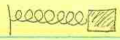
HWK 1 Solutions

13.14

a)



$m = 1.5\,kg$ $\quad$ $k = 2{,}000\,N/m$

use conservation of energy: $PE_f + KE_f = PE_i + KE_i$

$KE = \frac{1}{2}mv^2 \qquad PE = \frac{1}{2}kx^2$

so $\frac{1}{2}kx_f^2 + \frac{1}{2}mv_f^2 = \frac{1}{2}kx_i^2 + \frac{1}{2}mv_i^2$

$x_i = 0.30\,cm = 3\times10^{-3}\,m \qquad v_i = 0$

$x_f = 0 \qquad v_f = ?$

$\cancel{\frac{1}{2}} m v_f^2 = \cancel{\frac{1}{2}} k x_i^2$

$v_f = \sqrt{k/m}\; x_i = \sqrt{\frac{2000\,N/m}{1.5\,kg}}\;(3\times10^{-3}\,m) = \boxed{0.11\ m/s}$

b) now some energy is lost due to friction

$(PE_i + \cancel{KE_i}) - (\cancel{PE_f} + KE_f) = W_{nc}$

where $W_{nc} = F_f\ (\Delta x)$

$\frac{1}{2}kx_i^2 - \frac{1}{2}mv_f^2 = F_f(x_i - x_f)$

$\Rightarrow \frac{1}{2}mv_f^2 = -F_f\ (x_i - 0) + \frac{1}{2}kx_i^2$

$\Rightarrow v_f^2 = (k/m)\,x_i^2 - 2(F_f/m)\,x_i$

$v_f^2 = \frac{(2000\,N/m)}{(1.5\,kg)}(3\times10^{-3}\,m)^2 - 2\left(\frac{2.0\,N}{1.5\,kg}\right)(3\times10^{-3}\,m)$

$= 0.004\ m^2/s^2$

$\Rightarrow v_f = \sqrt{0.004\,m^2/s^2} = \boxed{0.063\,m/s}$

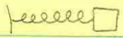
c) now $F_f = ?$ and $v_f = 0$

$(\frac{1}{2}kx_i^2 + \underbrace{\frac{1}{2}m\cancel{v_i^2}}_{0}) - (\underbrace{\frac{1}{2}\cancel{kx_f^2}}_{0} + \underbrace{\frac{1}{2}\cancel{mv_f^2}}_{0}) = F_f\ (x_i - \underbrace{\cancel{x_f}}_{0})$

$\Rightarrow \frac{1}{2}kx_i^{\cancel{2}} = F_f\,\cancel{x_i}$

$\Rightarrow F_f = \frac{1}{2}kx_i = \frac{1}{2}(2{,}000\,N/m)(3\times10^{-3}\,m) = \boxed{3N}$

②

13.18 

$m = 50.0\,g = 5.00\times10^{-2}\,kg \qquad k = 10.0\,N/m$

again, use conservation of energy

$\frac{1}{2}kx_i^2 + \frac{1}{2}mv_i^2 = \frac{1}{2}kx_f^2 + \frac{1}{2}mv_f^2$

$x_i = 25.0\,cm = 0.25\,m \qquad x_f = (x_i/2) = 0.125\,m$

$v_i = 0 \qquad v_f = ?$

$\cancel{\tfrac{1}{2}}kx_i^2 = \cancel{\tfrac{1}{2}}k\,(x_i/2)^2 + \cancel{\tfrac{1}{2}}\,m\,v_f^2$

$\Rightarrow \quad m v_f^2 = k x_i^2 - k\,(x_i^2/4)$

$= k x_i^2\,(3/4)$

$\Rightarrow v_f = \sqrt{(3/4)(k/m)}\;x_i$

$= \sqrt{\frac{3}{4}\,\frac{(10.0\,N/m)}{(5\times10^{-2}kg)}}\;(0.25\,m) = \boxed{3.06\ m/s}$

