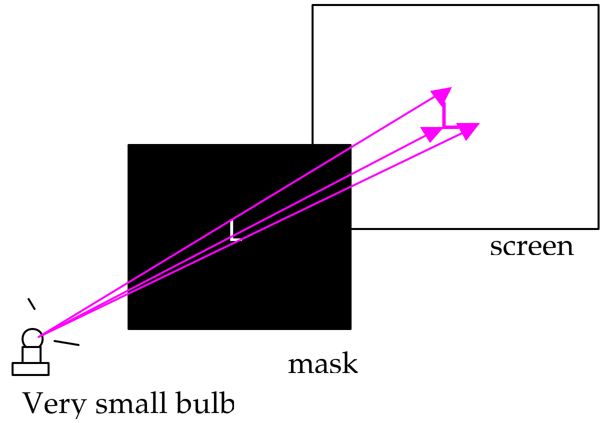
Physics 122 Assignment 8 Solutions
Laura Lising Due April 10, start of class

Reference: Cutnell & Johnson, sections 25.2-25.3. (Beginning geometric optics.)

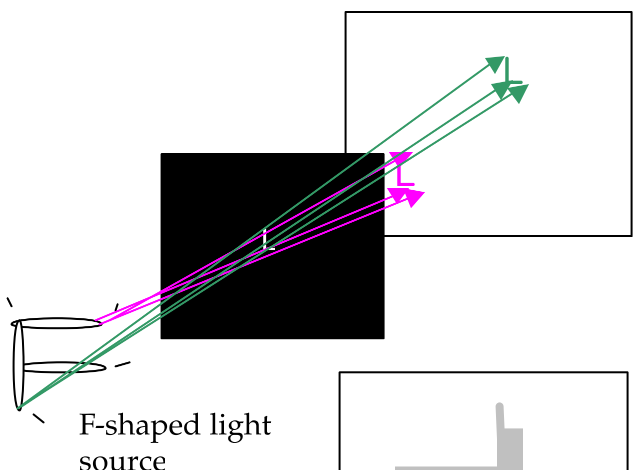
1)
A mask containing a hole in the shape of the letter L is placed between a screen and a very small bulb as shown at right.

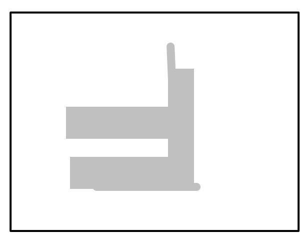
a) On the diagram, sketch what you would see on the screen when the bulb is turned on.
Following the rays through the aperture, you can see that the bright spot will be shaped like an L that is upright and bigger (about twice as big, as the aperture.)



b) the small bulb is replaced by three long-filament bulbs that are arranged in the shape of the letter F as shown at right.

On the diagram, sketch what you would see on the screen when the bulbs are turned on. The scale of your sketch should be consistent with your answer to part a. Explain how you determined your answer.

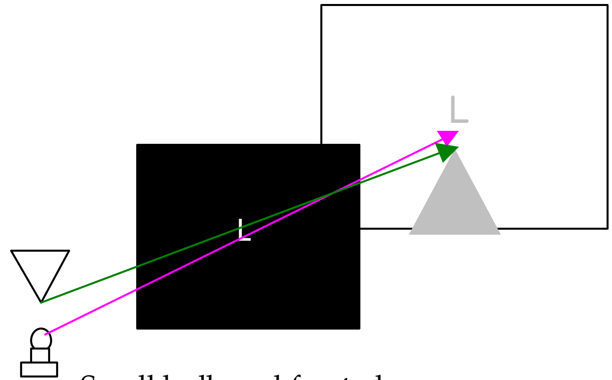




Our model says you can treat any extended source like a bunch of point sources. The pink lines show the rays from one point, which gives an L as before. The green lines show rays from another point, which gives an L elsewhere. Notice, the pink L from the top right of the F is on the bottom left of the image. And the green L from the bottom left of the F is on the top right. There are many many points you could do this for and what you'd find is an F that is kind of wide and blurry and inverted left-right and top-bottom. It's like an F drawn with an L-shaped magic marker.

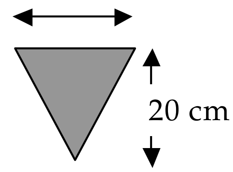
c) the three long-filament bulbs are replaced by a small bulb and a large triangle-shaped frosted bulb as shown at right.

