

Reference: Cutnell & Johnson, 18.1-18.5. (charge, Coulomb's Law, conductors/insulators, induction)

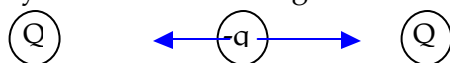
For this and all assignments, please be sure that what you hand in is neat, clean, and easily legible. **If your assignment isn't easily legible, we will not accept it.** (The reason for that is simple consideration for your TA, who has a lot of assignments to read...)

And, for every question, you should explain your thinking. You can explain using mathematics or words or diagrams, and in many cases the explanation can be very brief. But for this and all assignments, I'm asking the TAs to give no credit for answers without any explanation, even if the answer is correct.

- 1) Suppose you have a particle with a negative charge $-q$ exactly between two identical particles with equal, positive charge Q , as shown.

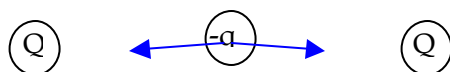


- a) If you moved the particle in the middle a tiny bit to the right, what direction would the total force be on it by the other two charges?



If you move it a bit to the right, then it will be a bit closer to the charge on the right and a bit farther from the charge on the left. The attraction toward the charge on the right would be a bit stronger; to the left a bit weaker, and so the total force on the charge in the middle would be to the right.

- b) Start with that particle back dead center again, and now move it a tiny bit up. In what direction would the total force be?



Now the two forces are equal in strength, but they're no longer in exactly the opposite direction: One's to the right and slightly down; the other's to the left and also slightly down. With both attractions pulling down a bit, the net force on $-q$ is down.

- 2) Rub a balloon on your head and stick it to the wall. And it sticks—very nice. We've talked in lecture about why it *sticks*. This question is about why, after a while, it *fall off*. Consider each of the following explanations.

- i) It falls because it uses up the energy you put on it when you rubbed it.
- ii) It falls because it uses up the charge you put on it when you rubbed it.
- iii) It falls because the energy you put on it slowly leaks off into the wall or the air.
- iv) It falls because the charge you put on it slowly leaks off into the wall or the air.

a) Which explanation fits best with the model we've discussed, for why the balloon stays on the wall?

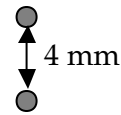
The model has two kinds of charge – positive (protons) and negative (electrons). They attract each other, and in neutral matter there's lots of each but in equal numbers. When they get separated objects end up with a net positive or negative charge. Nothing about that model has charge get "used up," only moved from one place for another. And nothing else gets used up, either: $F_e = \frac{kq_1q_2}{r^2}$ depends only on the charges and the distance between them. So the only things that could change, for the balloon, are the distance (you could knock the balloon away!) and the charge.

b) Given your choice for part a, would it be possible, in principle under ideal circumstances, for a balloon to stick to a wall indefinitely?

If you can keep the charge from leaving the balloon, there's no reason for the balloon ever to fall! (In very dry weather, the charge stays on the balloon much longer. Humid air lets charge move – conducts – better than dry air. And, of course, you'd want the wall to be a very good insulator, too.)

3) Someone just showed me this last week: If you go to a crafts store and buy "glitter beads," you get a little plastic tube of tiny, metal covered glass or plastic beads. On a nice dry day, you can get a charge on the beads, and some of them will "hover" in the air inside the tube, maybe 4 mm apart from each other. It's pretty cool.

a) To make it even more cool, let's estimate how much charge there is on a bead. For that, let's just assume there are only two beads, one under the other, and it's the electrostatic repulsion that's holding the top bead up. If the beads are 4 mm apart, they each have a mass of (I'm guessing) 0.1 grams, and we can assume they have the same charge, about how much is that charge?



A bead with mass 0.1 grams = 10^{-4} kg is pulled by the earth with a force (weight) of $(10^{-4}\text{kg})(10 \text{ m/s}^2) = 10^{-3}$ Newtons.

