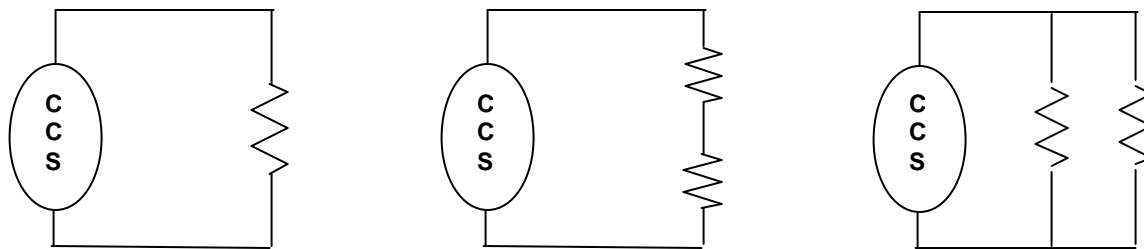


Reference: Cutnell & Johnson, 19.4-19.7 (electric potential, capacitance)

1) We have studied batteries, which provide a fixed voltage across their terminals. In that case, we had to examine our circuit and use our physical principles in order to calculate the current through the battery. In neuroscience, it is sometimes useful to use a constant current source (CCS), which instead provides a fixed amount of current through itself. In this case, we have to use our physical principles in order to calculate the voltage drop across the source.

(a) Suppose we have a fixed current source that always provides a current of $I_0 = 10^{-6}$ amps. For the three circuits shown below, find the voltage drop across the current source. Each resistor has a resistance $R = 2000 \Omega$. (If you prefer, you may leave your answer in terms of the symbols I_0 and R .)



We assume ideal resistanceless wires and perfect ohmic resistors, as usual.

In the example on the left, we have a given current and a given resistance. Across the resistor $\Delta V = I_0 R$. Since the resistor and the CCS are connected by conductors, the voltage drop across the resistor is equal to the voltage increase over the CCS.

In the middle example, the same current will flow through two resistors, making the total voltage drop across the two twice as big as in the left example. Thus $\Delta V = 2I_0 R$.

In the right example, the current from the CCS will split evenly into the two resistors. Each will get $I_0/2$ and will have a voltage drop of $I_0 R/2$, which will be the increase over the CCS.

(b) Constant voltage sources (batteries) get into trouble if their terminals are connected through a very low resistance (short circuit). Constant current sources get into trouble if their terminals are connected by a very high resistance (open circuit). Explain why and explain what “get into trouble” means.

A battery provides a constant voltage and adjusts the current it produces to maintain that voltage difference. If a battery has a low resistance wire connected across its terminals it will try to generate $I = \Delta V/R$. If R is very small, a very large current will be drawn and the battery will discharge quickly since the rate at which the battery uses up energy is $P = I\Delta V$. (Also, although the resistance is tiny in both the wires and the battery itself, because the current is so high the power is very high as we've just said, and both the battery and wires will get very hot.)

A CCS provides a constant current and adjusts the voltage it creates to maintain that current. If a CCS is connected to an open circuit (equivalent to a very high resistance - it would have to jump a

spark through the air) it tries to produce a fixed current through that very large resistance. Thus it generates a voltage $\Delta V = I_0 R$. If R is very large then the voltage it needs to produce is very large, which it just can't do.

2) In this class we have introduced the concept of *electrostatic potential*. Define what this means, explain its relationship to potential energy, and give an example.

The electrostatic potential is just the amount of potential energy per unit charge. In the same way the electric field is not the force but the force a charge *would have* if put at that point in space, the potential is the potential energy a charge would have at that point in space. It's all about describing what you know about that point in space, taking into account where you know there are charges nearby. It's just like gravitational potential. You know intuitively that an object placed at a greater height *would have* more potential energy than an object placed at a lower height. You don't know exactly how much potential energy it would have, because that depends on the mass of the object, but you know it would be more. You know this because you know about gravitational forces and that height means farther from this large massive object, the earth. If you had a big charge out in space, you'd have more potential going farther away from it, because you know that a charge placed farther away would have more potential energy, proportional, of course to the magnitude of its charge. If the charges are opposite, for example, this means that the farther away you go, put it, and let go, the faster it will be when it reaches the big charge. (The force on it farther away is less, but still, it gains some kinetic energy going from the weak force region to the high force region where it picks up even more speed...)

