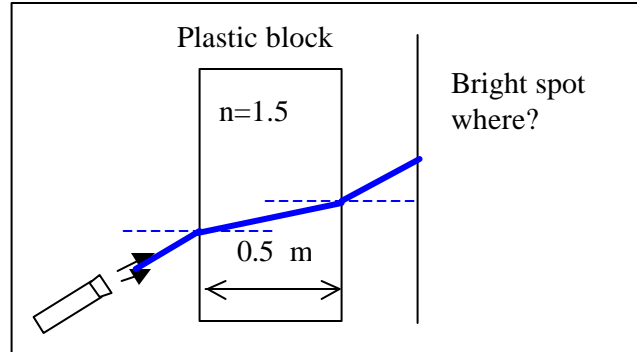
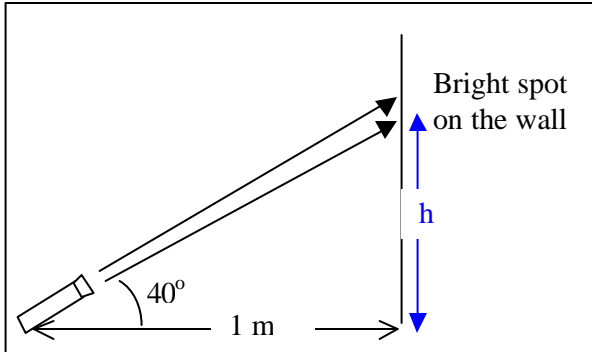


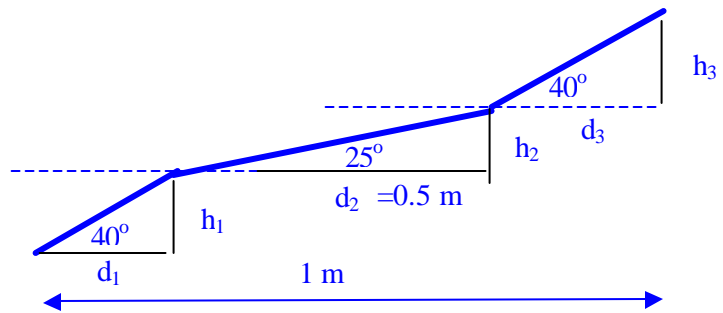
1) Revisiting the last question from the problem set before. Suppose you have a flashlight or a laser and you hold it at some angle and shine it on the wall, observing a bright spot at some height.



a) Suppose you know put a clear plastic block between the flashlight and the wall. You should have found that the bright spot was lower. How much lower is it? Tell me the original and final heights of the bright spots. I've given you the distances, indices of refraction, and angles you need (assume n of air is 1.) I haven't given you the exact location of the block from the wall because you don't need to know it. (Can you figure out why?)

Originally, $\tan(40^\circ) = h/(1\text{ m})$. $h = 84\text{ cm}$.
 In the new medium the angle of the light is given by $(1)(\sin 40^\circ) = (1.5)(\sin \theta)$. $\theta = 25^\circ$.
 And then the light comes out at 40° again.

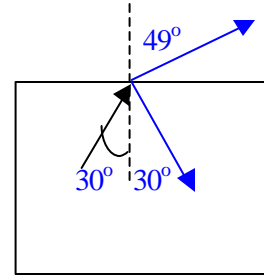
Looking at the triangles we see that the heights the light is traveling outside the block are $h_1 = d_1 \tan 40^\circ$ and $h_3 = d_3 \tan 40^\circ$, so that means $h_1 + h_3 = (d_1 + d_3) \tan 40^\circ$. But we also know that $d_1 + d_3$ is equal to $1\text{ m} - 0.5\text{ m} = 0.5\text{ m}$. So $h_1 + h_3 = (0.5\text{ m}) \tan 40^\circ = 42\text{ cm}$. (Makes sense, half of the total height without the block. That's why you don't need to know where the block is...) And inside the block the light travels $(0.5\text{ m}) \tan 25^\circ = 23\text{ cm}$. So the total height is $23\text{ cm} + 42\text{ cm} = 65\text{ cm}$, lower than without the block.



2) Suppose you have a ray of light traveling inside a plastic block with index of refraction 1.5 (assume air has $n=1$.) The ray of light will bend at the surface.