So we need an electric repulsion that strong, to hold the top bead up:

$$F = \frac{kq_1q_2}{r^2} = \frac{9 \times 10^{10} \text{ N} \cdot \text{m}^2/\text{C}^2 q_1q_2}{(0.004 \text{ m})^2} = 10^{-3} \text{ N}$$

Most of that's units. If $q_1=q_2$ then you can solve this to get $q_1=q_2 = 4 \times 10^{-10}$ Coulombs. Or thereabouts. Anyway, it's not much charge! It makes sense that it's much less than the amount of charge we found for the aluminum pie plate in lecture, since the beads are so much smaller and they're much closer together.

b) If the amount of the charge on the beads were to drop by 1/2, what would you expect to happen to the distance between them?

If the amount of charge on each bead drops by 1/2, then the numerator of $\frac{kq_1q_2}{r^2}$ would drop by 1/4. So the denominator has to drop by the same amount, because we know the

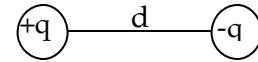
force up still has to be equal to the weight of a bead. So the distance would go down by $1/2$. It makes sense that it would be a smaller distance, since reducing the charge would make the force weaker, so we need to bring the beads closer to make up for that.

- 4) One more on those beads, another estimate: How much of a difference in the *mass* of a bead does that charge make?

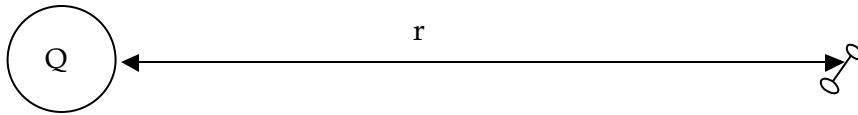
If the charge is 4×10^{-10} C, and there's 1.6×10^{-19} C per electron, then there are 3×10^9 electrons on a bead (or that many *fewer*, we don't know if they're + or -). And since an electron has a mass of 10^{-30} kg, then we're talking about a difference of 3×10^{-21} kg.

- 5) Lots of things that are electrically neutral overall have one side that's electrically negative and one side positive (a water molecule, for example).

We call such things "electric dipoles," and we can model them as pairs of charged particles separated by a distance d . Usually d is a very small distance. (For water it would be around 10^{-11} m, thinking of a few protons worth of charge on one end – about 6×10^{-19} C -- and a few electrons worth at the other.)



Suppose you have a dipole that's free to move in any way (including rotate – imagine it floating in space). And there's an object with charge Q a distance r away. That distance r would be much larger than d , the distance of the dipole, so I've drawn the dipole small.



- a) Would the dipole end up attracted to the charge, repelled, or neither?

It would be attracted: If Q is positive, then the negative end of the dipole will be pulled toward it, and the positive end pushed away. But the negative end would end up just a bit closer, so the attraction by Q on that side would be just a bit stronger than the repulsion by Q on the positive side.

- b) Would the charge Q be attracted, repelled, or neither to the dipole?

Newton's Third Law applies again! Q would be attracted to the dipole—the force on Q by the dipole would be equal and opposite the force on the dipole by Q . You could also think of it again in terms of the + and - ends of the dipole: The - end is closer, so it pulls a bit harder.

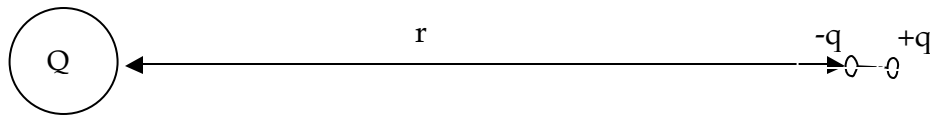
- c) Does the answer to parts a and b depend on the sign of Q ?

Only in the details: If Q were negative, then it would be the + end of the dipole pulled toward it and the - end repelled, again with the attraction a bit stronger than the repulsion.