13.26 $x = (0.30\,m)\cos(\pi t/3)$

make sure your calculator is in radian mode

a) $x(t=0) = (0.30\,m)\cos[(\pi/3)(0)]$

$= (0.30\,m)(1) = \boxed{0.30\,m}$

$x(t=0.60\,s) = (0.30\,m)\cos[(\pi/3)(0.6)]$

$= (0.30\,m)(.81) = \boxed{0.24\,m}$

b) $x = A\cos(\omega t) \Rightarrow \boxed{A = 0.30\,m}$

c) $x = A\cos(\omega t) = (0.30\,m)\cos[(\pi/3)t] \Rightarrow \boxed{\omega = \pi/3\ rad/s}$

d) recall that : $\omega = 2\pi/T$

multiply by (T/ω) to get: $T = (2\pi/\omega)$

$\Rightarrow T = \frac{2\pi}{(\pi/3)} = \boxed{6\,s}$

13.32

a) when the pendulum gets cold it shrinks

the period of a pendulum is: $T = 2\pi\sqrt{L/g}$

since L gets smaller in the cold room, this means the period gets shorter so the clock ticks more than once a second

$\Rightarrow$ $\boxed{\text{the clock gains time}}$

b) from pg. 328 the change in L is

$L - L_0 = \alpha L_0 (T - T_0)$ (here T is temperature!)

$\alpha = 24 \times 10^{-6}\ [^\circ C]^{-1}$

$\Rightarrow L = L_0 + \alpha L_0 (-25.0^\circ C) = L_0 (1 - 25\alpha)$

what is L_0?

we know $\overset{\text{period}}{T_0} = 2\pi\sqrt{L_0/g}$ is $1s$

$\Rightarrow L_0 = \frac{(1s^2)}{(2\pi)^2} g = 0.25 m$

so $L = (0.25m)\left[1 - 25(24 \times 10^{-6})\right] = 0.25$

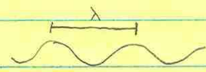
now $T = 2\pi\sqrt{L/g} = \boxed{0.9997 s}$

13.38 $v = 340\,m/s$ $f = 60.0\,kHz$ $\lambda = ?$

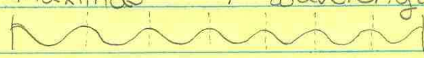
use eqtn 13.17

$v = f\lambda$

$\Rightarrow \lambda = v/f = \frac{(340\,m/s)}{(60.0\times10^{3}\,s^{-1})} = \boxed{0.0057\,m = 5.7\,mm}$

13.40  $\lambda = 1.20\,m$

8 maximas = 7 wavelengths



$v = \frac{\text{distance}}{\text{time}} = \frac{7\,(1.20\,m)}{12\,s} = \boxed{0.7\,m/s}$

13.44 $m = 0.350\,kg/m$

$v = \frac{\text{distance}}{\text{time}} = \frac{20.0\,m}{0.800\,s} = 25\,m/s$

$v = \sqrt{F/\mu} \Rightarrow v^2 = F/\mu \Rightarrow F = v^2\mu$

so $F = (25\,m/s)^2\,(0.350\,kg/m) = \boxed{219\,N}$

Oscillating graphs

a) D

b) C

c) N - force exerted by the spring oscillates around the value +mg, opposite the position

d) N - the kinetic energy is max whenever the position is equillibrium, zero whenever the position is a max. or min.

e) B or E (they're identical)

f) F

Where's the Force?

a) U - it must counter the force of gravity

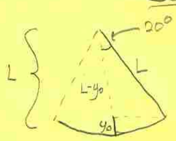
b) U → again, strictly b/c of gravity

c) zero → at the equillibrium position all the forces cancel

d) D → the spring is compressed and wants to expand

e) D → gravity and the spring both push/pull downward on the mass

Swingin' in the rain



$$T = 2\pi\sqrt{L/g} = (8s)$$

$$\Rightarrow \sqrt{L/g} = (8s)/2\pi$$

$$\Rightarrow L/g = \left(\frac{8s}{2\pi}\right)^2$$

$$\Rightarrow L = (8s/2\pi)^2\, g = \boxed{15.9\ m}$$

to find the speed use conservation of energy

1) define the vertical distance from the bottom of the arc to the top as y_0

2) find y_0

$$\cos(20^\circ) = (L - y_0)/L$$

$$\Rightarrow y_0 = -L\cos(20^\circ) + L = L(1 - \cos(20^\circ))$$

$$= 0.959\ m$$

3) use $KE_f + PE_f = KE_i + PE_i$

$$\frac{1}{2} m v_f^2 + mg\underbrace{\cancel{y_f}}_{0} = \frac{1}{2} m \underbrace{\cancel{v_i}}_{0}^2 + mgy_i$$

