

1. What is a BAR Magnet.?

Q2. GAUSS' LAW FOR \underline{B} -FIELD

Any object which has a non-zero magnetic moment ($\underline{\mu}$) can be designated as a BAR MAGNET because it will experience a torque

$$\underline{\tau} = [\underline{\mu} \times \underline{B}]$$

when placed in a \underline{B} -field.

To begin, let us recall that a current carrying loop of area \underline{A} experiences a torque

$$\underline{\tau} = I A \hat{n} \times \underline{B}$$

because it has a magnetic moment

$$\underline{\mu} = I A \hat{n}$$

Incidentally, it also has potential energy

$$U_B = -\underline{\mu} \cdot \underline{B}$$

so that \hat{n} wants to line up along \underline{B}

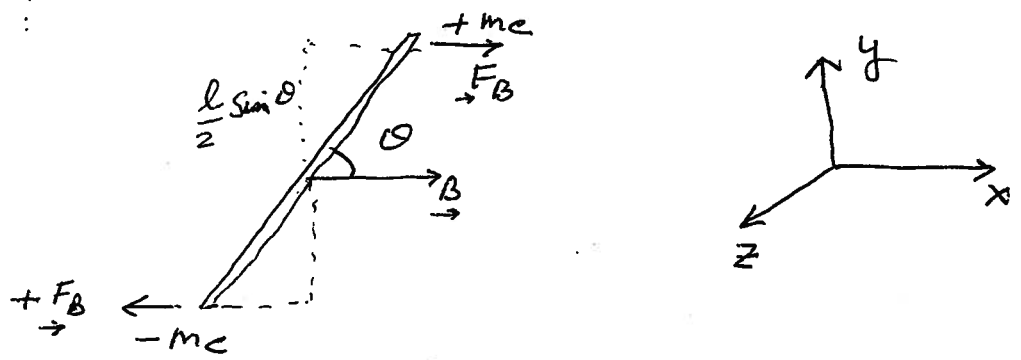
Note, the similarity to an Electric Dipole of moment $\underline{p} = q \underline{l}$ placed in \underline{E}

Torque is $\underline{\tau} = [\underline{p} \times \underline{E}]$.

Potential Energy is $U_E = -\underline{p} \cdot \underline{E}$

Next, let us take a compass or a store bought bar magnet. If you suspend it in Earth's magnetic field; the compass or bar will line up along the Earth's \vec{B} field. Again, this happens because of the torque on the compass (B.M.).

To understand this let us pretend that the B.M. can be imagined as consisting of magnetic "charges" $\pm mc$ separated by l .



And we find that it experiences a Torque

$$\vec{\tau} = - mc l B \sin \theta \hat{z}$$

or
$$\vec{\tau} = [\vec{\mu} \times \vec{B}] \quad \text{with } \vec{\mu} = mc \vec{l}$$

Question: Is the pretense justified?

Answer: A FIRM NO!

Why?: If you break the B.M. in to two parts you will not separate $+m_e$ and $-m_e$. You will get two bar magnets. Keep breaking and you get more and more bar magnets

1 → 2 → 4 → 8 → ... → Single atom → ELECTRON.

Ultimately, you will discover that a SINGLE Electron is a complete bar magnet because it has a magnetic dipole moment

$$\mu_B = 9.27 \times 10^{-24} \frac{N \cdot m}{T}$$

which is called a Bohr Magnetron (μ_B).

Thanks to quantum mechanics we now know that in addition to charge e ($= -1.6 \times 10^{-19} C$) and mass m_e ($= 9 \times 10^{-31} kg$) an electron has an intrinsic property called spin S ($= \frac{1}{2}$) which endows it with a magnetic moment

$$\mu_e = \mu_B = \frac{2 e \hbar S}{2 m_e}$$

where $\hbar \approx 10^{-34} J \cdot s$.

Indeed, even nuclei have a magnetic moment but since mass of proton is about 2000 m_e , nuclear moments are much smaller.

