

TRIG IDENTITY

$$\begin{aligned}
 \sin A + \sin B &= \sin\left(\frac{A+A+B-B}{2}\right) + \sin\left(\frac{B+B+A-A}{2}\right) \\
 &= \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right) \\
 &\quad + \sin\left(\frac{B+A}{2}\right) \cos\left(\frac{B-A}{2}\right) + \cos\left(\frac{B+A}{2}\right) \sin\left(\frac{B-A}{2}\right)
 \end{aligned}$$

Now $\cos\left(\frac{B-A}{2}\right) = \cos\left(\frac{A-B}{2}\right)$

But $\sin\left(\frac{B-A}{2}\right) = -\sin\left(\frac{A-B}{2}\right)$

Hence

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right).$$

Similarly

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right).$$

So when you superpose

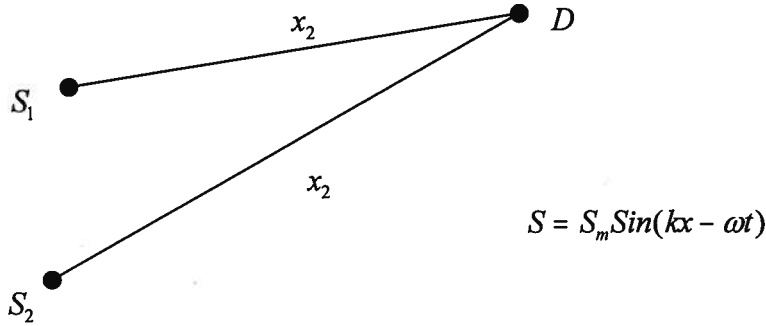
$$y_1 = A \sin(kx_1 - \omega t) \text{ and } y_2 = A \sin(kx_2 - \omega t)$$

you get $y = 2A \cos\left(\frac{k(x_1 - x_2)}{2}\right) \sin\left[\frac{k(x_1 + x_2)}{2} - \omega t\right]$

That is, a wave whose amplitude is controlled by $(x_1 - x_2)$ leading to INTERFERENCE.

SOUND WAVES-INTERFERENCE

What happens if two sound waves start in phase but have traveled different distances before they arrive at the detector? S_1, S_2 are two sources of sound, which emit waves in phase.



Let us put both phases equal to zero at the starting points. Note: when a wave travels a distance λ its phase must change by 2π . Hence, the wave from S_1 is when it arrives at D will have the phase $\Phi_1 = \frac{2\pi}{\lambda} x_1$.

When this wave arrives at D there is nothing there. However, when the wave from S_2 arrives at D its phase will be $\Phi_2 = \frac{2\pi}{\lambda} x_2$ and since wave from S_1 already there, the two waves superpose. If $(\Phi_1 - \Phi_2) = 0, 2\pi, 4\pi, 6\pi$ the two waves will be in phase at D and will combine to produce a maximum at D .

We call this constructive interference.

CONDITION FOR MAXIMUM AT D:

$$(\Phi_1 - \Phi_2) = 2M\pi \quad M = 0, 1, 2, \dots$$

$$\frac{2\pi}{\lambda}(x_1 - x_2) = 2M\pi \tag{I}$$

$$\text{or } (x_1 - x_2) = M\lambda \quad M = 0, 1, 2, \dots$$

In other words, if the path DIFFERENCE is a whole # of λ 's, the waves which started in phase will again be in phase at D and produce a maximum there.

However, if

$$(\Phi_1 - \Phi_2) = \pi, 3\pi, 5\pi \dots$$

or equivalently

$$(x_1 - x_2) = (2m + 1) \frac{\lambda}{2}$$

(II)

with $m=0, 1, 2, \dots$

When the waves meet at D , they will be exactly out of phase
And cancel each other.



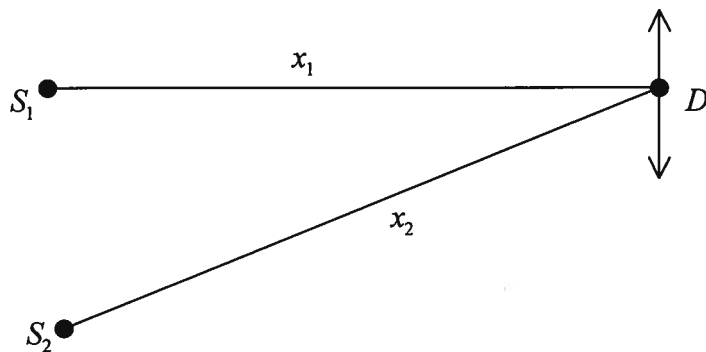
DESTRUCTIVE INTERFERENCE

CONDITION FOR MINIMUM

$$(x_1 - x_2) = (2m + 1) \frac{\lambda}{2}, m = 0, 1, 2, \dots$$

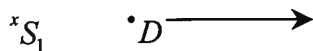
(II)

If you keep S_1 and S_2 fixed and move D up and down as shown, $(x_1 - x_2)$ will vary and you will encounter a series of maxima (loud sound) and minima (no sound) as you go alternately from Eq I to Eq II and vice versa.



INTERESTING CASE

S_1 and S_2 are separated by $M\lambda$ and D (your ear) moves along line joining S_1 and S_2 .



xS_2

a) When D is at S_1 :

$$x_1 = 0$$

$$x_2 = M\lambda$$

$$(x_1 - x_2) = -M\lambda \quad \text{MAX}$$

b) When D is at Mid-point:

$$x_1 = x_2 = \frac{M\lambda}{2}$$

$$(x_1 - x_2) = 0 \quad \text{MAX}$$

c) When D is at S_2 :

$$x_1 = M\lambda$$

$$x_2 = 0$$

$$(x_1 - x_2) = M\lambda \quad \text{MAX}$$

In all you hear $(2M+1)$ Maxima

In between where will be $2M$ minima

N.B. CRUCIAL QUANTITY IS PATH DIFFERENCE!!!