Physics 122, sections 502-4 and 8101 **Assignment 6**
Laura Lising **Due March 20, start of class**

Reference: Cutnell & Johnson, 19.4-19.7 (electric potential, capacitance), 16.1-16.3, 16.5 (beginning waves and sound).

1) In class we decided that since capacitance of a parallel plate capacitor increases with area (A) and decreases with distance (d) between them, a formula for it could be of the form C=(constant)A/d. The constant is called ε_0, and is about 9 x 10^{-12} Farads/meter. So $C=\varepsilon_0 A/d$.

a) Estimate the capacitance of two pie plates held at a distance of 1 cm from each other.

A pie plate is around 10 cm = 0.1 m in radius, so the area is about πr^2 = 0.03 m^2. So the capacitance is about $C = \varepsilon_0 A/d = \varepsilon_0$ (0.03 m^2/0.1 m) = 3 x 10^{-12} Farads. That's not a whole lot of capacitance. Most useful capacitors have more area (often rolled up like I showed you) and very small spacings.

b) What would be the effect of the capacitance of putting Styrofoam plates between the pie plates? (I heard some of you guys making the argument that the fact that the Styrofoam cup in the Leyden jar problem had something to do with the ease with which it held charge.

First think back to what happens when we bring the plates closer together. This increases the capacitance. One way to think about the increase in capacitance from bringing plates closer together is that the opposite charges on the two plates attract each other more strongly and thus it is easier for more charge to build up because although the like charges on the plate are repelling, the opposite charges on the other plate are attracting and this effect is stronger the closer the plates are together. With the same potential difference from a battery, more charge could get on the plates. Putting an insulator made up of dipoles has a similar effect, because the charge on the plates can pull the opposite charged ends of the dipoles toward it, pulling more charge on to the plate.

c) The dielectric constant for insulators is a constant that takes into account how well an insulator can be polarized. The constant gets bigger if the insulator has bigger dipoles (ex, more separation between the charges in the dipoles.) This constant is a unitless number that is 1 for an insulator that really doesn't polarize, and gets larger for more polarizable insulators. You could modify the equation for a capacitor to take into account the dielectric constant of whatever material separates the two conductors of a parallel-plate capacitor. Considering what you said in part b, would you multiply or divide the right side of the equation above by the dielectric constant? Why?

More dipoles or more separation for the dipoles means more of the effect that increases capacitance. Thus, it seems to make sense to multiply to get C = (dielectric constant)ε_0A/d. So, when designing a capacitor, if you want to make a smaller one with a bigger capacitance, you can choose a dielectric with a big constant.

d) Think back to the membrane in previous problems. We said the thickness of the membrane was 8 nm and the capacitance was 10^{-4} µF. On what size area of the membrane is charge collecting? Does this make sense to you? Why or why not? (The dielectric constant of membranes is on the order of 10. How does that change your answer. Does your result make more or less or the same amount of sense to you now?)

10^{-4} μF = (10) ε_0 A/8nm. Solve for A = 10^{-8} m^2. If that were a square, it would be about 10^{-4} meters on a side, or about 0.1 mm, which is smaller than the 0.3 mm that we would calculate without the dielectric constant. Still that's a pretty darn big cell! Hmm, I wonder if that 10^{-4} μF is too large?

2) The AA batteries in a walkman might last about 20 hours when playing tapes, and the player says (in very tiny print!) that it uses a maximum of 0.15 Amps of current.

a) Estimate the potential energy stored in an average 1.5 Volt AA battery.

If it plays 0.15 amps for 20 hours, that's
0.15 Coulombs/seconds x 20 hrs x 3600 seconds/hour = 11000 Coulombs.
That much charge went down a potential difference of 1.5 Volts = 1.5 Joules/Coulomb.
So it's a total of 11000 Coulombs x 1.5 Joules/Coulomb = 16000 Joules

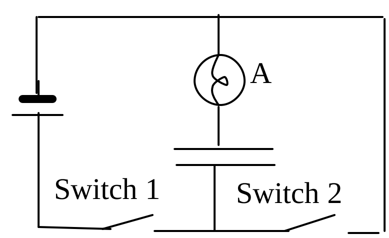
b) How high off the ground must a AA battery be so that its gravitational potential energy equals its stored electrical potential energy?

