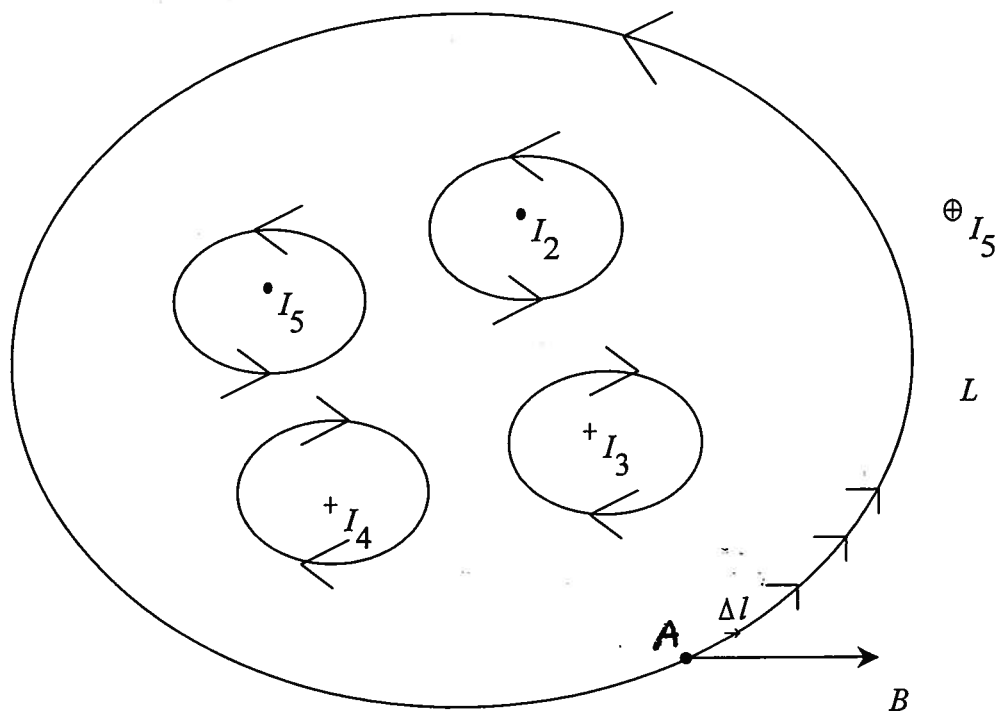


Ampere's Law - Applications

Consider the situation shown schematically in the diagram. Currents I_1, I_2, I_3, I_4, I_5 are flowing out of (\cdot) or (+) into the paper. The corresponding B -fields swirl around their sources as shown. The main point is that the B -field lines circulate around the currents. Choose a closed loop (L). Start at A , measure B , choose small step Δl along loop. Calculate Dot product (component of B along Δl multiplied by Δl)

$$\vec{B} \cdot \vec{\Delta l} = B \Delta l \cos(\vec{B}, \vec{\Delta l}).$$

If $\vec{B} \perp \vec{\Delta l}$, $\vec{B} \cdot \vec{\Delta l} = 0.$



Repeat this calculation at every step as shown. $\vec{B} \cdot \vec{\Delta l}$

Write out the sum

$$\sum_c \vec{B} \cdot \vec{\Delta l};$$

c : closed loop.

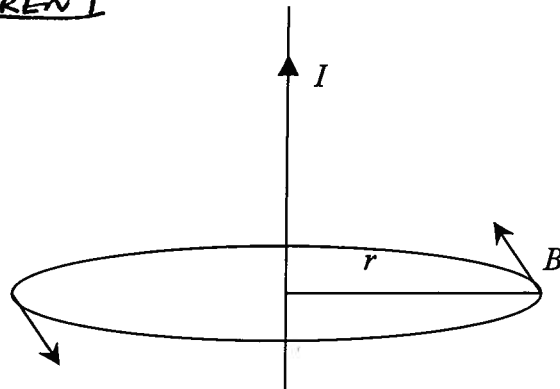
This sum is called circulation of \vec{B} around closed loop and Ampere says that it is determined solely by currents threading through the surface on which the loop is drawn & only currents within the loop contribute, i.e. exclude I_5 . The mathematical Equation is:

$$\sum_c \vec{B} \cdot \vec{\Delta l} = \mu_0 \sum I_i, \mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$$

In words, circulation of \vec{B} around closed loop is proportional to the algebraic sum of the currents threading the ~~the~~ surface on which the loop is drawn.

Note: As in case of Gauss' Law, Ampere's law gives Circulation but not \vec{B} . To get \vec{B} you need high symmetry.