- d) Since water molecules are dipoles, you can test your idea: Rub some wool or flannel on a hard rubber comb to get a charge on it, and then hold it near a thin stream of water coming from a faucet. What happens?

Well, I hope you saw the stream of water pull slightly toward the comb! And if you held the wool near instead, that should also produce an attraction.

- 6) OK, we're not done with that dipole. Think of the dipole as lined up the way I've drawn here, so the $-q$ end is a distance r from Q , and the $+q$ end is a distance $r+d$.



- a) Write a formula for the total force on the dipole by the charge Q .

Just use $\frac{kq_1q_2}{r^2}$ for each pair, Q and $+q$ and then Q and $-q$, and add what you get. The second would be negative, an attraction; the first positive:

$$F_{\text{net}} = \frac{kQq}{(r+d)^2} - \frac{kQq}{r^2}$$

- b) If you were to double the distance r between Q and the dipole, what would happen to the magnitude of the force? Would it double? Cut in half? Something else?

You could work on this in a couple of ways; pick whichever one you prefer. One way would be to try numbers. You could use the water molecule if you like, $q = 6 \times 10^{-19} \text{ C}$ and $d = 10^{-11} \text{ m}$. A reasonable amount of charge for Q might be 10^6 Coulombs , and r could be $1/2 \text{ cm} = 5 \times 10^{-3} \text{ m}$. Find F , and then double r . Or you could make up your own numbers – you could pick easier numbers – just be sure d is much smaller than r .

The other way would be to simplify the formula you found in part a. The tricky part about that is figuring out what you can ignore along the way. Remember that d is a very small number, and so d^2 is a very very small number. When your adding numbers, and you know one is very tiny compared to the others, you can often ignore that one – just cross it out. (So $1,000,000 + 2$ is about equal to $1,000,000$; $a + b \sim a$ for very small b .)

You can plug in numbers and calculate; I'm sure of that. So I'll show the other way. I'll

start with $F_{\text{net}} = \frac{kQq}{(r+d)^2} - \frac{kQq}{r^2}$.

Expand $(r+d)^2 = r^2 + 2rd + d^2$. But if d is much less than r , or $d \ll r$, then $d^2 \ll 2rd$, so we can ignore it d^2 . (Yes, we could also ignore the $2rd$, since it's $\ll r^2$ but that would be to ignore the small difference between the attraction and the repulsion, and that's what we want to find out! So we only ignore the smallest term, not the next smallest.)

So now we have: $F_{\text{net}} = \frac{kQq}{(r^2 + 2rd)} - \frac{kQq}{r^2}$

Now you've got to make a common denominator:

$$F_{\text{net}} = \frac{r^2 kQq}{r^2(r^2 + 2rd)} - \frac{kQq(r^2 + 2rd)}{r^2(r^2 + 2rd)}$$

And then simplify:

$$F_{\text{net}} = \frac{r^2 k Q q}{r^2 (r^2 + 2rd)} - \frac{k Q q (r^2 + 2rd)}{r^2 (r^2 + 2rd)} = \frac{-k Q q 2rd}{r^2 (r^2 + 2rd)} = \frac{-2k Q (qd)}{(r^3 + 2r^2 d)}$$

One more time, ignore the smallest term, since $2r^2 d \ll r^3$:

$$F_{\text{net}} = \frac{-2k Q (qd)}{r^3}$$

So... if you double the distance, the force should go down by $2^3 = 8$, as long as $r \gg d$.

(Notice also that the force only depends on the product qd –that’s the size of the charge on either end of the dipole times the separation. This product is called the “dipole moment.” Chemists measure tiny dipole moments in Debyes, $1 \text{ D} = 3.335 \times 10^{-30} \text{ C}\cdot\text{m}$. E.g. a water molecule has a dipole moment of 1.85 D. In some later course in chemistry or physiology you may see that unit again... we won’t use it here.)