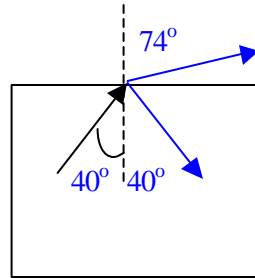
a) If θ_1 is 30 degrees what direction will the reflected and refracted light rays go? Calculate the angles and draw the rays.

(1.5) $\sin(30^\circ) = 0.75 = (1) \sin(\theta_2)$, so $\theta_2 = 49^\circ$.



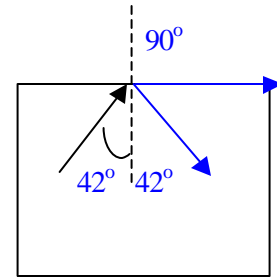
b) If θ_1 is 40 degrees, what direction will the reflected and refracted light rays go?

(1.5) $\sin(40^\circ) = 0.96 = (1) \sin(\theta_2)$, so $\theta_2 = 74^\circ$.



Note, if $\theta_1 = 42^\circ$, $(1.5) \sin(42^\circ) = 1 = (1) \sin(\theta_2)$, so $\theta_2 = 90^\circ$.

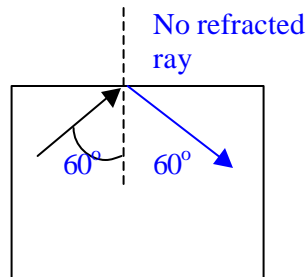
The refracted ray comes out along the surface of the plastic.



c) If θ_1 is 60 degrees, what direction will the reflected and refracted light rays go? Calculate the angles and draw the rays. If you got a funny result, how might you interpret this?

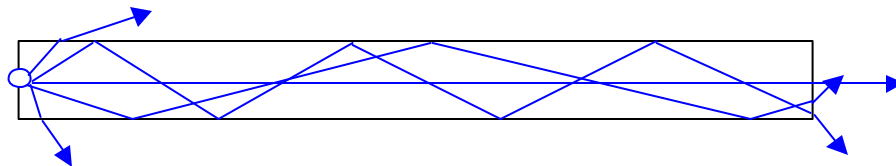
(1.5) $\sin(60^\circ) = 1.3 = (1) \sin(\theta_2)$, so $\theta_2 = \text{Error}$.

This equation can't be solved for the angle. In reality what happens is that there is no refracted ray.



c) It turns out that any time you can't actually solve the equation for Snell's Law, it means that there is no refracted light - all the light gets reflected internally. This is called total internal reflection. Think of at least one application for this and describe how it might work.

In class we talked about fiber optics, the most common application for this. Light enters a long, usually cylindrical piece of plastic or glass at all angles. Some of it is at a steep enough angle to be refracted out and escape. But other rays come in at shallow enough angles (42° or more if $n=1.5$) and is totally internally reflected. Since the angles of reflection are equal, it hits the surface on the other side at the same angle and as such is also totally internally reflected there. Thus, all the light that reflected inside once keeps reflecting, bouncing back and forth down the cylinder and finally comes out the other end, sometimes miles away.



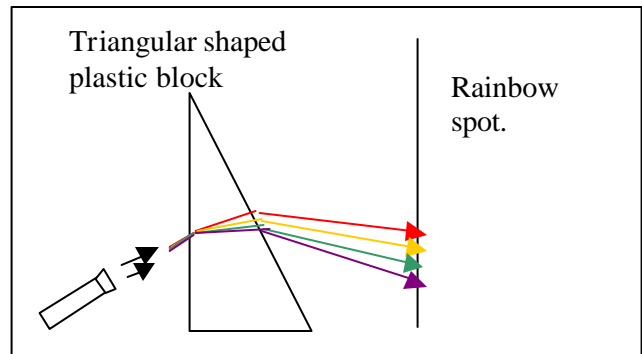
3) Think again about the triangular shaped plastic block in between the flashlight and the wall. If you tried this at home, what you would see is a bright spot much lower than with out the block, but instead of being a white spot, if your flashlight beam is very narrow you may actually get a rainbow spot. A triangle shaped block is also called a "prism."

a) If I told you that most things actually have a different index of refraction for different colors, would this explain what you see? How?

If different colors had different indices of refraction, different colors would bend more or less going in and coming out and thus end up at different spots on the wall.