1.) SINGLE CURRENT



Single wire with current I , there is cylindrical symmetry so B can be a function of r only & must encircle I . [\vec{B} and $d\vec{\ell}$ are parallel to one another]

Appropriate loop is circle of radius r centered on the wire

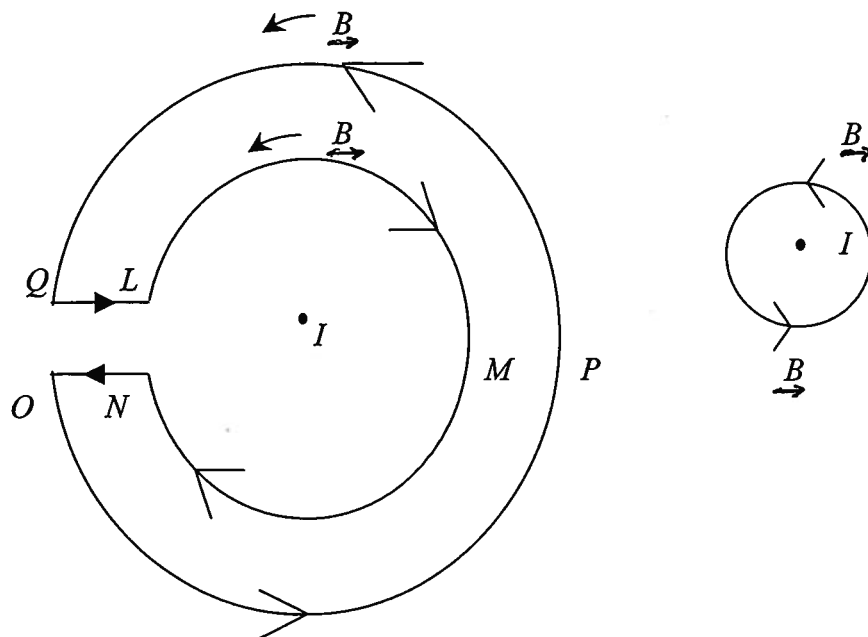
$$B \cdot 2\pi r = \mu_0 I$$

$$\text{so, } \vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$$

as claimed previously

2.) Next, we begin by showing that if current is outside the loop it contributes nothing to the circulation. Choose $LMNOPQ$ with I at the center of the circles of radii r_1 , and r_2

I is
out of
paper.



$$\vec{B}(r_1) = \frac{\mu_0 I}{2\pi r_1} \hat{\phi}$$

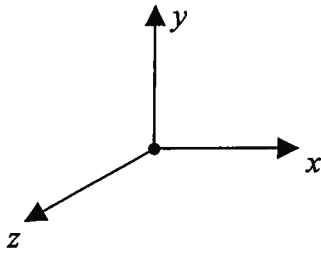
$$\vec{B}(r_2) = \frac{\mu_0 I}{2\pi r_2} \hat{\phi}$$

$$\begin{aligned} \sum_{c \rightarrow \rightarrow} \vec{B} \cdot \Delta \vec{l} &= \frac{\mu_0 I}{2\pi r_1} \cdot 2\pi r_1 + 0 + \frac{\mu_0 I}{2\pi r_2} \cdot 2\pi r_2 + 0 \\ &= L \rightarrow M \rightarrow N + N \rightarrow 0 + 0 \rightarrow P \rightarrow Q + Q \rightarrow L \\ &= 0 \end{aligned}$$

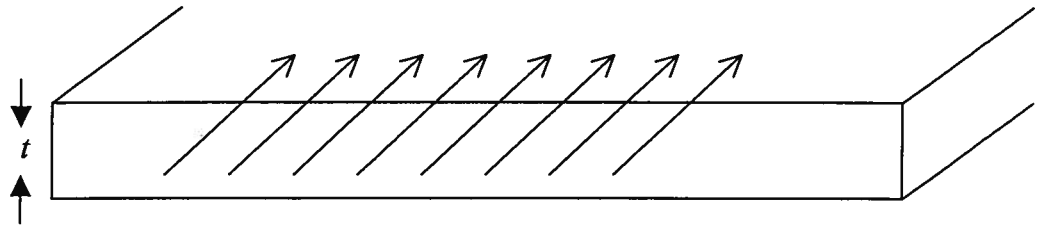
$\vec{B} \perp \Delta \vec{l}$

[The first term is -ive]
 \vec{B} and $\Delta \vec{l}$ are opposite to one another

