 d}{2\varepsilon_0 A}$$

$$= \frac{1}{2}\varepsilon_0 E^2 Ad \qquad E = \frac{\sigma}{\varepsilon_0}$$

Energy density = $\frac{U_E}{vol} = \frac{U_E}{Ad} = \frac{1}{2}\varepsilon_0 E^2$ this is like a "Pressure" $\eta_E = \frac{1}{2} \in_0 E^2$

||-Plate [Dielectric]

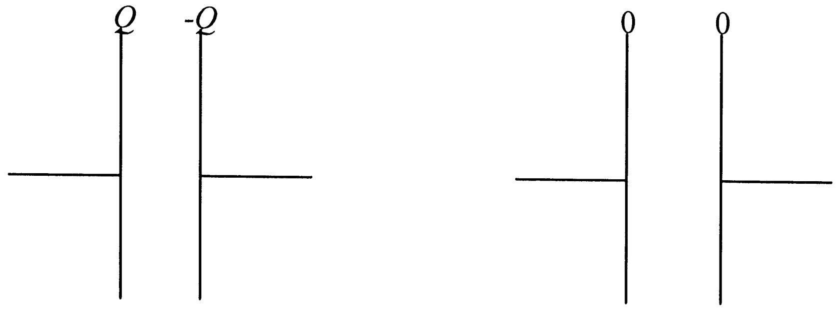
$$U_E = \frac{Q^2}{2C_k} = \frac{\sigma^2 A^2 d}{2k\varepsilon_0 A}$$

$$= \frac{1}{2} k\varepsilon_0 E_k{}^2 Ad$$

$$\eta_E(k) = \frac{1}{2} k\varepsilon_0 E_k{}^2$$

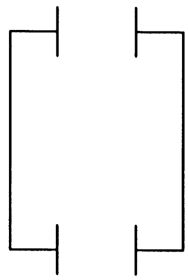
Next, consider the Expt: two identical capacitors.

First, put charges $\pm Q$ on one $U_E = \dfrac{Q^2}{2C}$



Next, connect left-to-left, right to right to make Equipotentials, now charge will be $\pm\dfrac{Q}{2}$ on each.

Total Energy $U_E = \dfrac{2\cdot 1}{2}\left(\dfrac{Q}{2}\right)^2 \cdot \dfrac{1}{C} = \dfrac{Q^2}{4C}$



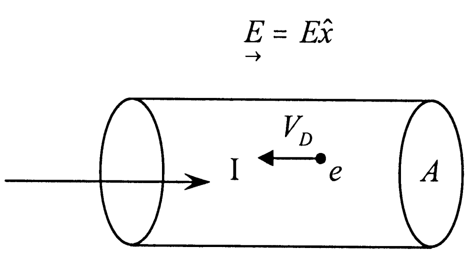
What happened to half of the energy? In the second half of the experiment charge was transported from one set of plates to the other. This experiment tells us that it costs energy to transport charge through a conductor. This leads us to our third device.

-RESISTOR

First we define current

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t}$$

as the amount flowing per second



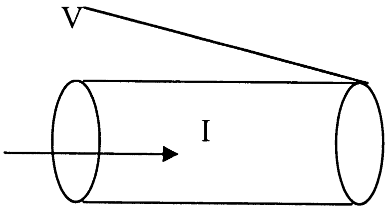
To calculate, quantity of charge flowing per second, note that cross-sectional area is A and since only electrons are mobile one can write $I = n_e(-e)\,A(-V_D)$

where n_e = # of mobile electrons/m^3

$-e = 1.6x10^{-19}\,C$

$V_D = drift\ speed$

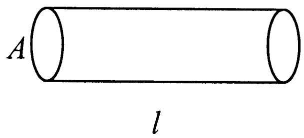
Notice: Direction of I is opposite to that of electron drift.
Recall: $V_D \cong 10^{-4} m/s$. While $V_{rms} \cong 10^5 m/s$ at 300K. This is because ions are stationary and act as scattering centers. Electron has a very tortuous path so although the speed between collisions is high the entire electron "cloud" drifts rather slowly.



Since transport costs energy, potential must drop. Hence, definition of resistance

$$R = \frac{V}{I}$$

For a particular piece of conductor $R = \frac{\rho l}{A}$



l=length, A= cross-section
ρ = resistivity (material property)
[cf. conduction of heat $\frac{DQ}{\Delta t} = -kA\frac{\Delta T}{\Delta x}$]

$I = \frac{V}{R} = \frac{\Delta Q}{\Delta t} = \frac{VA}{\rho l} = \sigma A \frac{V}{l}$ σ = electrical conductivity

It is instructive to write
$I = \underset{\rightarrow}{J} \bullet \underset{\rightarrow}{A}$ $\underset{\rightarrow}{J}$ = current density vector

and we know that

$$E = \frac{V}{l}$$

so $\underset{\rightarrow}{J} = \sigma \underset{\rightarrow}{E}$

That is, if you apply an $\underset{\rightarrow}{E}$ -field to a conductor it responds by setting up a current density proportional to E, the proportionality factor being the conductivity (electrical).