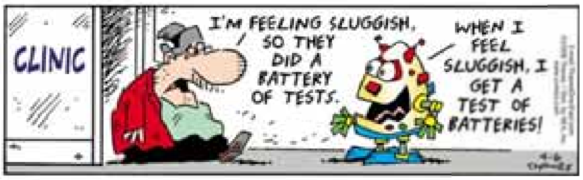
April 25, 2011 Physics 122 Prof. E. F. Redish

- **Theme Music:** Joni Mitchell
  You Turn Me On, I'm a Radio
- **Cartoon:** Bob Thaves
  Frank & Ernest



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Outline

- Quiz 9
- Recap
  - Current – Foothold ideas
  - Ohm's Law
- Electrical Analogies
- Foothold ideas for circuits (Kirchoff's principles)
  - Ohm's law
  - Flow rule
  - Loop rule
- Examples

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Foothold Idea: Local Neutrality

- Most matter is made of of an equal balance of two kinds of charges: positive and negative.
- Since the electric force is very strong, mostly the + and - charges overlap closely and cancel each other.
- Small imbalances in the cancellation leads to:
  - polarization forces
  - potential drop across a resistance
  - observed electric forces.

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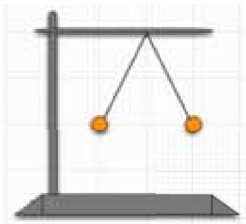
How small an effect? Estimation

- Two small aluminum spheres of mass 1 gram hang by 23 cm long non-conducting threads and are negatively charged. Suppose that the two spheres are equally charged.
- Estimate what fraction of the aluminum atoms in a sphere have an extra electron.



Answer: 1 in 30 billion

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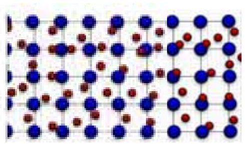
Picture of current flow

- In wires (and in most other substances), the neutral matter breaks up into positive and negative bits but remains neutral.
- Current flow corresponds to (usually) one of the kinds of charge moving through the rest.
- The charges tend to not build up anywhere since the moving charges repel each other if not balanced by the other kind of charge.



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Foothold ideas: Currents

- Charge is moving: How much? $I = \frac{\Delta q}{\Delta t}$
- How does this relate to the individual charges? $I = q\,n\,A\,v$
- Constant flow means pushing force balances the drag force

$$ma = F_e - bv$$
$$a = 0 \quad \Rightarrow \quad v = F_e/b$$

- What pushes the charges through resistance? Electric force implies a drop in *V*!

$$F_e = qE$$
$$\Delta V = -\frac{E}{L}$$

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Ohm's Law

- Current proportional to velocity $I = qnAv \Rightarrow v = \frac{I}{qnA}$
- Due to resistance, Electric force proportional to velocity. $qE = bv$
- Force proportional to "electric pressure drop" = "electric PE" $\Delta V = EL \Rightarrow E = \frac{\Delta V}{L}$
- Therefore, current proportional to "electric PE" $\Rightarrow \frac{q\Delta V}{L} = \frac{bI}{qnA}$

$$\Delta V = IR \qquad \Delta V = I\left(\frac{bL}{q^2 nA}\right) \equiv IR$$

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Resistivity and Conductance

- The resistance factor in Ohm's Law separates into a geometrical part (*L/A*) times a part independent of the size and shape but dependent on the material.
- This coefficient is called the *resistivity* of the material (ρ). It's reciprocal (g) is called *conductivity*.

$$R = \left(\frac{bL}{q^2 nA}\right) = \rho\frac{L}{A} = \frac{1}{g}\frac{L}{A}$$

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Electric Power Dissipation

- We can figure out the energy needed to push the electrons through the material against the resistance using the WE theorem.

$$P = \text{ rate of doing work (using energy)} = \frac{\Delta W}{\Delta t}$$

$$= \text{(number of charges moved)} \times \text{(force)} \times \text{(distance moved in a time } \Delta t)$$

$$P = \frac{(nAL)(qE)(v\Delta t)}{\Delta t} = (nAL)qv\frac{\Delta V}{L} = (qnAv)\Delta V = I\Delta V$$

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Units of power

- Since the units of work (energy) is the Joule, the unit of power is the Joule/second.
  - 1 Watt = 1 Joule/second (definition)
- Our analysis shows that current x voltage = power.
- 1 Watt = 1 Ampere x 1 Volt

$$P = I\Delta V$$

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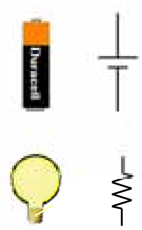
Electric circuit elements

- *Batteries* —devices that maintain a constant electrical pressure difference across their terminals (like a water pump that raises water to a certain height).
- *Resistances* —devices that have significant drag and oppose current. Pressure will drop across them.
- *Wires* — have very little resistance. We can ignore the drag in them (mostly – as long as there are other resistances present).