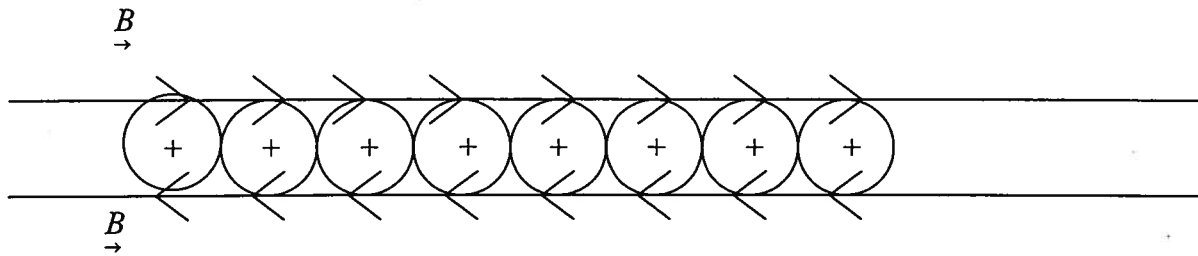
3.)



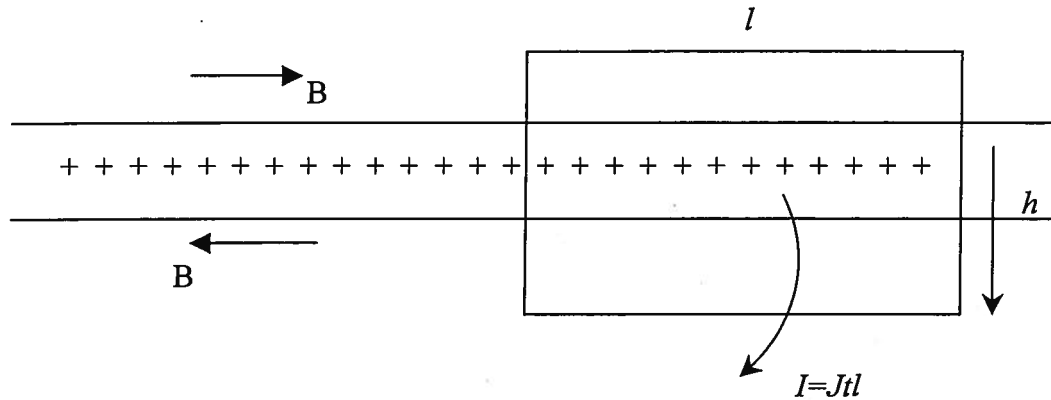
\vec{J}



At $y=0$ there is a Current sheet of thickness t carrying current density $\vec{J} = -J\hat{z}$. Looking ^{at} it end-on we see sources as



and we see that y -components of \vec{B} cancel out. $\vec{B} \parallel \hat{x}$ survives. Let us take loop of width l and height h .

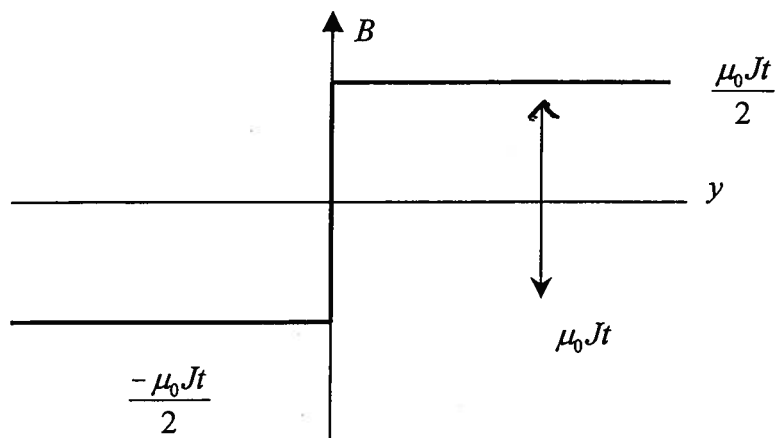


$$\begin{aligned}\Sigma_c \vec{B} \cdot \Delta \vec{l} &= Bl + 0 + Bl + 0 \\ &= 2Bl \\ &= \mu_0 Jtl\end{aligned}$$

so,

$$\begin{aligned}B &= \frac{\mu_0 Jt}{2} \quad \& \quad B = \frac{\mu_0 Jt}{2} \hat{x} \quad y > 0 \\ &= -\frac{\mu_0 Jt}{2} \hat{x} \quad y < 0\end{aligned}$$

That is, \vec{B} -field will jump by $\mu_0 Jt$ on crossing the current sheet from $y < 0$ to $y > 0$.



4.) Hollow Cylindrical Conductor- Radius R , carries uniform current. We want \vec{B} at a distance r from its axis. Since there is a cylindrical symmetry we should use circles centered on the axis for our closed loop.