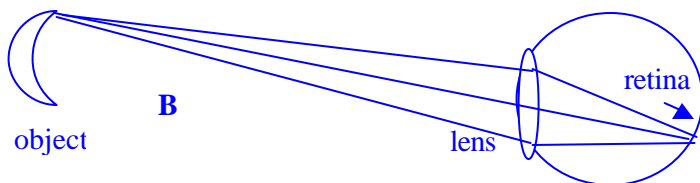
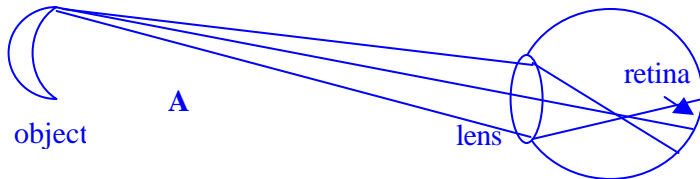
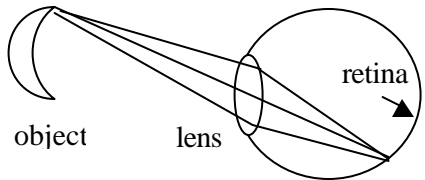
b) If I told you that the index of refraction was higher (oops should be lower) for red than for blue, is the red on top of the rainbow spot on the wall or at the lower point?

If red has a lower n than blue, it bends less going in and coming out and thus ends up on top. This is how a prism produces a rainbow on the wall.



4) In your eye, you have a converging lens that focused the light from objects onto your retina. (Actually you have two lenses, your cornea and the lens inside. Both are converging and work together.)

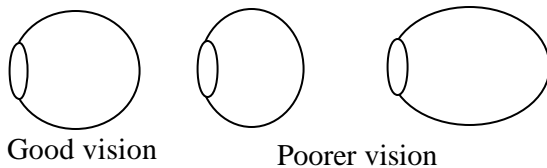
a) The lens inside is made up of flexible material that allows it to be stretched by muscles around it, effectively changing it's shape. The muscles can make it flatter, for instance. Why is this necessary?



The lens is there to focus the image onto the retina. If the image isn't focused, meaning that light from each point on the object hits a single point on the retina, you will see stuff very blurry. Since the distance from lens to retina is fixed, if you try to look at a more distant or closer object with the same lens (drawing A), the image will not be focused on the retina. However, if your muscles stretch the lens the focal point moves farther away from the lens and the image also moves farther away. Your eye adjusts the lens so that you get the clearest image (drawing B). You've probably noticed that you can't focus on far things and close things at the same time. If you focus on something near (like a finger held close to your face), everything far behind the finger is out of focus. And if you focus far, the finger becomes blurry.

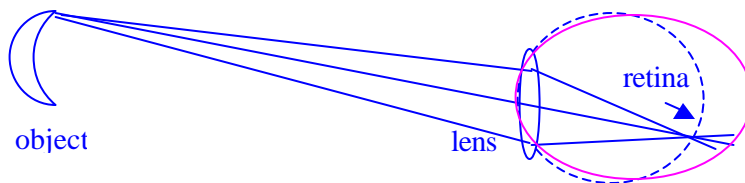
We didn't talk in class explicitly about why a thinner lens has a longer focal length. We did talk about this for a mirror - the less the curvature the farther away the focal point. The trend is exactly the same for a lens, although the physics is different, of course, because we are refracting instead of reflecting. You should figure out why, from how a lens bends each individual light ray, a flatter lens has a longer focal length. Also, try to figure out how the focal length would change for the same lens shape with a different index of refraction. You should be able to figure this out and it is possible it might show up on the final...

b) People with poor vision actually have eyeballs that are a little too long or too short.

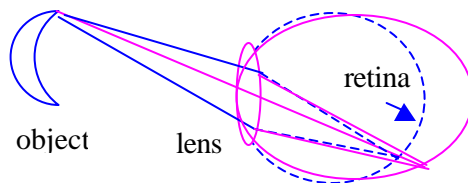


Explain why this makes the vision blurry without glasses.

Okay this is a tough one, even tougher than the one above. You may have been asking yourself, hopefully, "What does it matter how the retina is shaped? Can't the lens just change to accommodate for this?" If you were asking this, fantastic! That's the kind of thinking I'm hoping you guys are doing at this point. Well, the answer is, the lens can't accommodate for that. It really can only change a little bit to allow it to focus on things of different distances. But there's a limit to how much the muscles of your eye can stretch and compress the lens. So let's say the picture I drew above for a lens focusing on a far object shows the flattest a lens can get.