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Analogy 1 (Drude model): Ping-pong balls and nail board

- In this analogy, we treat the electrons as small particles that can move freely through the conductor. (ping-pong balls)
- The ions that form the fixed body of the conductor are treated as fixed. (nails)
- The electron move freely between the ions until they hit them. Then they scatter in a random direction.

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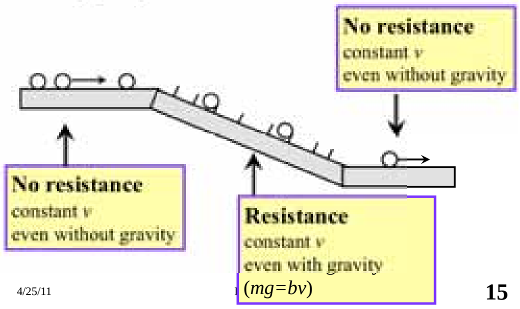
Nail Board: Two kinds of regions

- Ping-pong balls and nail board



Properties of analogy 1

- While running horizontally in a region with no resistance (aligned nails), the balls continue with a constant velocity without need for pushing.
- Running downhill through a region with resistance (random nails), the balls continue at the same constant velocity, but are pushed by gravity.
- To get back they have to be carried up by some kind of pump that restores the balls from a region of low PE back to a region of high PE (the demonstrator lifts the balls back up to the top).

Analogy 2: The rope model

- Since like charges repel strongly, there can't be a buildup of charge anywhere in the circuit (unless we make a special arrangement -- capacitance).
- Moving charges push other movable charges in front of them. The electrons move like links in a chain or rope.

Properties of analogy 2

- In a simple loop, the rope keeps moving around.
- A battery is like a person holding the rope and pulling it, causing a tension throughout the rope.
- A resistor is like a person squeezing the rope, having it pulled through her hands. The friction generates heat.
- The more people are squeezing, the slower the rope goes, even if the battery pulls with the same tension.

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Analogy 3: Water flow

- The rope analogy fails because electrons can go either way at a junction. A current can split in a way a rope cannot.
- Water flow is a useful analogy because water
  - can divide
  - is conserved and cannot be compressed.

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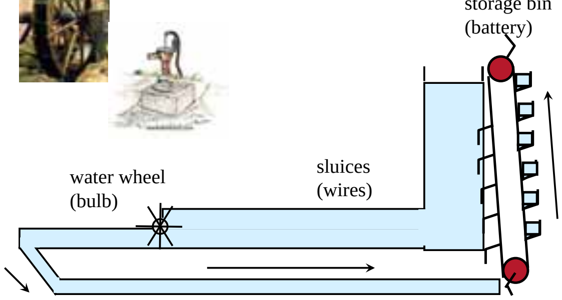
Water Model

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Properties of analogy 3

- Water can divide and recombine.
  But: total flow into any volume = total flow out of any volume – no build up.
- Going around any loop – sum of potential rises (positive Δh) equals sum of potential drops (negative Δh).

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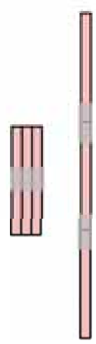
Analogy 4: Air flow

- Pressure is analogous to electric potential.
- Pressure drop produces flow.
- Amount of flow depends on what is connected across a pressure drop.



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Foothold ideas (Kirchoff's Rules)

- Flow Rule
  - The total amount of current flowing into any point in a network equals the amount flowing out (no significant build-up of charge anywhere).
- Potential Rule
  - Following around any loop in an electrical network the potential has to come back to the same value (sum of drops = sum of rises).
- Ohm's Rule
  - When a current *I* passes through a resistance *R*, there is a voltage drop across the resistor of an amount $\Delta V = IR$

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Very useful heuristic

- The Constant Potential Trick (CPT)
  - Along any part of a circuit with 0 resistance, then $\Delta V = 0$, i.e., the voltage is constant since in any circuit element

$$\Delta V = IR$$

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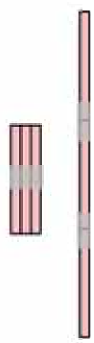
What happens when we combine resistors?

- Consider increasing the width or the length of a resistor.
  - Analogy with air flow
  - Using the equation for resistance
  - Analyze in terms of potential drops and current flow



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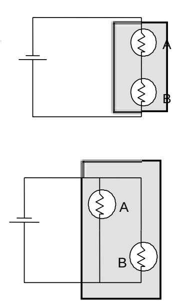
Series and Parallel Rules

- In series, the current through each element is the same.

$$\Delta V = \Delta V_A + \Delta V_B$$
$$IR_{eff} = IR_A + IR_B$$
$$R_{eff} = R_A + R_B$$

- In parallel, the pressure drop across each element is the same.

$$I = I_A + I_B$$
$$\frac{\Delta V}{R_{eff}} = \frac{\Delta V}{R_A} + \frac{\Delta V}{R_B}$$
$$\frac{1}{R_{eff}} = \frac{1}{R_A} + \frac{1}{R_B}$$



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