PHYS 122

Dr. Noyes

HW II

A1. The speed of wave on a string is given by

$$v = \sqrt{T/\mu}$$

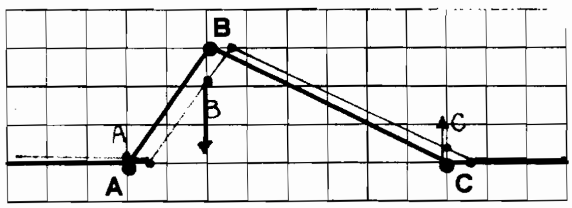
linear density

Tension in the string

In the second spring the speed of the wave is half compared to the original spring. Assuming that both springs have the same linear density (mass per unit length), then we conclude that the second spring has less tension. More specifically, Spring 2 has only 1/4 of the tension in spring 1, so the speed of wave is spring 2 is 1/2 compared to spring 1.

A2. Note that the speed of wave pulse is independent of the shape of the wave. So to have the same speed in spring 2 we need to increase the tension in spring 2 by a factor of 4.

B1. At a later time (just slightly after the current snapshot), we have



So we see that point A has not moved, Point B has moved down, and Point C moved up. Furthermore, Point B has moved down more than the amount by which Point C moved. So we conclude that the speed is proportional to the slope of the piece of string

Equations for Sinusoidal Waves

$$y_m \sin k(x - vt)$$
$$= y_m \sin(kx - kvt), \quad v = \frac{\omega}{k}$$
$$= y_m \sin\left(kx - k\frac{\omega}{k}t\right)$$
$$= y_m \sin(kx - \omega t)$$

$$y_m \sin 2\pi\left(\frac{x}{\lambda} - ft\right)$$
$$= y_m \sin\left(2\pi\frac{x}{\lambda} - 2\pi f t\right) \quad \frac{2\pi}{\lambda} = k, \ 2\pi f =$$
$$= y_m \sin(kx - \omega t)$$

$$y_m \sin \omega\left(\frac{x}{v} - t\right)$$
$$= y_m \sin\left(\omega\frac{x}{v} - \omega t\right)$$
$$= y_m \sin\left(\omega\frac{x}{\omega/k} - \omega t\right), \quad v = \omega/k$$
$$= y_m \sin\left(\frac{x}{1/k} - \omega t\right)$$
$$= y_m \sin(kx - \omega t)$$

$$y_m \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$
$$= y_m \sin\left(2\pi\frac{x}{\lambda} - 2\pi\frac{t}{T}\right),$$
$$= y_m \sin(kx - 2\pi f t), \quad \frac{2\pi}{\lambda} = k, \ \frac{1}{T} =$$
$$= y_m \sin(kx - \omega t)$$

λ is the wavelength and T is the period. The wavelength is the length of one cycle of the wave. If we concentrate on one piece of the string, the period is the amount of time it takes for one cycle of the wave to pass through that point.

To show this, note that for a fixed time, for any x, if we displace x by λ (so $x \to x + \lambda$), we have

$$y_m \sin\left[2\pi\left(\frac{x+\lambda}{\lambda} - \frac{t}{T}\right)\right] = y_m \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + 2\pi\right]$$
$$= y_m \sin\left[2\pi\left(\frac{x}{\lambda} - t/T\right)\right]$$

So if we displace x by λ, it's as if we have not displaced x.

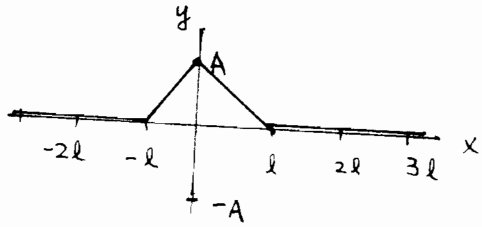
Similarly, for a fixed point on string (that is, fix x), for any time t, let us compare the displacement of the fixed piece of string at time t and time $t + T$:

$$y_m \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t+T}{T}\right)\right] = y_m \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) - 2\pi\right]$$
$$= y_m \sin\left[2\pi\left(\frac{x}{\lambda} - t/T\right)\right]$$

So the displacement of the piece of string is the same for time t and $t + T$.

Making a Pulse Move

(a) $y = f(x,0) = \begin{cases} A\left(1 - |x/\ell|\right) & |x| \leq \ell \\ 0 & |x| \geq \ell \end{cases}$

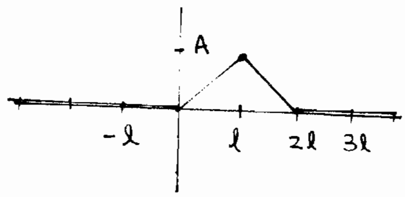


(b) The equation for the wave as a function of time is

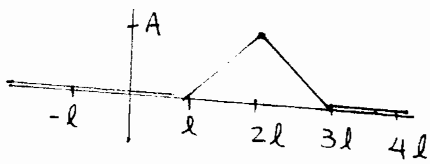
$$f(x,t) = A\left(1 - \left|\frac{x - vt}{\ell}\right|\right)$$

(1) So for $t = \ell/v$, $f(x, \ell/v) = A\left(1 - \left|\frac{x - \ell/v \cdot v}{\ell}\right|\right)$

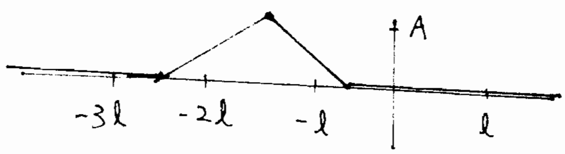
$$= A\left(1 - \left|\frac{x-\ell}{\ell}\right|\right)$$