If a person's eyeball is too long, the lens can't get any flatter to focus the image on the retina. Thus the image for far objects is blurry. For near objects, the lens for a normal eye would need to be compressed (more curved) to focus on a the retina. But it can get flatter here and focus even though the retina is farther away. This is why a longer eyeball makes for a nearsighted person. We can focus near objects but not far ones.



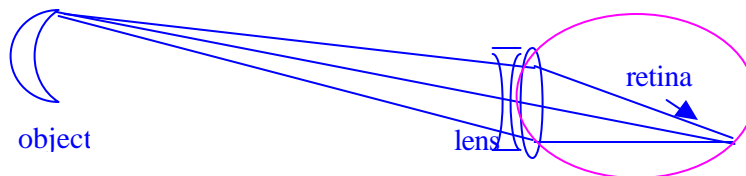
The opposite is true for farsighted folks. For far objects the lens can get fatter to focus, but it can't get fat enough for near objects to focus.

c) Eyeglasses are made of either diverging ($\left[\left[\right] \right)$ or converging ($\left(\bigcirc \right)$) lenses. Which do you need

for which type of vision problem? Does the diverging lens go with the shorter eyeball or the longer one, and which does the converging lens go with, or is there a single lens that works for both and which is it?

Okay, here's one way to think about this. If you are nearsighted (long eyeball), your lens can't get flat enough to focus far objects, and it doesn't need to be as fat for near ones, so you'd be better off with a lens that started less fat and went much flatter than yours can. Adding a

diverging lens to a converging one is kind of like making a flatter converging lens. That's one way to think about it. [Another way to think about it is to think about what the diverging lens does to the rays and then how that changes what the converging lens will do. Another way to think about it is to figure out where the image of the diverging lens is and see what happens when the second lens makes a second image from the first one's image. You should be able to figure this out these three ways and maybe more!]

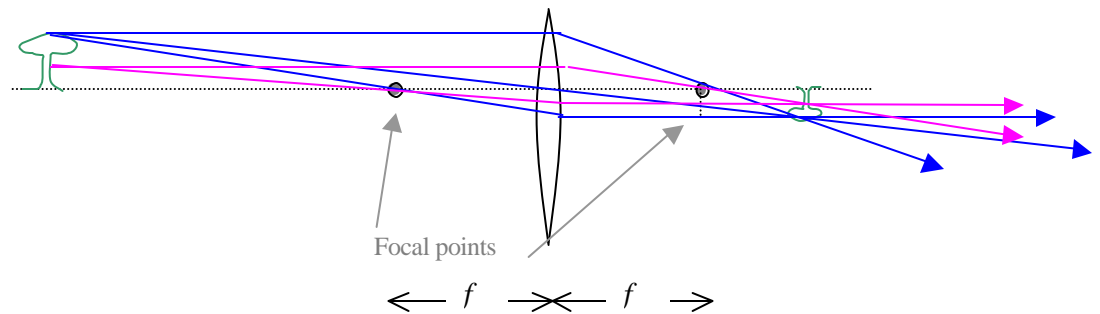


So the diverging lens makes it possible to focus on the retina of a nearsighted eye.

Adding a converging lens to another converging makes for an even fatter converging lens, equivalently, so the analogous reasoning works for the shorter eyeball.

5) You have a tree and a thin converging lens with focal points as shown.

a) Find the image location for the tree. Is the image real or virtual? Is it magnified or reduced?



Drawing the three convenient rays from the top of the tree (blue lines) gives the location for the top of the tree in the image. Repeating for the middle of the tree (pink lines) and so on defines the image. From this we can see that the image is on the opposite side of the lens, past the focal point and that it is inverted and reduced.

b) The distance from the lens to the focal points is called the focal length, f . Call the distance from the object (the tree) to the lens d_o , and the distance from the lens to the image d_i . Come up with a formula for the relationship between the image distance and object distances.

The figure shows two of the light rays from a point on the top of the object that pass through a lens and converge to an image point. One goes through the focal point on the way in, and comes out parallel to the central axis of the lens; the other comes in parallel to the central axis and goes out through the other focal point. As with the derivation of $h_i = h_o d_i / d_o$, which you've done in a couple of versions, we use similar triangles. This time we use similar triangles made out of the rays that go through the focal points.