3) A positive charge is placed at rest at the center of a region of space in which there is a uniform electric field. (A uniform field is one whose strength and direction are the same at all points within the region.)

a) When the positive charge is released from rest in the uniform field, what will its subsequent motion be?

The charge will feel a force in the direction of the electric field ($F = Eq$) and thus it will accelerate with a constant acceleration. (Just like releasing an object above the earth's surface - constant acceleration, at least until it hits something.)

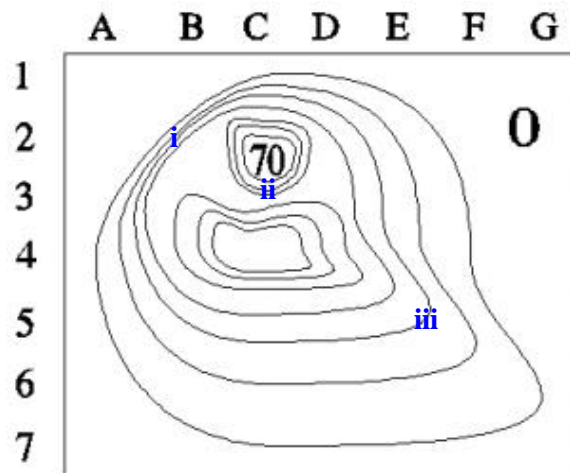
b) What happens to the electric potential energy of the positive charge, after the charge is released from rest in the uniform electric field?

The potential energy decreases, converted to kinetic energy.

4) (a) The figure on the right shows a contour plot of a piece of a range of hills in Virginia. The outer part of the figure is at sea level (marked 0). Each contour line from the region marked zero shows a level 10 m higher than the previous. The maximum height is 70 m and is shown by the number 70.

Answer the following questions by giving the pair of grid markers (a letter and a number) closest to the point being requested.

i. Where is there a steep cliff?



Oh, I should have just asked you to mark it on the plot. There are a few place; one is where I put an "i." It's where a small horizontal motion along the terrain makes for a big change in height.

- ii. Where is there a pass between two hills? The ii.
- iii. Where is the easiest climb up the hill?

The iii – that looks like the shallowest slope to me – the most gradual change in height.

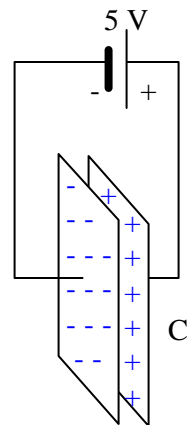
(b) Now suppose the figure represents instead a plot of the electric equipotentials for the surface of a glass plate. (Equipotentials are the places where the potential is equal.) The numbers now represent voltage. The maximum is 70 V and each contour line from the region marked zero shows a level 10 V higher than the previous.

- i. Where would a test charge placed on the glass feel the strongest electric force? In what direction would it point?

Same place as the i for the first part, where there's a steep cliff. That's where a small change in position makes for a large change in potential, and $\Delta V = E\Delta x$ That is, big change in potential for small change in $x \rightarrow$ big force per unit charge = E.

Is there a place on the glass where a charge could be placed so it feels no electric force? Where? Looks like at ii. At that point, you can walk in any direction, just for a few steps, and not be going uphill or downhill. (Well, we can only tell to the precision the plot gives us, in steps of 10.) Similarly, a charge can move a small distance and not change potential.

5) Suppose you have a battery in a circuit with a battery and a "capacitor" that is made of two metal plates, parallel to each other and somewhat close, but unable to pass a charge between them. (Perhaps they are glued to opposite sides of a very good insulator.)



- a) What is the voltage difference ΔV between the two plates of the capacitor? The two plates of the capacitor are each connected by conductors to the sides of the battery, so the voltage difference across the capacitor must equal the voltage difference across the battery, 5 V.

b) Draw in the what the excess charges on the capacitor plates will look like.

- c) If we say the two excess charges are Q and -Q, we can define a value called the capacitance C that is related to Q and ΔV . If I told you that C was either defined as $\Delta V/Q$, $\Delta V/Q$, or $Q/\Delta V$, which would you say made more sense and why?