If you think about drawing a point and a triangle with an L-shaped magic marker and inverting the two, which is what you get when your trace the rays, you get this.

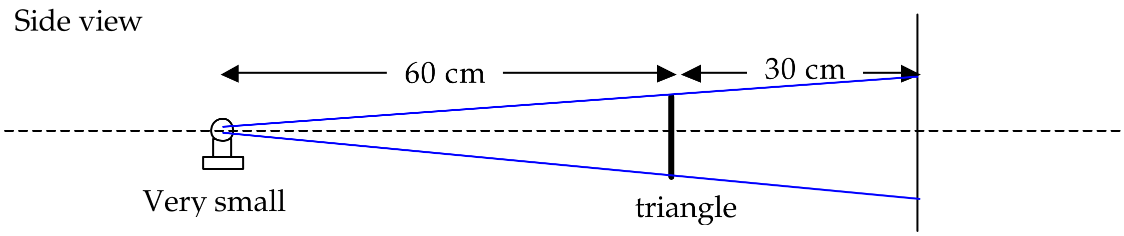


Small bulb and frosted, triangle-shaped bulb

2) A piece of cardboard has been cut into the shape of a triangle. The dimensions of the triangle are shown at right.



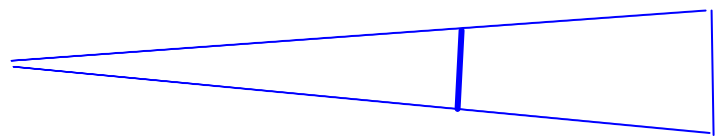
a) Predict the size and shape of the shadow that will be formed on the screen when a lit bulb, the cardboard triangle, and a screen are arranged along a line as shown. Draw a sketch to illustrate your prediction. Assume the bulb is small enough that you can consider it a point source.



Light sprays out from the bulb in all directions, but the triangle blocks some of that light from reaching the screen. The lines of light I've drawn show where the shadow begins – any light that leaves the bulb between those lines hits the cardboard triangle and stops.

So they show the height of the shadown view – this is the side view, so one line is at the top and one at the bottom. (A figure showing the view from above would look just the same! Then the lines would show the size side-to-side – one line would be on the right and one on the left.)

To find the height of the shadow on the screen, notice the similar triangles. (If you never understood high school geometry, don't worry – you can now!) When two shapes are similar, everything about them is in the same proportion. One triangle has corners at the bulb and the top and bottom of the cardboard (which is also a triangle,which could make writing about this confusing! I'll just call it the cardboard). The long sides of that triangle are the limiting rays of light from the bulb, that mark where the shadow starts. The larger triangle has corners at the bulb and the top and bottom of the shadow.



The height of the large triangle is 90 cm (the distance from the bulb to the shadow), the height of the small triangle is 60 cm (bulb to cardboard). So the large triangle is 1.5 x the side of the small one, and so the height of the shadow is 1.5 x the height of the cardboard: 30 cm. All of that works for the top view, so 30 cm is also the width of the shadow.

b) Is it possible to place the bulb in another location along the horizontal line so that the shadow of the triangle is twice as tall as in part a? If so, where? If not, why not?

Move the bulb toward the cardboard, and the lines of light. If you do, the shadow will get bigger. Using the same reasoning as before, you now want the distance from the bulb to the shadow to be 3 x the distance from the bulb to the cardboard. If the distance from the bulb to the cardboard is d, then we want $30 + d = 3d$, which gives $d = 15$ cm.

c) Suppose that the bulb were placed along the dashed line very far away from the triangle and the screen. What would be the approximate shape and size of the shadow? Explain.

If the bulb is very far away, then the lines coming from it get closer and closer to parallel. And if they are parallel, then the shadow is the same size and shape as the cardboard triangle!

d) Write a formula for the size of the shadow on the screen, in terms of how far you place the bulb from the triangle.