Well… we need to estimate the mass of a battery. I'll guess 50 g (I'm thinking a package of 20 feels like it could be a kilogram). Then mgh = 16000 Joules gives me about 30 kilometers! Does this surprise you? The electric force is so much stronger than the gravitational force that it is easy to store more energy in a small mass.

c) Estimate the energy costs from your average 1.5 volt AA battery compared to energy from wall socket. In other words, how much does it cost for say, 1000 Joules of energy from batteries versus from the wall. Wall socket energy costs about 10 cents a kilowatt hour.

If a AA battery costs, say, $1.60 (easy number for estimation!), then that's 10¢/1000 Joules. For the wall socket, 1 kilowatt-hour (check an electric bill, that's the unit of energy)
= 1000 Watts x 1 hr = 1000 Joules/second x 3600 seconds = 3,600,000 Joules for that same dime.
Batteries are very expensive energy!

3) The circuit at right contains a battery, a bulb, a switch, and a capacitor. The capacitor is initially uncharged.



a) describe the behavior of the bulb in the two situations below.

i) Switch 1 is closed. Describe the behavior of the bulb from just after the switch is closed until a long time later. Explain.

When you close switch 1, making the connection, the circuit is the same as one we talked about in lecture with a battery and a capacitor, this time with a resistor in series. It's also the same as that situation with the two metal boxes, which together make kind of capacitor, and it's similar to the question from one assignment with the two aluminum pie plates.

So: At the start, there's no charge on the capacitor. When you make the connection, current flows to give one plate of the capacitor a negative charge and the other a positive charge, until the potential difference across the capacitor is the same as the potential difference across the battery. In order to flow, the current has to go through the resistor – if it were a bulb, we'd see it light for a little bit and then get dimmer and go out.

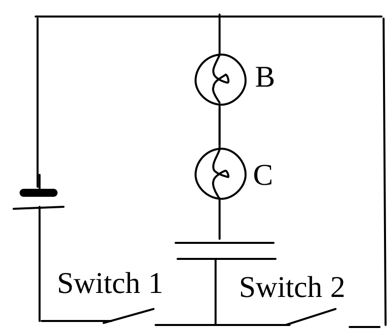
In class we discussed hooking up a capacitor to a battery without a bulb and it was obvious that the capacitor reached the same voltage as the battery by our "conductors touching have equal voltage" rule. It is not as obvious with the bulb. The bottom of the capacitor will obviously have the same voltage as the bottom of the battery, and the top must too, because the resistor must have the same voltage on both sides OR CURRENT WOULD FLOW THROUGH IT. Thus we know that when the current stops flowing the voltage across the capacitor is equal to the voltage of the bulb. We could also have gotten this by doing a Kirchoff's loop. +(Battery voltage difference) -IR - (Capacitor voltage difference)=0 implies the battery and capacitor voltage differences must be the same when the current finally goes to zero.

ii) Switch 1 is now opened and switch 2 is closed. Describe the behavior of the bulb from just after the switch is closed until a long time later. Explain your reasoning.

Opening switch 1 disconnects the battery; closing switch 2 connects another wire that would allow current to flow from one plate of the capacitor to the other, again through the resistor. That happens until the capacitor plates return to neutral. This is just like the demo I did in lecture with a light bulb: I charged a capacitor, then connected a bulb which lit for a while.

How long would the bulb light during discharge compared to during charging? Well, we know that it's how big the resistor is and how big the capacitor is that matters. If there were no resistor during charging, the charging would happen immediately. But the resistor slows the process down with a time constant RC. It's the same time constant for both charging and discharging, so if you were just looking at the bulb, you'd have no way of distinguishing charging from discharging.

b) a second identical bulb is now added to the circuit as shown. The capacitor is discharged.



i) Switch 1 is closed and 2 left open. Describe the behavior of bulbs B and C from just after the switch is closed until a long time later. Explain. How does the brightness of bulb C compare tot he brightness of bulb A in question i of part a? A long time after the switch 1 is closed, is the potential difference across the capacitor greater than, less than, or equal to the potential difference across the battery?

