

1) Suppose you have an object 12 inches tall that's 10 meters away from your lens ( $d_o = 10$  m), and the lens has a focal length of  $f_1 = 1.00$  m.

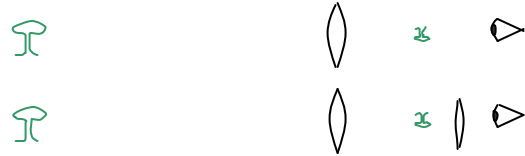
a) Find the distance from the lens to the image ( $d_i$ ), and the height of the image ( $h_i$ ).

First:  $1/1.00 \text{ m} = 1/10 \text{ m} + 1/d_i$ . That gives  $d_i = 10/9$  m, or about 1.1 m from the lens to the image.

The height of the image is then  $h_i = h_o d_i / d_o =$

12 inches  $(1/9) = 1.3$  inches.

b) Now, you're going to look at that image with another lens. Suppose this lens has a focal length of  $f_2 = 10.00$  mm, and you place it a distance of 9.99 mm from the image from the first lens. In other words, the image from the first lens is now the object for this lens—it's just the same as if you had a little tiny object there, and you're looking at it with a magnifying glass. Where is the image from the second lens, and how big is it?



Thin lens formula again:  $1/10.00 \text{ mm} = 1/9.99 \text{ mm} + 1/d_i$ . Here that gives  $d_i = -9990 \text{ mm} = -10$  m.

That's 10 m behind the second lens, which puts it out near the original object. But now the final image is

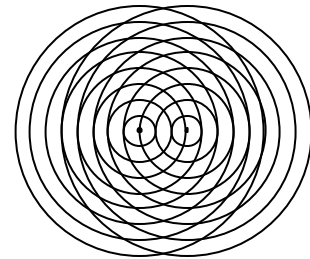
$h_i = h_o d_i / d_o = 1.3$  inches  $(9990/9.99) = 130$  inches. So the final image is still far away, but now it's very large.

This is a telescope. It's not a great telescope: The final image is upside-down. That's ok for looking at stars, but to look at a tree far away it would be better to flip the image right-side-up again. One way to do that is with another lens in the middle.

2) We started with a particle model of light, thinking of light as objects that bounce. Then we found some phenomena this model couldn't explain and discovered that a wave model of light was able to explain these phenomena well. Explain in your own words how these two models fit together. Give an example of a foothold idea that arises out of both models and an example of a foothold idea that could arise only out of the wave model. Give examples for which one or the other model is more convenient to use, and for the particle model example, show how the wave model could have given the same prediction.

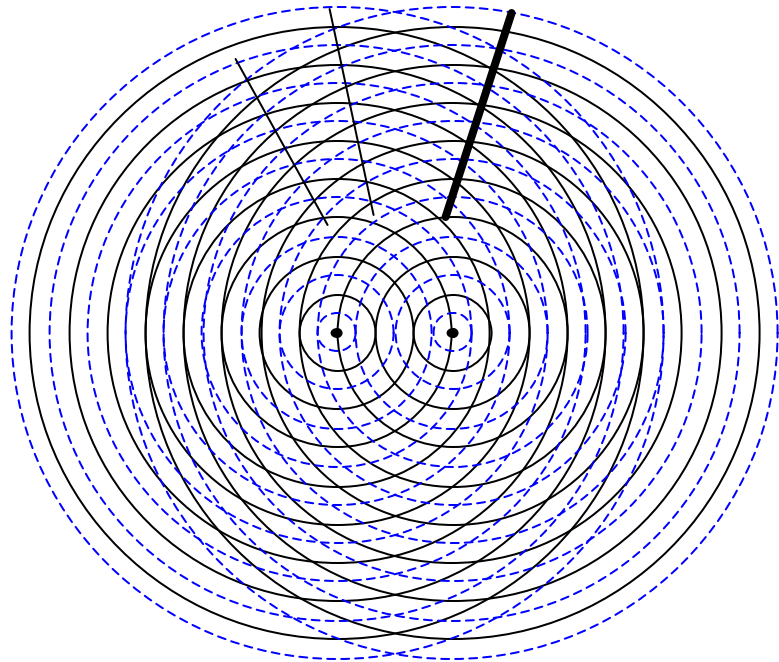
For most things, we found that the particle and wave models gave the same predictions once you realize that the rays we drew in the particle model (for light traveling in straight lines) could be drawn as the lines perpendicular to the wavefronts in the wave model. Then most things - mirrors, non-mirrors, images, light hitting you in the eye - were the same for both models. Looking at light passing through apertures, the particle model seemed to work better for big apertures (point sources giving spots shaped like the apertures) and the wave model seemed to work better for small apertures (multiple-spot interference patterns). But we saw that the wave model worked for both of these. The wave model also worked better for refraction, since the speed of light in water is less than air and the wave model predicts the bending (Snell's law) from the slowing. However, the particle model worked better for predicting the force that light exerts on objects and for thinking of light as photons. In the end, we seemed to have come up with a hybrid model of light. It's not exactly like little particles with mass, nor is it perfectly like waves propagating on a medium. It's more like particles with no mass that have wave properties. Keeping that in mind, sometimes you use your massive particle ideas, sometimes your wave on medium ideas. Often you can use either - they'll predict the same thing. And sometimes (as in explaining the photoelectric effect or understanding how a photon has a wavelength!) you need to use BOTH.