(2) $f(x, 2\ell/v) = A\left(1 - \left|\frac{x - v\,2\ell/v}{\ell}\right|\right) = A\left(1 - \left|\frac{x - 2\ell}{\ell}\right|\right)$

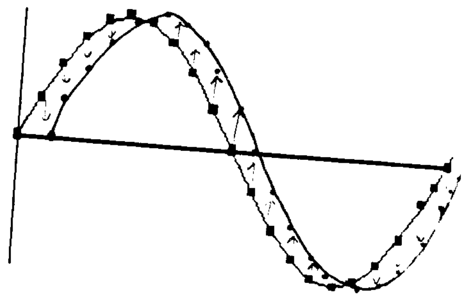


(3) $f(x, -3\ell/2v) = A\left(1 - \left|\frac{x + v\,\frac{3\ell}{2v}}{\ell}\right|\right) = A\left(1 - \left|\frac{x + \frac{3}{2}\ell}{\ell}\right|\right)$



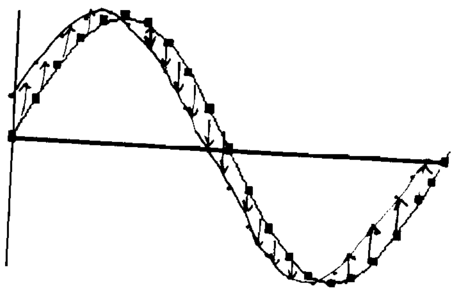
comparing waves

(a)

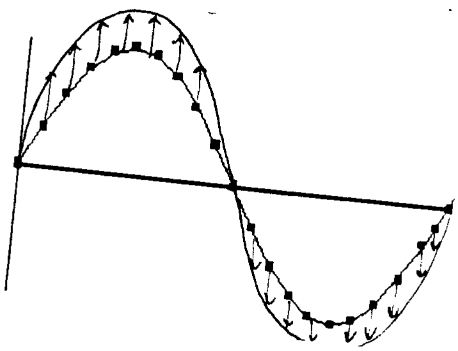


.e marked points at time $t = 0$

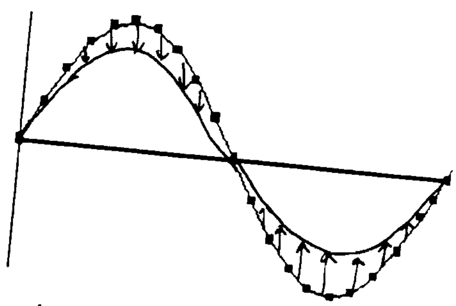
(b)



(c)



(d)



he marked points at time $t = 0$

combining pulses

(a) Graph B

(b) Graph D.

At time $t_0 + s/v_0$, the waves has each traveled

$$\begin{aligned} & v_0 (t_f - t_i) \\ &= v_0 (t_0 + s/v_0 - t_0) \\ &= s \end{aligned}$$

And since they are separated by 2s at t_0, they have now met at $t = t_0 + s/v_0$

(c) Graph A.

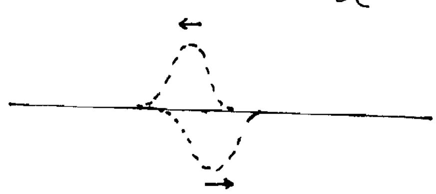
The waves pass by each other and reach the opposite ends.

(d) Graph D.

Remember from previous problem that velocity of pieces of particles on the string (<u>NOT</u> the velocity of the wave!) is proportional to the slope.

In this case, the slope of Graph D is zero everywhere.

(e) None. it should be



Modified Harmonics

(a) The speed of waves is decreased by $1/\sqrt{2}$, and since $v = \frac{\lambda}{T}$ (in this case λ does not change), so T is increased by $\sqrt{2}$.

(b) We are now in the third harmonic. The wavelength is now 1/3 of the original, and since the speed of the wave remains the same, the period must also be 1/3 of the original.

(c) The period is independent of the amplitude, so "1".

(d) Since $v = \lambda/T = \sqrt{F/\mu}$, for tension decreased by ½, v decreases by $1/\sqrt{2}$, so T increases by $\sqrt{2}$.

Speed of Sound vs Light

(a)
$$d = vt$$
$$= (343 \text{ m/s})(3.5 \text{ s})$$
$$= 1200 \text{ m.}$$

(b) Speed of light is $c = 3\times10^8$ m/s.
$$t = d/c = 1200 \text{ m} / 3\times10^8 \text{ m/s}$$
$$= 1.2\times10^3 \text{ m} / 3\times10^8 \text{ m/s}$$
$$= 4\times10^{-6} \text{ s}$$
This is very small compared to 3.5 s, so the approximation in part (a) is valid.

Doppler - Shifted Sounds of the City

(a)
$$f = \left(\frac{V_{sound} + V_{police}}{V_{sound}}\right) f_{stationary}$$
$$= \left(\frac{343 \text{ m/s} + 20 \text{ m/s}}{343 \text{ m/s}}\right)(3500 \text{ Hz}) = 3700 \text{ Hz}$$

(b)
$$f = \left(\frac{V_{sound}}{V_{sound} - V_{amb}}\right) f_{stationary} = \left(\frac{343 \text{ m/s}}{343 \text{ m/s} - 20 \text{ m/s}}\right)(3500 \text{ Hz})$$
$$= 3720 \text{ Hz}$$

(c)
$$f = \left(\frac{V_{sound} - V_{police}}{V_{sound} + V_{amb}}\right) = \left(\frac{343 - 20}{343 + 20}\right)(3500 \text{ Hz}) = 3110 \text{ Hz}$$