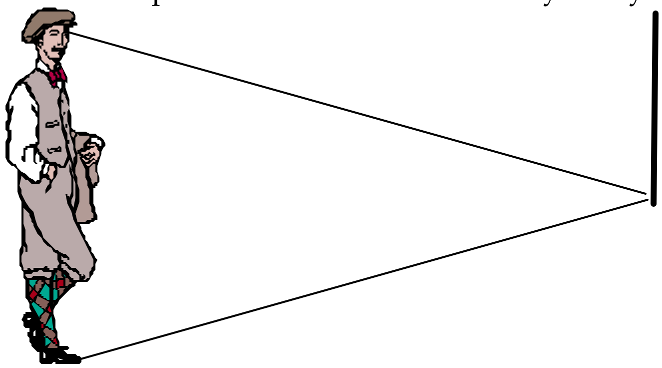
The distance from the bulb to the shadow is going to be $30 + d$, so the shadow is going to be $(30 + d)/d$ times bigger than the cardboard: $h_{shadow} = (20\text{ cm})\,(30 + d)/\,d$. Check it for very large d – does it give an answer of 20 cm? (Yes.)

3) Here's the problem I promised in lecture.

a) Stand in front of a bathroom mirror, a foot or two away, and use soap to draw horizontal lines on the mirror where you see your eyebrows and your mouth. Now back away from the mirror. Do you see more of your face between the lines, less of your face or the same?

I hope you found the same – it's not easy to do this without letting your prejudice affect how you see what's going on! But why is it the same?

b) Measure the distance between the two soap lines you drew. How does that distance compare to the distance between your eyebrows and mouth?



It should be about half the distance: You need a mirror half the size of your face to see your whole face; or half your height to see your whole body. Why? This comes right from our model. You see your feet in the mirror when some of the light from them bounces from the mirror and hits you in the eye. That light bounces out at the angle it came in, so the part of the mirror that reflects the light from your feet is exactly half way, in height, between your eyes and your feet.

c) The most important part! Most people (over 90% when I asked the clicker question in lecture) think that you see more of your face when you back away. But that's not what most people see when they try it, if they're careful. What were you (or they) thinking , and why didn't that reasoning work? When *would* it work?

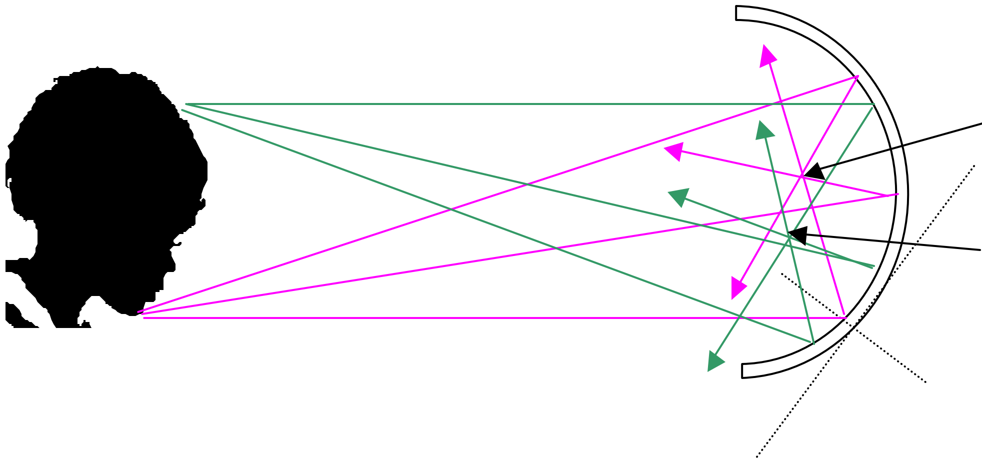
Many people try this in the bathroom where there's a sink or counter blocking part of their view. When they back up, the counter doesn't block any more, so they see more. Or when they're up close there's a part of the mirror they're not using, and backing up lets them use more of it. Or it's that they can take in more of the image at once: When you back up, you don't have to scan your eyes up and down as much to see your whole image. Up close it's harder, and if you get really close (examining a pimple?), you only notice the part you're staring at…

4) Look at your reflection in a spoon (you'll need a shiny one), or a curved mirror. (If it's curved inward we call it *concave* and if it's curved outward we call it *convex*. Use a concave mirror for this – your reflection off the *inside* of a spoon.)

a) When you look at your reflection, is it magnified or reduced? Can you account for that, using our model of light? Draw a diagram.