When the switch is first closed the current through the bulbs is half the current through bulb A in part a. This may seem obvious - twice the resistance, half the current. But if you are having trouble thinking about this with the capacitor there, go back to Kirchoff's loop. +(Battery voltage difference) -IR -IR- (Capacitor voltage difference)=0. When the switch is just closed, there is no charge on the capacitor and thus no voltage drop, so I=(battery voltage)/2R, just the same as if there was no capacitor there. But this current immediately starts to drop as the capacitor charges up, although the current lasts longer than that in bulb A because there is less current = slower charging.

So, the bulbs are dimmer but light longer and in the end when there is no current the capacitor voltage difference equals the battery difference just like in a.

ii) Now switch 1 is opened and 2 is closed. Describe the behavior of bulbs B and C from just after the switch is closed until a long time later. Explain. How does the initial brightness of bulb C compare to the initial brightness of bulb A in question ii of part a? A long time after the switch is closed, is the potential difference across the capacitor greater than, less than, or equal to the potential difference across the battery?

Again, the bulbs look exactly the same during discharge as they do in charging, dimmer than A but lasting longer. (Note, if wire on the right connecting the bulbs through switch 2 had a resistor or another capacitor or battery, the discharge would be different! Discharges aren't always the same as charges; they just are in this case since R and C are the same….) At the end, the capacitor gets completely discharged and has zero voltage difference.

4) In class we decided that a capacitor discharges more slowly if it has a large capacitance (easily keeps it's charge) or if it is discharging through a large resistance. This gave us a time constant τ = 1/RC. Oops, that τ = RC!

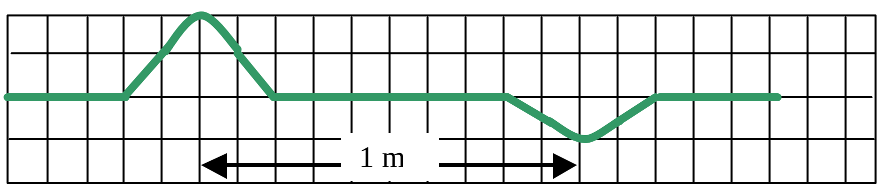
a) If it takes about a millisecond to discharge the membrane, what is the resistance of the ion channels through which the sodium ions travel?

.001 sec = R(10^{-4} μF) gives us R= 10 MΩ.

b) A defibrillator requires a pretty big current to be delivered for a very short time to a the heart of a victim of cardiac arrest. . The defibrillator is a large capacitor that is charged up and the disconnected from the power supply when it's ready. It is then connected to the patient's chest with the kind of electrolytic gel that you measured in lab. Explain why a capacitor is used rather than a direct connection to a power supply and why it is important to use low-resistance gel.

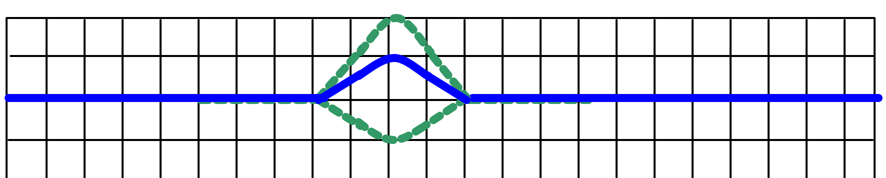
Okay, so it needs to be for a short time, largely because you want to deliver a limited amount of energy to the patient. You don't want to kill the patient after all so you certainly wouldn't want to hook him/her up to the wall power! A charged capacitor that's disconnected has a limited charge. However, it needs to be enough current to deliver the shock, so you want the smallest resistance you can get and thus need the electrolytic gel. For a smaller resistance, you get a larger current AND a smaller time constant since τ = RC.

