## Assignment 7 Due April 3, start of class

Reference: Cutnell \& Johnson, 16.1-16.3, 16.5 (beginning waves and sound). And/or see the links on the web pages, to chapters from Prof. Redish's book.

1) Think of two or three different types of waves (at least one should be something not from class.)
a) For each of these, make a list of things that you think should affect how fast the wave propagates (travels) and a list of things that you think shouldn't affect the propagation speed.
a) I don't think we talked about ocean waves in class! Things that might affect how quickly:

- the weight of the water (salt water is a bit heavier than fresh, so I'd guess salt water waves move a bit more slowly);
- the depth of the water (thinking of surface waves, this is just something I happen to know - when the water is shallow compared to the size of the waves, the bottom plays a role in how fast waves travel...)
- the pressure (now I'm thinking of waves deep undersea - not surface waves - say the ripples of pressure a fish makes as it swims, and the higher the pressure, the faster the wave moves).
Things that shouldn't affect the speed:
- the amplitude of the wave - height for a surface wave, maximum strength for a pressure wave.
- the duration of the wave - how long the pulse lasts.
b) "Amber waves of grain." I'm thinking of the ripples you see in a field when a gust of wind blows by - you see can see the "pulse" travel along. Well, now that I think of that, I'm not sure it's really a "wave": The air is actually moving, and what you're seeing is the wind pushing on the grass. On the other hand, it's a pulse of wind strength traveling, and maybe that's a kind of disturbance traveling. Back to the first hand, if I think of a bit of dust caught in the gust, the dust could keep moving along with it all the time. So, in the end, I don't think that's a good example of a wave.
c) An earthquake: I'm thinking of how the shaking propagates from the epicenter of a quake. When I lived in California, people talked about being able see the ripple move along the ground, if they were in an open space. Things that might affect how quickly:
- The density of the rock/ground the wave is moving through. More dense $\rightarrow$ slower wave.
- The firmness of the rock - more firm $\rightarrow$ faster wave. (The rock is more tightly connected, so motion in one place has a quicker effect on motion in the next place.)
Things that shouldn't affect the speed:
- again, I'd think the amplitude and duration of the pulse shouldn't affect how quickly it travels through the ground.
b) Take the things in these two categories and re-sort them into two different categories, labeled "properties of the disturbance (or the pulse)" and "properties of the medium." They don't seem to make for different lists! In general, properties of the disturbance wouldn't affect the speed; properties of the medium (depth of water, density of rock, etc) would.
c) Are the two sets of categories the same? Does looking at list (b) make you want to rethink any thing about your sorting for (a)?
It doesn't for me, but I knew what was coming...

2) A student has a rope tight to a wall, and flicks her wrist to make a small pulse as shown in the figure. The rope has a red dot painted on it. For each of the following, try to come up with a few different ways,
a) What could she do to make the pulse get to the wall more quickly?
Use a lighter rope; hold it so that there's

more tension. (Make it so that it takes
less time for the next piece of rope to react.)
b) What could she do to make the pulse wider?

Flick her hand more slowly, so the pulse lasts longer; or do either of the things in part a (so the front of the pulse has moved further down the string before she's made the back of the pulse. Remember why we said $\ell=v t$, the pulse length depends on the wave speed and the flick time. )
c) What could she do to make the red dot move more quickly?

Make a bigger (higher) flick in the same amount of time (so the dot moves up and down a greater distance); make the same sized pulse in a shorter time (so the dot moves up and back in a shorter time). I was hearing a lot of arguments in course center about wave speed, but these all eventually went back to something different about the flick. Notice that the red dot is going to do (a little bit later) what the hand does. How soon it does that depends on the wave speed, but what the motion looks like is exactly the same as the hand's flicking motion. If this is not obvious to you try to figure out why it must be true.
3) In lecture and tutorial we've talked about the fact that when these two pulses pass each other the spring would become flat.

a) Explain, in your own words and with a diagram why the waves reappear after the string is flat. (Yes, I gave an explanation in lecture - but I wasn't sure how many people "got it.")
The string is in a flat position, but it's only passing through that position for an instant. Particular piece of the string have a velocity up or down, and in the next instant those parts of the string will have moved.


A lot of folks were having trouble thinking about the individual bits of string moving because it is so hard not to think of how the waves would have moved. But the pulses have disappeared, only the energy remains in the form of kinetic energy. That means actual physical stuff moving, not an effect. KE is always $1 / 2 \mathrm{mv}^{2}$, and that means only real things with mass can have KE.
b) A puzzle to try to reconcile: According to the rule from tutorial and what we said in lecture, the piece of spring right in the middle never moves. But if it never moves, how can the waves go through it? Shouldn't it have to move for the waves to pass by? In fact, if it never moves, it shouldn't change anything if we were to tie it down at that point, and then the waves would have to stop there, wouldn't they?
Of course you know that the waves don't stop there, but how do you respond to this thinking?
The piece of string at that point doesn't have to move to be under tension - it's just that the tension has to be the same, equal and opposite, from both sides for that piece of string never to have a net force on it. It can still be part of transferring the tension from one side to the other.