3) The diagram below is a representation of air pressure waves at an instant in time from two speakers that are in phase and playing a tone at a frequency of 400 Hz. (1 Hz = 1 Hertz = 1 cycle per second) The lines depict maxima in pressure (the peaks); I'm leaving out the dotted lines you had in tutorial that depicted the minima (troughs) between the peaks.



a) What would the diagram look like to depict the waves 1/400 of a second later? Exactly the same as it does now (the original lines). It's a frequency of 400 cycles per second, so it takes 1/400 of a second for a wave to move one wavelength, and if you move each peak one wavelength, it will look like this again.

b) What would the diagram look like to depict the waves 1/800 of a second later? It would look similar, but instead of the peaks where they are now, they'd each move 1/2 a wavelength out – I've put blue dashed lines there. Notice that the peaks after 1/800 seconds are where the troughs used to be.



c) You hear things if the air pressure is vibrating at your ear, and bigger amplitudes of vibration mean louder sounds. Show on the diagram lines where the sound would be the loudest.

I showed one line with a thick line – this is where at any instant the peaks of the two sources line up, or the troughs line up. (Any point where troughs line up at one instant will be a point where peaks line up 1/800 seconds later.)

d) Show on the diagram lines where there would be no sound.

I showed two with thin lines, where peaks of one source line up with the troughs of the other, at any instant.

e) If you were at a point where there was no sound, what would happen if one of the speakers turned off? It would get louder! You'd have eliminated the destructive interference from the other source, so the remaining source would make a tone.

4) The speed of sound in air varies with the air temperature, humidity, density, but it's typically around 340 m/s.

a) What would be the real distance in the air between the lines in the diagram? That is, how far apart are the peaks of the air pressure?

$v = f\lambda$ , so  $340 \text{ m/s} = (400 \text{ Hz}) \lambda$  gives  $\lambda = (340 \text{ m/s}) / (400 \text{ 1/s}) = 0.85 \text{ m}$

b) The nodal lines are curved, but far from the sources they straighten out. Look at two adjacent nodal lines in the previous diagram and estimate the angle between them.

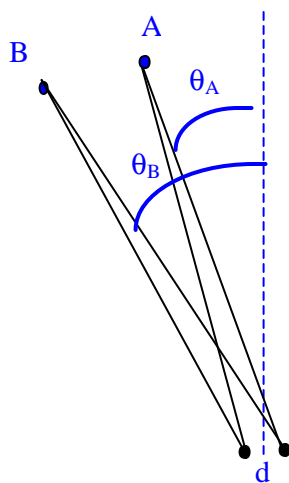
Well... extend the lines until they intersect, and it looks like about  $\alpha = 20$  degrees to me.

c) Now calculate the angle between the nodal lines, when they've straightened out far from the sources, using the formula from problem 6 in assignment 10.

