Measuring the speed of light a la Galileo

(a) The time it takes light to travel 5 km is

$$\frac{5 \times 10^3 \text{ m}}{3 \times 10^8 \text{ m/s}} = 1.67 \times 10^{-5} \text{ s}, \quad \Rightarrow \text{total round trip time is } 3.3 \times 10^{-5} \text{ s}.$$

(b) This is not a good way to measure speed of light because of human reaction time. Let's assume the reaction time is 0.2 s, this is much larger than the precision required to measure the speed of light. Light travels almost instaneously compared to reaction time, so we would measure the speed of light to be

$$\frac{10 \times 10^3 \text{ m}}{2 \times 10^{-1} \text{ s}} = 5 \times 10^4 \text{ m/s}$$

and anything travelling faster than this can not be measured precisely.

Speed of light in GPS systems

(a) Using kepler's law, we have

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg s}^2}\right)\left(5.9 \times 10^{24} \text{ kg}\right)}{4.23 \times 10^7 \text{ m}}} = 3.05 \times 10^3 \text{ m/s}$$

(b) The time it takes light to travel 20,000 km is

$$\frac{(2 \times 10^4) \times 10^3 \text{ m}}{3 \times 10^8 \text{ m/s}} = 6.7 \times 10^{-2} \text{ s}$$

(c) In this time, the satellite moves

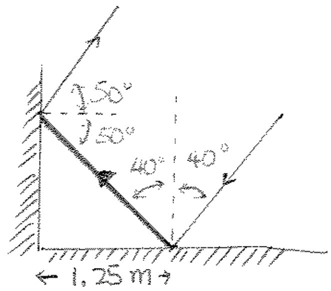
$$3.05 \times 10^3 \text{ m/s} \times 6.7 \times 10^{-2} \text{ s} \cong 200 \text{ m}$$

So without taking speed of light into account, a GPS system would be useless.

Ch. 22

6, 8, 22, 28, 32, 40, 56, 60

(6)



The distance of the bolded portion of light path is

$$\frac{1.25\text{ m}}{\sin 40°} = \frac{1.25\text{ m}}{\cos 50°} = 1.94\text{ m}$$

(8)

Treat $n_{air} = 1$.

$$n_{Lucite} = \frac{c}{(\text{speed of light in lucite})} \Rightarrow (\text{speed of light in lucite}) = c/n_{Lucite}.$$

The time it takes to travel the double layer is

$$\frac{1.00\text{ cm}}{(3\times10^8\text{ m/s})/(n_{water})} + \frac{0.500\text{ cm}}{(3\times10^8\text{ m/s})/(n_{lucite})}$$

The time it takes to travel the same distance in vacuum

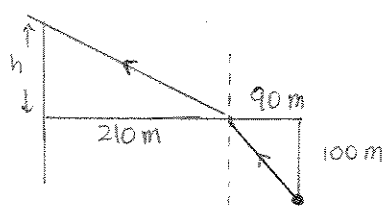
$$\frac{1.00\text{ cm}}{(3\times10^8\text{ m/s})} + \frac{0.500\text{ cm}}{(3\times10^8\text{ m/s})}$$

So the difference in time is

$$\frac{1.00\text{ cm}}{3\times10^8\text{ m/s}}\left(n_{water} - 1\right) + \frac{0.500\text{ cm}}{3\times10^8\text{ m/s}}\left(n_{lucite} - 1\right)$$

$$= 2.08\times10^{-11}\text{ s}$$

(22)



Incident angle $\Rightarrow \tan^{-1}\frac{90}{100} = 42°$

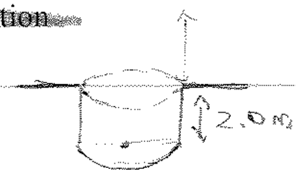
Refracted angle $\Rightarrow$

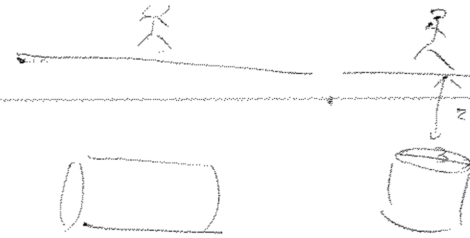
$$n_{water}\sin 42° = n_{air}\sin\theta_f$$

$$\theta_f = 62.8°$$

Height of Building

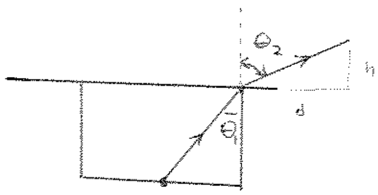
$$\tan 62.8° = \frac{210\text{ m}}{h} \Rightarrow h = 108\text{ m}$$