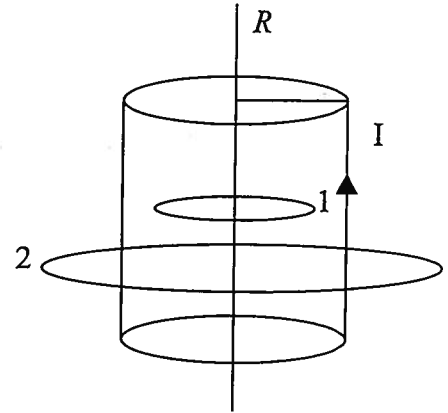
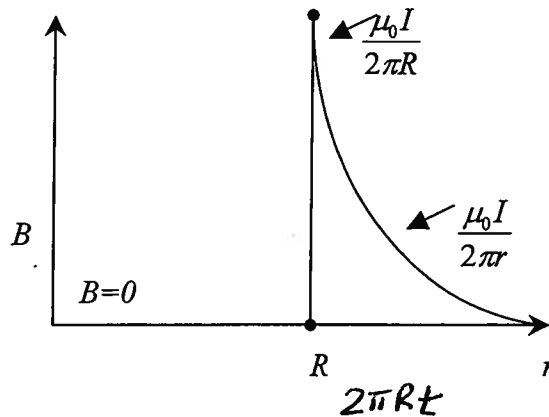
For $r < R$. use loop 1.

$B \cdot 2\pi r = 0$. No Current threads through loop 1.

for $r > R$

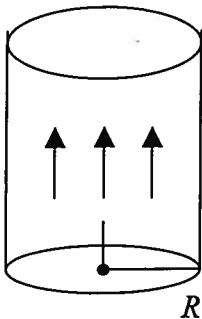
so, $B \cdot 2\pi r = \mu_0 I$, the entire current threads loop 2.

$$\text{so, } \vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$$



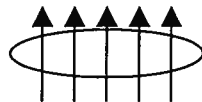
Note: if cylinder has wall thickness t : $I = J \cdot 2\pi R t$ and field at surface would be $\mu_0 J t$. Again field would jump by $\mu_0 J t$ on crossing a current sheet.

5.) SOLID CYLINDRICAL CONDUCTOR – with uniform current



$$\text{Define } J = \frac{I}{\pi R^2}$$

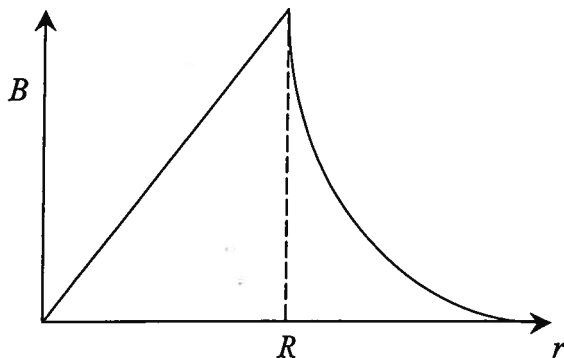
Now for $r < R$ $I = J\pi r^2$



$$B \cdot 2\pi r = \mu_0 J \pi r^2$$

$$\vec{B} = \frac{\mu_0 J r}{2} \hat{\phi} \quad r < R$$

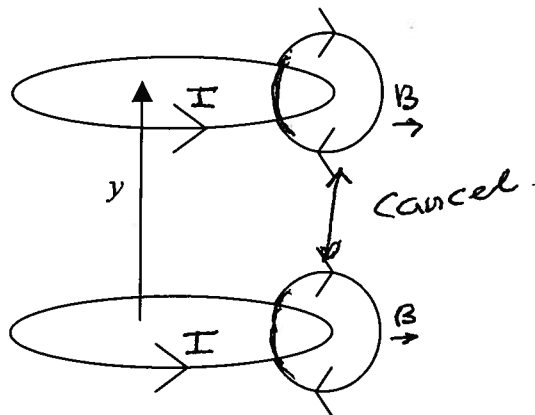
For $r > R$, entire I contributes $B = \frac{\mu_0 I}{2\pi r} \hat{\phi}$



6.) Solenoid: Tightly wound, small radius, length much larger than radius:

N turns, L long, $n = \frac{N}{L}$ = # of turns per meter.

Look at two neighboring turns



r -component cancels
 B_y inside survives.

Long-narrow solenoid. $B \approx 0$ just outside. B_z field lines must come out of top, loop around and enter at bottom with no breaks or bends allowed.

Take loop as shown $Bl = \mu_0 nIl$
 $B = \mu_0 nI$

For case shown $B = \mu_0 nI \hat{y}$