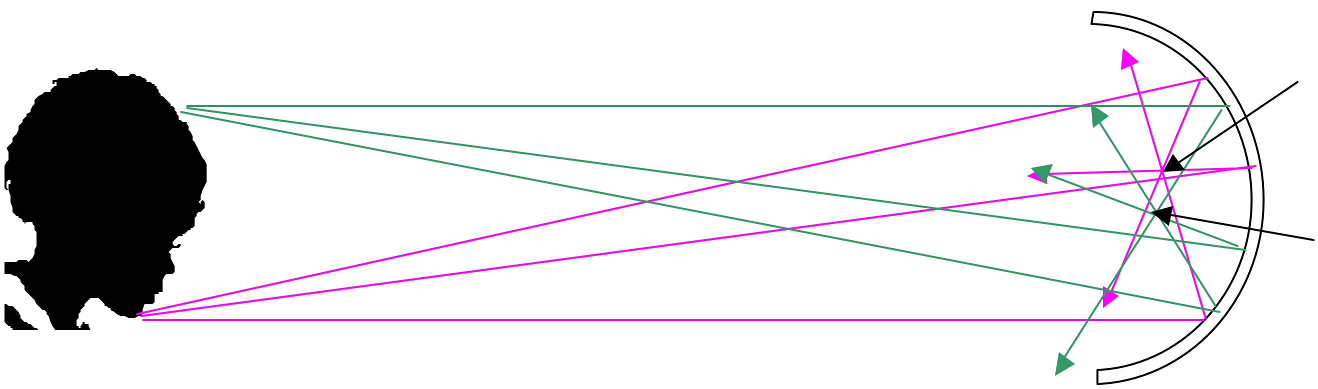
This is difficult and we didn't expect you to get it perfectly. The curved mirror makes the light bounce up if it hits the bottom of the mirror and down if it hits the top of the mirror. This is one way you might know that the image gets inverted.

To think more carefully about the inversion or the size and about where exactly the image is, we need to trace the rays. Each point of your face is like a point source of light, meaning rays go out in all directions. Looking at your chin first, we can look at a couple of rays that hit the mirror (pink lines.) These bounce off at equal angles (you can think of equal angles like a plane mirror at the tangent line.) The reflected rays all cross at one point in front of the mirror. Similarly, the rays from your forehead (green) reflect and cross at a different point below. Because this point is below, we know the image is inverted (thinks on the top have rays cross below and vice-versa). Because the points are closer together than your actual chin and forehead, the image is reduced. We also know that the image is front of the mirror.

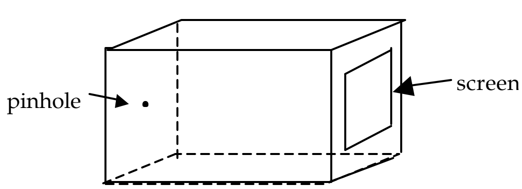


b) Move the spoon or curved mirror away. What happens to your reflection? Can you account for that, using our model of light? Draw a diagram.

The farther away you go, parallel the rays get and one is hitting the top of the mirror will be reflected downward even more and vice versa. So the image is even closer to the mirror, although still in front, and the image is smaller.



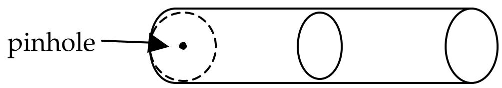
5) Make a pinhole camera.



You could do this with a small cardboard box, something like a shoebox, a pair of scissors, and some paper towel (or some other translucent sheet - wax paper would be ok, too). Poke a hole with a pin (or a pen tip can work) on one side, and cut a screen on the other side.

To use that version, cover your head and the screen end with a blanket, and point the pinhole end at some well-lit object. You could also put the screen in the middle of the box, leaving just a little hole to look through at the end – that's more pleasant than covering your head with a blanket! The idea is to make it dark around the screen, so that most of the light that hits the screen is the light from the pinhole.

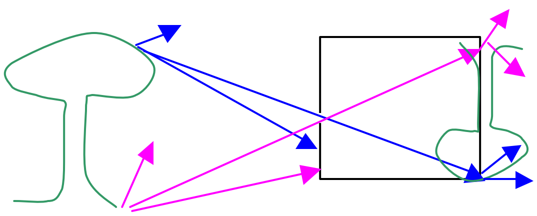
Or use a cardboard tube – this time I'm showing the screen in the middle of the tube.



However you make it, you may need to play around with it a bit to get it to work.

For the assignment you hand in, draw a diagram of the pinhole camera that explains how it works.