The height of the larger triangle is the height of the object, h_o , and the base is d_i , the distance from the lens to the image. The height of the smaller triangle is the height of the image, h_i , and the base is $d_i - f$, the distance of the image from the focal point, where f is the focal length of the lens. Those triangles are similar, so they have the same ratio of height to base:

$$h_o / f = h_i / (d_i - f)$$

We already know that $h_i = h_o d_i / d_o$, so we can substitute for h_i to get:

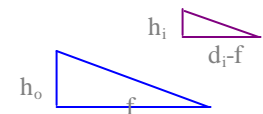
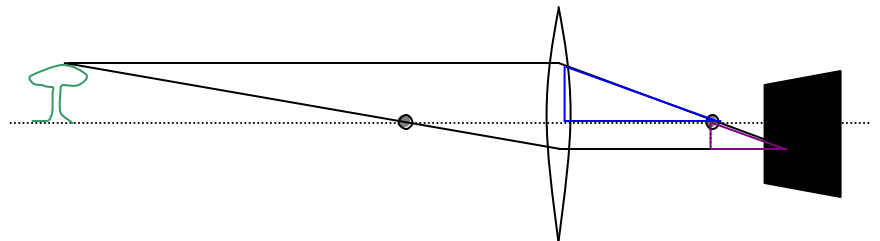
$$h_o / f = h_o d_i / d_o (d_i - f)$$

Divide both sides by h_o , and multiply both sides by $(d_i - f)$:

$$d_i / f - 1 = d_i / d_o$$

Rearrange and divide by d_i to get:

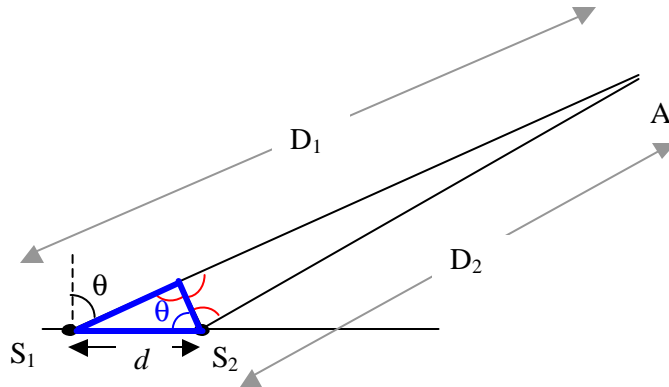
$$\boxed{1/f = 1/d_i + 1/d_o}$$



That's called the "thin lens equation," because it assumes the lens is so thin we don't have to pay attention to its thickness. (We didn't.)

We can apply this equation to the problem to check and see if it fits with our ray conclusions. (It always should!!) The object looks to be about $3f$ away from the lens. Thus $1/d_i = 1/f - 1/3f = 2/3f$, giving $d_i = 3f/2$. Looks about right!

6) Consider a point A very far away from two sources of waves (both in-phase with wavelength λ , like two sources in the ripple tank) separated by a distance d . The distance from one source (S_1) to the point A is D_1 and from S_2 to A is the distance D_2 . To figure out whether the two waves will interfere constructively or destructively at point A, we need to know the difference in the two distances $\Delta D = D_1 - D_2$.



a) Three angles are shown in the drawing (in red), but not labeled. (A fourth angle is labeled θ .) If the point A is very far away from the sources, these three angles all become right angles. In this limit, find an expression for ΔD in terms of the angle θ and the source separation d . (Hint look at the triangle that's a right triangle with hypotenuse d . One of the angles in that triangle is θ . Which one?)

In the limit of A far away, that little triangle is a right triangle, with the marked angle equal to θ . And the side opposite that angle θ is ΔD (that's how much D_1 is longer than D_2). And in a right triangle the sine of an angle is the opposite side / hypotenuse: $\sin \theta = \Delta D/d$.

So $\Delta D = d \sin \theta$.

b) For what values of ΔD (in terms of λ) will there be maximum constructive interference? Complete destructive interference?

There'll be maximum constructive interference when D_1 is longer than D_2 by a wavelength or an integer number of wavelengths (or $1\lambda, 2\lambda, 3\lambda \dots$), because then the waves will be exactly in sync. Thus there is constructive interference for $m\lambda = d \sin \theta$, where m is an integer.

There'll be destructive interference when D_1 is longer than D_2 by half a wavelength (or $1.5\lambda, 2.5\lambda, 3.5\lambda \dots$), because then the waves will be exactly opposite. The smallest difference would be $1/2$, which would be: $\Delta D = \lambda/2 = d \sin \theta$.

So for destructive interference $(m+1/2)\lambda = d \sin \theta$, where m is an integer.