Now, if it were tied, say, to a wall, then no, it wouldn't transfer tension from one side of the string to the other. But it would do something else interesting: It would return the tension you put on it, because the wall will pull back on that point just as hard as you pull on it. It would be as if there's an exactly opposite pulse coming in from the other side! In fact, if you tie one end of a string to a wall and send in a pulse, what happens is the pulse reflects and comes back from the wall upside-down, just as if that tied point to the wall were the center point between opposite pulse.
4) At the time $t=0$, a string has the shape shown below. The pulse on the left is moving towards the right and the pulse on the right is moving towards the left. Each box in the grid has side of 1 cm .
a) The leading edges of the pulses will meet at time $t=0.05$ seconds. What is the speed at which each pulse is traveling?
They're 6 cm apart, and they'll meet in 0.05 seconds. So they're approaching each other at a speed of $6 \mathrm{~cm} / 0.05 \mathrm{~s}=120 \mathrm{~cm} / \mathrm{s}$. That means each of the pulses is moving at a speed of 60 $\mathrm{cm} / \mathrm{s}$.
b) Two points on the string are marked with heavy black dots and with the letters A and B. At the instant shown, what are the velocities of the dots? Give magnitude and direction (up, down, left, right, or some combination of them).


Dot B is moving up. (Think about where the pulse will be a fraction of a second later - it will have moved to the right. That means B moves up and A moves down. Eventually the peak reaches B and after that it starts to move down. ) At the moment shown B is in the middle of moving up from the zero line to the peak at 2 cm , and the time it takes to do that is the time it takes the pulse to travel forward 3 cm :
$(3 / 120) \mathrm{sec}=(1 / 40) \mathrm{sec}$. So B is moving $2 \mathrm{~cm} / .025 \mathrm{~s}=80 \mathrm{~cm} / \mathrm{s}$.

Dot A is moving down. It's in the middle of returning from the peak of the pulse to the zero line, and the time it takes to do that is the time it takes the pulse to travel forward 1 cm : $1 / 120 \mathrm{sec}$. So A is moving more quickly than B, a speed of $240 \mathrm{~cm} / \mathrm{s}$.

Does this make sense? Remember, the dots on a string have to make the same movement as your hand does to make the pulse. You just figured out that a dot will move up slowly, like B, then down quickly, like A. Does it make sense to you that flicking your hand slowly up and faster down would make a pulse like this?
c) Draw what the string would look like when the two waves are on top of each other - the leading edge of each wave lined up with the trailing edge of the other.


This is just applying the rules you learned in tutorial - go point by point, and add how much each wave "tells" the string to move from the zero line - positive for up, negative for down. The thin dotted lines show what the waves each would do alone. So, e.g., look at the point that would be the top peak of the wave coming in from the left. That waves tells the
string to go up 2 cm . At that same place, the wave coming in from the right tells the string to go down a bit less than 1 cm . ( $2 / 3$ of a cm , to be precise) So the sum is a bit higher than 1 cm (at $11 / 3 \mathrm{~cm}$ ) for the peak of the overlapped waves.
5) In class we found that mass of the bars had an effect of the wave propagation speed. This is also true for a wave pulse on a stretched spring like the ones you used in tutorial. Assuming you can make two springs with different masses but the same stiffness and put them under the same tension, the pulses will travel faster on the lighter one. ("Lighter" means that the spring has a smaller linear mass density, $\mu$, which is the mass per unit length. Thus for the same length of spring, there is less mass.)

In tutorial we found that the pulse traveled faster if you stretched the spring more, that is, applied more tension. Considering this and your answer to part (a), come up with a formula that relates the wave speed to the mass density, m , and the tension T . (The waves speed formula should give you the units of speed, so if you need to add a constant for units, state this and what the units of that constant are.)

So you want a formula for velocity $v$ that gets bigger if T goes up and if $\mu$ goes down. $v=\mathrm{T} / \mu$ would be a reasonable guess, based on that thinking.

We could do better: The units don't work out in that - Tension is in units of force, Newtons, and $\mu$ is in mass/ length. Newtons / (mass/length) has dimensions of (velocity) ${ }^{2}$, so, just thinking of dimensions, we might suppose that it's $v^{2}=\mathrm{T} / \mu$.

Another way to fix the dimensions problem would be to have a constant in the formula, with dimensions of $1 /$ velocity. Then we could suppose it's $v=($ constant $) \mathrm{T} / \mu$.

If you're solid on the ideas from 121 (optional reading here!): $v=\mathrm{T} / \mu$ would say that doubling the tension should give you double the velocity. But if you double the tension, for the same size of pulse, you double the force exerted for the same displacement, which means you double the work done. That means twice the energy, not the velocity; energy scales with $v^{2}$. So that's another reason to guess $v^{2}=\mathrm{T} / \mu$, which we could show is actually the correct formula.
6) Suppose it takes $t$ seconds for me to flick my wrist to make a single pulse on a slinky that is, suppose the time t goes by from when I start my hand moving to when I stop my hand from moving. And suppose the speed of the wave on the string is v. Write a formula for the length of the wave pulse I make along the string.

If the front of the pulse is moving at a speed $v$, then it travels a distance $v T$ during that time, T , so the length of the pulse must be $\ell_{\text {pulse }}=v t$.

Very often we think of sinusoidal waves, like the periodic wave I drew in class. In that case the time between pulses is the period, $\tau$. Actually, people more often use $f=$ frequency $=$ the \# of cycles per unit of time, measured in Hertz = cycles per time. But that's just $1 / \tau: f=1 / \tau$. (Right? Check that! If the period is $1 / 40$ of a second, how many cycles is it per second?) So then the formula you figured out, $\lambda=v \tau$ would just be $f \lambda=v$.