HW 8
page 3

(28)



$\theta_1 = \tan^{-1} \frac{1.5m}{2.0m} = 36.87°$

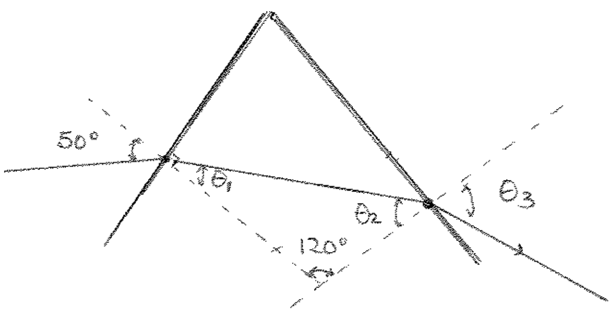
$(1.5) \sin 36.87° = (1) \sin \theta_2 \Rightarrow \theta_2 = 64.15°$

$\tan \theta_2 = d/h \Rightarrow d = h \tan \theta_2$

$= (1.2\ m) \tan 64.15°$

$= 2.48\ m$

(32)



For purple, we have,

$\sin \theta_1 = 0.46 \Rightarrow \theta_1 = 27.48°$

$\Rightarrow \theta_2 = 32.52° \Rightarrow \sin \theta_2 = 0.5376$

$\Rightarrow \sin \theta_3 = 0.8710 \Rightarrow \theta_3 = 60.57°$

From geometry, we have $\theta_2 = 60° - \theta_1$

For red light, we have,

$n_{air} \sin 50° = n_{red} \sin \theta_1$

$n_{red} \sin \theta_2 = n_{air} \sin \theta_3$

Putting in the numbers, we have

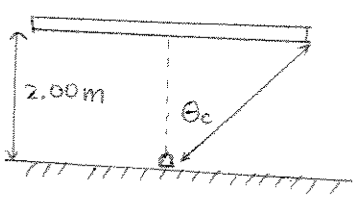
$\sin \theta_1 = 0.47 \Rightarrow \theta_1 = 28° \Rightarrow \theta_2 = 32°$

$\sin \theta_2 = 0.53 \Rightarrow \sin \theta_3 = 0.858$

$\Rightarrow \theta_3 = 59°$

The angular dispersion is 1.57°

(40)



To find θ_c, we need

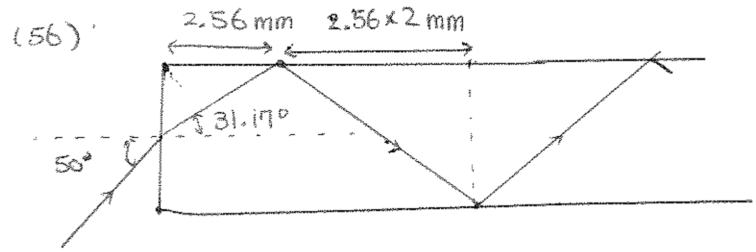
$n_{water} \sin \theta_c = n_{air} \sin 90° \Rightarrow \theta_c = 48.75°$

Then we have

$\tan \theta_c = \frac{d/2}{2.00m} \Rightarrow d = (4.00\ m)(\tan \theta_c)$

$= 4.56\ m$

(56)

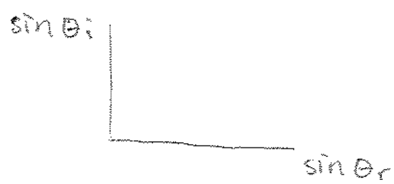


$$420\text{ mm} - 2.56\text{ mm} = 417.44\text{ mm}$$

$$\frac{417.44\text{ mm}}{2 \times 2.56\text{ mm}} = 81$$

So there are a total of 82 total internal refractions.

(60) If we plot $\sin\theta_i$ vs $\sin\theta_r$, we have



and the slope should be $n_{air} \sin\theta_i = n_{water} \sin\theta_r$

$$\sin\theta_i = \underbrace{\frac{n_{water}}{n_{air}}} \sin\theta_r$$

Using best-fit line from Excel, the slope using the data turns out to be 1.334. Of course, you know it's the error that counts, and fortunately I don't have to do that here.