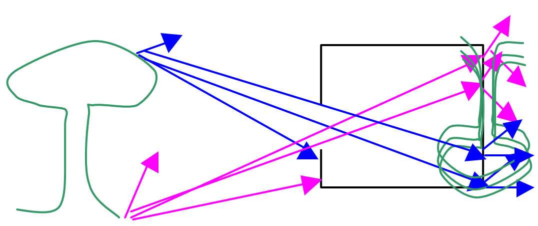
This works just like the frosted bulb in the tutorial. The object you are looking at can be thought of as a bunch of point sources of light. Each of those point sources has lots of light going in all directions, but only one direction allows the light to pass through the pinhole and hit the screen. Then the screen scatters the light so it can get to your eyes from wherever you are looking. But each point on the screen only has light from one point on the object, so you get an image, which is inverted (and smaller.)



6) Now, use your explanation of how the pinhole camera works to make some predictions. Test your predictions with your camera, and try to reconcile any inconsistencies.

   a) What do would be the result of widening the pinhole? Will it make the image brighter? Bigger? Sharper or blurrier?

Widening the pinhole lets more light through so the image would be brighter. But now it will also be blurrier because the light from each point on the object is reaching more than one point on the screen. It's like instead of making the image with a ball point pen, you are making the image with a big round magic marker. It's got more ink on the page, but the picture isn't as sharp. The size of the image doesn't really change, though.

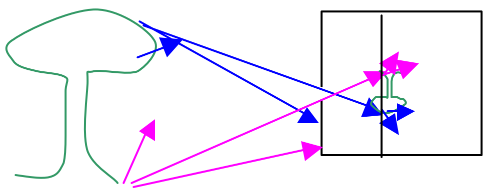


b) What would be the result of changing the shape of the pinhole – e.g. using a triangular hole as in tutorial, or a star shape, etc?

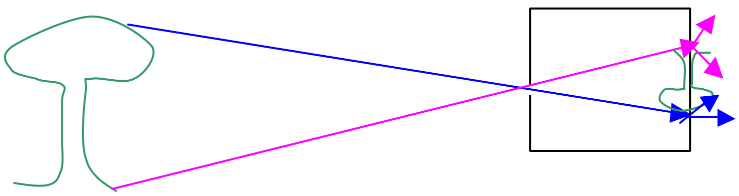
Just like in above, blurrier than the pinhole and brighter, but still tree-shaped, inverted and about the same size. Now you are trying to draw a tree with a triangle-shaped or star-shaped magic marker and it's just not as sharp. [One thing I noticed in course center was that folks were thinking about the specific shape of the funny-shaped pinhole while not thinking about the round shape of the bigger pinhole. So some said more light and blurry for a and overlapping triangles for b. If you did that, think about why you might have thought about the shape only if it was funny-shaped and more about the rays of light explicitly when it was round. Might you still be doing some "remember the rule" thinking?]

c) What affects the size of what you see on the screen — e.g., suppose you're using the pinhole camera to look at a 5 foot tall window (the *object*). What affects how big the window looks on the screen?

How far away the screen is from the pinhole matters. Look how big the object is on a closer screen.

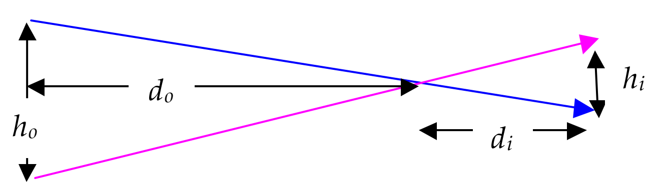


Similarly, if you move the camera farther away from the object, it gets smaller.



d) Formalize that last part. If you're looking at an object of height h_o that's a distance d_o away from the camera, what would be the height h_i of the "image"[1] you see on the screen. Call the distance from the screen to the pinhole d_i. (Start with a diagram!)

The rays at the top and the bottom form two triangles that are similar. Thus the ratio of h_i to d_i is the same as the ratio of h_o to d_o. Thus, $h_i = h_o d_i / d_o$. This makes sense, because if the object gets taller, so does the image. If the object gets closer the image gets taller and if the screen gets farther the image gets taller.



[1] In quotes, because we're going to define "image" in a technical way, and by that definition this won't count as an image. Don't worry about this for now.