So now that we have the elementary building blocks let us start "constructing" a bar magnet beginning with electrons. The next constituent is the atom. Here, to get a non-zero μ_A we need unpaired electrons. That is, if the electrons are arranged in pairs so that they look like

$$\uparrow\downarrow \uparrow\downarrow \uparrow\downarrow \text{ then } \mu_A \equiv 0$$

and that ^{atom} is no good to make a B.M.

Iron, on the other hand, is a "good" atom. Fe^{3+} has five unpaired electrons.

$$\uparrow \uparrow \uparrow \uparrow \uparrow \quad \mu_{Fe} = 5\mu_B$$

at the simplest level.

Next, put these atoms into a solid. At high temperatures, the thermal energies make these atomic moments wobble rapidly so that the time average

$$\langle \mu_A \rangle \equiv 0$$

and all we get is a paramagnet. There is no net moment so no bar magnet.

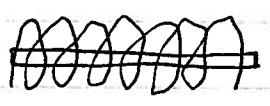
However, [recall the dielectric in an E_s -field] if we put the material in an applied magnetic B_s it will cause all the moments to line up and the solid will ~~have~~ ^{acquire} a magnetic moment M_{sol} . We can define the magnetisation

$$M = \frac{M_{sol}}{Vol.}$$

It has units of $\frac{A \cdot m^2}{m^3}$ or $\frac{A}{m}$ which are the same as units of $\frac{B}{\mu_0}$.

Simple Expt. Take a solenoid. Pass a current I . you will get

$$B_0 = \mu_0 n I = \mu_0 H$$



This can be applied to our paramagnet. We will find that the B_s field enhances to

$$B = \mu_0 n I + \mu_0 M = \mu_0 (H + M) = \mu_0 H (1 + \chi_m)$$

which defines the magnetic susceptibility

$$\chi_m = \frac{M}{H}$$



In the paramagnet, χ_m is a constant, independent of H . M is linear in H .

However, χ_m is a function of temperature. On reducing T , χ_m increases

according to

$$\chi_m = \frac{C}{T}$$

and this will hold if the atomic moments do not "talk" to one another and we will never get a bar magnet.

Fortunately, again thanks to quantum mechanics, in some materials (Fe and Ni are most familiar) the atomic moments interact with one another and produce an internal field proportional to M. In such systems one can write

~~$$M = \chi_m (H + \lambda M)$$~~
$$M = \chi_m [H + \lambda M]$$

where λ is a constant.

Again
$$M = \frac{C}{T} (H + \lambda M)$$

and we should find:
$$\chi'_m = \frac{M}{H} = \frac{C}{T - \lambda C}$$

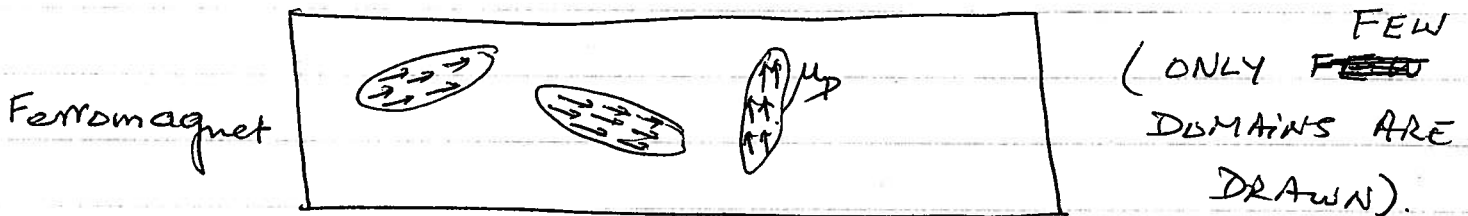
and we note that if T is reduced until $T = \lambda C$, χ'_m will blow up. That is, the material will have a spontaneous magnetic moment M. We have succeeded in making a

FERROMAGNET.