5) This figure shows two wave pulses of different amplitudes at time t=0. The pulses are moving toward each other, each traveling at a speed of 10 m/s. At t = 0 seconds, they are 1 meter apart.



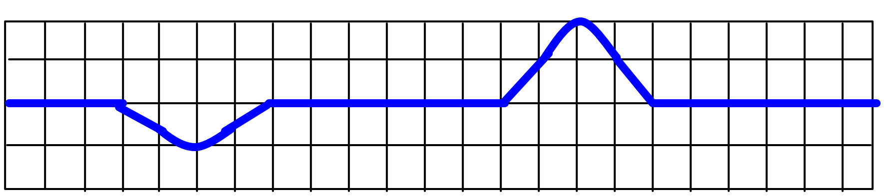
a) Sketch the shape of the string at time t = 0.05 seconds.

The pulse moves at 10 m/s so since each square is .1 ms, it travels 1 square each 0.01 seconds. And by "travel" we mean the whole thing travels, which means each point travels. If the edge is at square 7, after 0.01 seconds it is at square 8, and if the peak is at square 5, after 0.01 seconds it's at square 6. So after 0.05 seconds the two pulses (dotted lines) would be completely overlapping and would combine as shown.



b) Sketch the shape of the string at t = 0.1 seconds.

Another .05 seconds later they have passed each other completely.



c) Think of the point on the string exactly halfway between the pulses. Describe its motion —try to be precise — from t = 0 until after the wave pulses have passed each other.

It starts from not moving, then as the pulse from the left reaches it so does the pulse on the right. The pulse on the left causes an upward force on that point on the string (because the piece of string just to the left is now up and tugging that way). The pulse on the left causes a downward force (because the piece of string on the right tugs that way). The tug upward is stronger than the tug downward, so the point on the string still accelerates upward, but at a smaller acceleration than it would by the pulse on the left alone. It moves upward, and when it's got more of a tug downward than upward it starts to slow, until it reaches a maximum height (at the peak of the pulse) where its velocity hits zero, and the point on the string starts to move downward. When there's more of a tug upward again, it starts to slow, until it comes to a stop—both pulses have passed.

Notice that we can look at the width of each pulse, 0.4 meters, and from the fact that the pulses travel 10 meters per second we can figure out that from the instant the two pulses reach the center point of the string it take 0.02 seconds for it to reach it's maximum height of 0.1 meters, and another 0.02 seconds to come back down.

6) Suppose you were to place a candle in front of a loudspeaker, and crank the volume up good and high. (Of course, make a prediction first, if you decide to try it.)

a) What would you expect to see the candle flame do? Would it bend away from the speaker? Toward it? Neither?

I expect the flame to flicker in place, not bending toward or away from the speaker. The sound from the speaker makes the flame (all the air, really) vibrate back and forth, but it's as much back as forth, so the flame shouldn't have any overall motion toward or away from the speaker. Maybe I'll be able to see it vibrate, if the sound is loud enough, and at a low enough frequency that the vibrations would be visible?

b) What other answer might someone give to part a, and how would they argue for it? (If you can't think of any, you might need to talk to other people!)

People often expect the candle flame to bend away from the speaker, and maybe blow out, as if there were a breeze. They will say that the sound is moving outward, which it is, of course, and moves the candle flame with it.

c) Can you respond to the reasoning in part b? Why doesn't it work? (When *would* it work, and how is this situation different?)

Sound IS moving, of course, but the air is not moving one way more than another. It's the same way that when you are out far from the beach bobbing up and down on a raft, the waves are obviously going to the beach, but you and the water around you are just moving up and down. Sound is like the water wave, the disturbance, not the water, the stuff, the air.
The reasoning would work if the speaker were a fan that blew air rather than a speaker. Speakers are like big flexible sheets that move in and out; they push out and then pull back in rapidly. This is unlike a fan which just pushes out continuously.

Actually, if it's a huge, very loud speaker, maybe the back and forth vibration could be enough to blow the candle out –? For a large enough amplitude of pushing in and out, maybe the back or forth would be enough to blow the candle out.