I like to think of this by thinking about Q, and I think $Q=C\Delta V$ makes sense. Higher voltages across the capacitor is like a higher pressure pushing the charge on leading to a higher Q. So ΔV must be in the numerator of my equation for Q. And the more capacitance something has, the more charge it can hold, so it also must be in the numerator. $Q=C\Delta V$ rearranges to $C=Q/\Delta V$.

d) Alright, now I'm telling you that $C = Q/\Delta V$ and capacitance is measured in Farads = Coulombs/Volt. If this is a 100 microFarad capacitor, find Q.

Just $Q = CV = (100 \mu F) (5 V) = 5 \times 10^{-4}$ Coulombs.

e) Explain: Why is it called “capacitance”?

It’s the “capacity” for holding charge – the higher the capacitance, the more charge it holds for a given potential difference.

f) Does the capacitance get larger or smaller as we bring the plates closer? Why? What about making the plates bigger? What effect will that have on C?

As we discussed in class, the closer the plates are the "happier" the like charges are squishing onto one plate, because the opposites are nearby. You can also think of the forces balancing. The likes on the plate repel, but the opposites nearby help pull more on. The farther the other plate is away the more this effect is reduced.

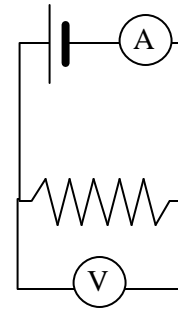
For bigger plates it's obvious: bigger plates are easier to put charge on because you don't have to squish them as much.

g) That cell membrane in the last problem set, question 2, has a capacitance of $10^{-4} \mu\text{F}$. And the potential difference from one side to the other is $70 \mu\text{V}$. How many sodium ions would have to move across the membrane to change the potential difference from $70 \mu\text{V}$ in one direction to $30 \mu\text{V}$ in the other direction (i.e. from $+70 \mu\text{V}$ to $-30 \mu\text{V}$)?

You could take this in two steps, if you want: How much charge is on the capacitor (membrane) for the first voltage, and then how much is on it for the second, and then find the change. (Just notice that the signs of the two sides have to switch, and keep that in mind in finding the change.)

So $CV = Q$: $(10^{-4}\mu\text{F})(70 \mu\text{V}) = 7 \times 10^{-15}\text{Coulombs}$ is the charge on one side at the start.

And then $(10^{-4}\mu\text{F})(-30 \mu\text{V}) = -3 \times 10^{-15}\text{Coulombs}$ is the charge on that side side at the end. That means $10 \times 10^{-15} = 10^{-14}$ Coulombs had to move from one side to the other.



Now all we have to do is figure out how many sodium ions that would be. Each ion has a charge of 1.6×10^{-19} Coulombs, so the number is $10^{-14} \text{ C} / 1.6 \times 10^{-19} = 0.6 \times 10^5 = 6 \times 10^4$ ions.

Well, that’s the right answer for the question as I asked it, but I made an error in asking it! It should have been 70 mV (milliVolts) and -30 mV , not μV (microVolts). That puts my answers off by a factor of 1000, and so in the more realistic cell it would be a change of 10^{11} Coulombs, which is about 6×10^7 sodium ions.

6) Your physics lab manual contains a lab similar to the following:

1. Connect the circuit shown in the diagram
2. Measure the current through the resistor for at least five different voltages. Change the voltage by adjusting the dial on the power supply. Read the voltage from the voltmeter, and read the current from the ammeter.
3. Construct a graph of current as a function of voltage. What is the relationship between I and V? Express this relationship in terms of an equation."

Discuss whether this procedure is appropriate for finding the relationship between I and V. (Would the relationship be convincing to you? Why or why not? Is there anything you'd change?)