Remember that in class we did an experiment to show that Ni becomes a ferromagnet at

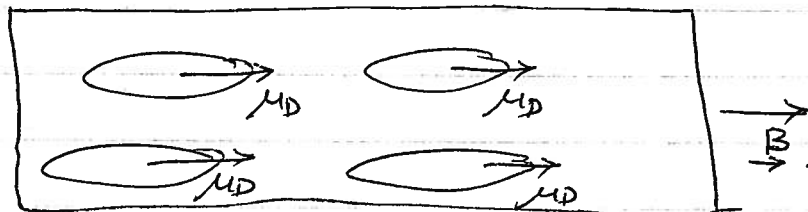
$T \lesssim 630 \text{ K.}$

At lower temperatures the system can be imagined as consisting of "domains" each of which contains billions of atoms with their ~~μ_A~~ μ_A 's aligned. These domains



have giant magnetic moments μ_D and the response to an applied field is greatly enhanced [Expt. in class: nickel was attracted strongly when it was ~~cold~~ ^{cold}]

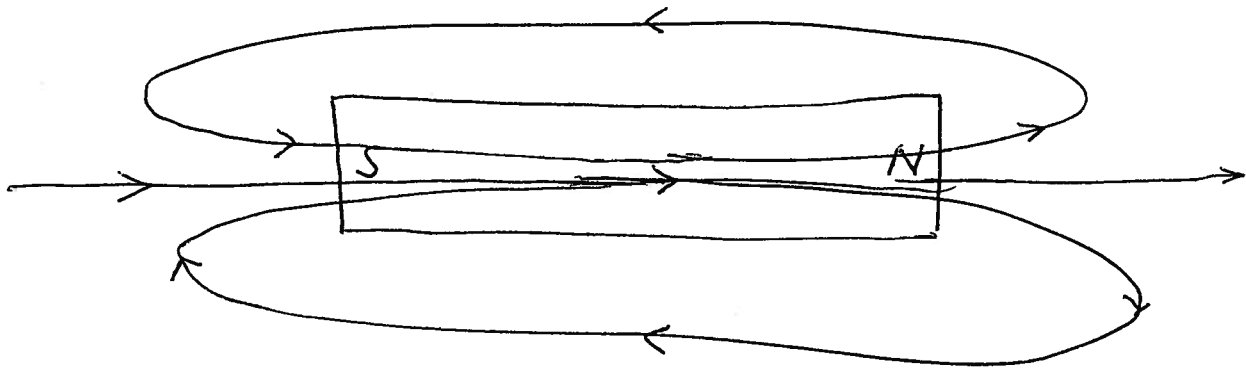
The last step in constructing a B.M. is to recognize that we need a material in which there is a preferred direction. That is, the μ_D 's prefer to lie along some axis. Let us assume that for our bar this is along \hat{x} . If we apply a $\vec{B} \parallel \hat{x}$ all the μ_D 's will align along \hat{x} and we get



and every domain has its M_D along a direction where it prefers to stay. So next if we remove \vec{B} , the M_D 's will stay put, the Bar has a "permanent" magnetic moment. Indeed, we just "constructed" a

BAR MAGNET

which has a \vec{B} field looking like



2. GAUSS'S LAW FOR \vec{B}

FACT A single electron is a complete magnet with a dipole moment μ_B .

CONSEQUENCE Elementary source of \vec{B} is a DIPOLE with effectively "zero" size. ∴ "sources" and "sinks" are coincident. When we use lines to map \vec{B} fields the lines must close ON THEMSELVES. There is no "beginning" and no "end" to a \vec{B} field line.

PROFOUND IMPLICATION Total flux of \vec{B} through any closed surface must be always equal to zero:

$$\boxed{\sum_c \vec{B} \cdot \Delta \vec{A} \equiv 0}$$

Every line that comes into enclosed volume must go out as \vec{B} field lines do not stop or start anywhere.

NOTE: The law tells you that total flux of \vec{B} is zero. It says nothing about \vec{B} .