$$\Rightarrow \frac{1}{2}\cancel{m} v_f^2 = \cancel{m} g y_0$$

$$\Rightarrow v_f^2 = 2gy_0$$

$$\Rightarrow v_f = \sqrt{2g(0.959m)} = \boxed{4.34\ m/s}$$

Catching a pellet and oscillating

a) use conservation of momentum

$p_f = p_i$

$\Rightarrow (m+M)\,v_f = mv$

$\Rightarrow v_f = \dfrac{mv}{(M+m)}$

b) use conservation of energy

$KE_f + PE_f = KE_i + PE_i$

here the "final" instant is when the spring is fully compressed

$\frac{1}{2}m\underbrace{\cancel{v_f}^2}_{0} + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}k\underbrace{\cancel{x_i}^2}_{0}$

$\Rightarrow kx_f^2 = mv_i^2$

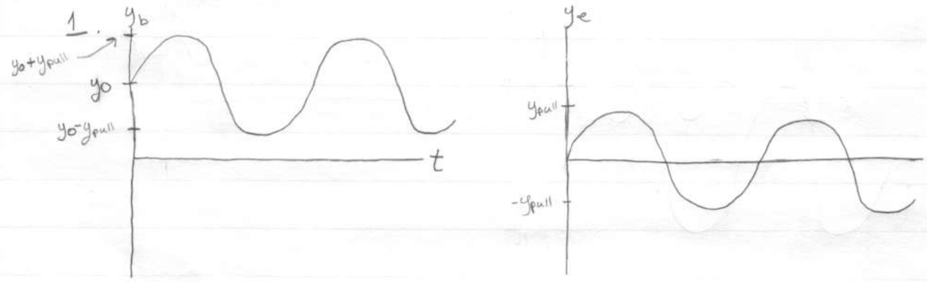
$\Rightarrow x_f = \sqrt{k/m}\;v_i$

$\Rightarrow \boxed{x_f = \sqrt{\dfrac{k}{m}}\;\dfrac{mV}{(M+m)}}$

c)
1) all of the clay sticks to the block after the collision
2) the clay hits the block head-on, so it doesn't twist or bunch up the spring
3) there is no friction (stated) or wind resistance (unstated)
4) the starts at rest (stated) at its equillibrium position (implied)

Harmonic Motion

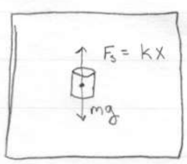
1.



y_b is shifted by a distance y_0 equal to the distance from the base to the equillibrium position

2. $y_e = (y_{pull})\sin(\omega t)$
3. $y_b = y_0 + (y_{pull})\sin(\omega t)$

4.



same in both systems

x here is the displacement from what would have been the equillibrium point if there were no gravity (i.e. $kx = mg$ so $x = mg/k$)

5. Newton's 2^{nd} Law: $F = ma$

in y_e system $F_{net} = F_{spring} + F_{gravity} = -k(y_{pull})\sin(\omega t) - mg$

using 2^{nd} Law $\Rightarrow a_{net} = F_{net}/m = -(k/m)(y_{pull})\sin(\omega t) - g$

in y_b system $F_{net} = -k(y_b - y_0) - mg = -k[\cancel{y_0} + (y_{pull})\sin(\omega t) - \cancel{y_0}] - mg$

using 2^{nd} Law $\Rightarrow a_{net} = -(k/m)(y_{pull})\sin(\omega t) - g$

since the acceleration must be the same in both systems, Newton's 2^{nd} Law is the same in both systems