(Solution by Ray Hodges)

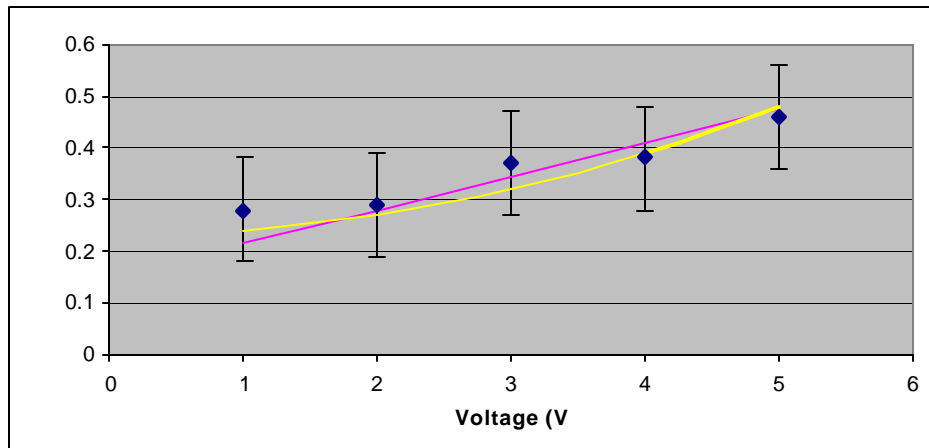
The good thing about this procedure is that it actually does measure the current and the voltage. It would be impossible to find the relationship between I and V without data. But the procedure doesn't give enough information to convincingly find a relationship between the two variables. In particular, we have no idea what the uncertainty is in the measurements of I and V. Without knowing how certain we are of these measurements it is very difficult to choose one relationship over another.

The procedure is not convincing because it does not explain how the relationship between I and V will be determined. Will the relationship be determined graphically or through some algebraic manipulation of the data? A graph of 5 points without error bars could possibly be fit to many different functions. We do not know how many data points will be sufficient to make a decision without knowing the uncertainty in those data points.

For example, examine the data below with an uncertainty of 0.1 A (100 milliamps) in the current measurements.

If the analysis method is "If the graph of the function stays inside all the error bars, the function fits the data." then the purple linear function and the yellow quadratic function both fit the data. Which is a better fit?

Voltage (V)	Current (I)
1	0.28
2	0.29
3	0.37
4	0.38
5	0.46

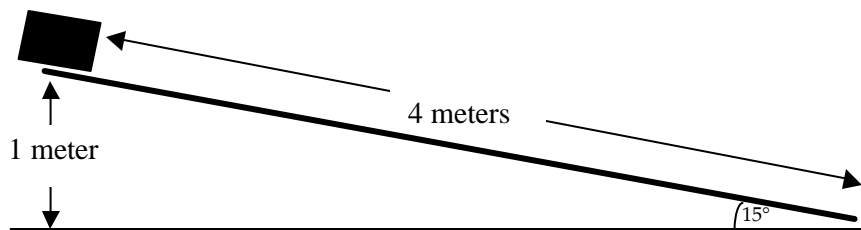


If the analysis method is "Find the value of R at each point for each function. Whichever R is most constant is the better fit." then we still have questions. This analysis method isn't quantitative. If "most constant" means smallest range of R, then the linear function is a better fit. If "most constant"

means smallest percent change in r , then the quadratic function is a better fit. (10.87 is about triple 3.57, but 23.63 is only about double 12.76)

$V=IR$	$V=(I^2)R$
$R=V/I$	$R=V/(I^2)$
3.57	12.76
6.90	23.78
8.11	21.91
10.53	27.70
10.87	23.63

7) (Not graded, just for review.) A box of mass $M = 20$ kg is at rest 1 meter off the ground and about to slide down a 4 meter ramp. Suppose has very low-friction wheels, so you can ignore friction.



a) Find the kinetic energy of the box at the bottom of the ramp.

This is a “basic idea” sort of question from Physics 121: Going from 1 meter high to 0 m is a loss of $mg\Delta h = (20 \text{ kg}) (10 \text{ m/s}^2) (1 \text{ m}) = 200$ Joules of potential energy. All of it goes into kinetic energy, so the answer is 200 Joules.

b) What values in the problem could change without affecting the answer to a? Explain how you know and why that makes sense.

All that matters is the change in height and the mass of the box. And, had I asked for the *speed* of the box at the bottom, only the height would have mattered: If the box starts from rest, $mg\Delta h = 1/2 mv^2$, so $g\Delta h = 1/2 v^2$. $1/2 v^2$ is the kinetic energy per unit mass; $g\Delta h$ is the change in gravitational potential energy per unit mass, or what we could call the “gravitational potential difference.”