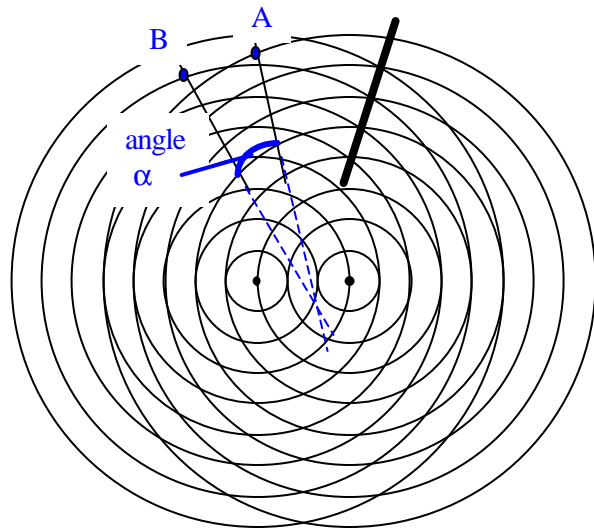
Far from the source, the lines of maximum constructive interference were at  $m\lambda = d\sin\theta$ , with  $d$  the distance between the sources and  $\theta$  the angle between the lines of maximum constructive interference and the vertical line. Nodal lines are lines of maximum destructive interference, so they are at  $(m+1/2)\lambda = d\sin\theta$ . The first node is at  $\sin\theta_A = \lambda/2d$ . Here,  $d = 3\lambda$ , so  $\lambda/2d = 1/6$ , and so  $\theta_A = 9.6$  degrees.

The second node is at  $\sin\theta_B = 3\lambda/2d$ , or  $\sin\theta_B = 1/2$ , so  $\theta_B = 30$  degrees. So the angle between the nodal lines is about 20 degrees.

d) Do your answers to parts b and c agree? They should! Mine do.



Looking at two points A and B on the nodal lines, we see that these each have  $\theta$  angles to the vertical given by the formula. The difference between these angles is  $\alpha = \theta_B - \theta_A$



5) A friend has 100 pennies and is going to throw them all in the air, let them land, and then count the heads and tails. He thinks they're going to be exactly 50 heads and 50 tails, and he wants to bet you \$5 he'll be right.

a) Give an argument someone might make for why you should take the bet (that is, why your friend is probably going to be wrong).

There are so many ways it could turn out – 49 heads and 51 tails, 48-52, 51-49, 47-53, etc etc – the odds of it turning out this particular way seem pretty small.

b) Give an argument someone might make for why you shouldn't take the bet (that is, why your friend is probably going to be right). The odds of it coming up heads are  $1/2$ , so  $1/2$  the time it ought to come out heads.

c) Explain what you think, including a response to the argument you disagree with.

You should take the bet. There really are lots of ways it could come out, and while it's true that 50-50 is the *most probable of the possibilities*, that doesn't make it *probable overall*. Think of a jar full jelly beans, 20 different flavors, but there's two limes and only one of every other. If you were to put your hand in and pick one, lime would be more likely than any other flavor, but you're still not very likely to get lime. Same thing with picking 50-50 out of all the other possibilities – it's more likely than the others, but still not likely.

6) Carbon 14 has a half-life of about 5700 years. It decays by beta-decay.

a) What does C14 decay to? Try to use only a periodic table and the fact that it's a beta-decay.

It's a beta decay, which changes a neutron to a proton. Nitrogen 14 has one more proton and one less neutron than carbon 14.

b) The amount of C14 found in a fossil is found to be 1/10 of what we expect it started with. About how old is the fossil?

How many half lives does it take to go down to 1/10<sup>th</sup>? Three half lives would get it to 1/8, which would be about 17000 years.

More precisely, we want to solve  $(1/2)^{t/5700} = 1/10$ . It's a problem to solve with logarithms:

$$\begin{aligned}\log (1/2)^{t/5700} &= \log (1/10) \\ - (t/5700) \log 2 &= -1 \\ \log 2 &= 0.3, \\ \text{so solve for } t &= 19000 \text{ years.}\end{aligned}$$

You could also do this with the exponential. If  $N=N_0e^{-t/\tau}$  and  $\tau = \tau_{1/2}/0.69$ , and  $N/N_0$  is 1/10, then  $e^{-(0.69t/5700 \text{ years})} = 0.1$ .  $(0.69t/5700 \text{ years}) = -\ln(0.1) = 2.3$ .  $t = (2.3)(5700 \text{ years})/(.69) = 19000 \text{ years}$ .

c) In lab you saw that the number of decays in a given time "fluctuated" considerably. Does that mean that the half-life of a radioisotope (the indium you were using or the C14 in this problem) fluctuates over time?

No. This is just the same as the pennies question above. The number of decays you'd calculate with the half-life for a given time is the most probable number to decay in that time, but being the most probable of the many (many many!) possible numbers doesn't make it probable for any given time interval. The half life of indium doesn't fluctuate over time any more than the probability of getting heads fluctuates with each coin toss.