

Name: _____

S.S.N.: _____

**Exam II,
Physics 122-Summer 2003, Thu. 8/8/2003
Instructor: Dr. S. Liberati**

GENERAL INSTRUCTIONS

- Do all the problems by writing on the exam book (continue to work on the back of each page if you run out of room).
- Write your name (in capital letters) on every page of the exam.
- Purely numerical answers will not be accepted. Explain with symbols or words your line of reasoning. Corrected formulae count more than corrected numbers.
- Use a calculator

Hints to do well

- Read the problem carefully before you start computing.
- Do problems with symbols first (introduce them if you have to). Only put in numbers at the end.
- Check your answers for dimensional correctness.
- If you are not absolutely sure about a problem, please write down what you understand so that partial credit can be given.

Honor Pledge: Please sign at the end of the statement below confirming that you will abide by the University of Maryland Honor Pledge

"I pledge on my honor that I have not given or received any unauthorized assistance on this assignment/examination."

Signature: _____

Short Answers

Q.1) _____

Q.2) _____

Q.3) _____

Q.4) _____

Short Exercises

E.1) _____

E.2) _____

Problems

P.1 _____

P.2 _____

P.3 _____

Total Score

Useful Constants & Formulae

Fundamental unit of charge : $|e| = 1.6 \times 10^{-19} \text{ C}$

Coulomb constant : $k_e = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$,

Constants:

Permittivity of vacuum : $\epsilon_0 = 1/(4\pi k_e) = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)$

Permeability of vacuum : $\mu_0 = 4\pi \times 10^{-7} (\text{T} \cdot \text{m}) / \text{A}$

Speed of light : $c = 1/\sqrt{\epsilon_0 \mu_0} = 3 \times 10^8 \text{ m/s}$

Coulomb Law:

$$F_e = k_e \frac{q_1 q_2}{r^2}$$

Electric field:

$E = F/q_0$ For a point charge Q : $E = k_e Q/r^2$ For a parallel plate capacitor: $E = q/(\epsilon_0 A) = \sigma/\epsilon_0$

Gauss' Law:

$$\oint (E \cos \theta) dA = Q/\epsilon_0$$

Electric potential energy:

$$W_{AB} = \int_A^B (EPE) = EPE_A - EPE_B$$

Electric potential:

$$V = EPE/q_0 \quad W_{AB} = \int q_0 dV = q_0 V_A - q_0 V_B$$

For a point charge : $V = k_e Q/r$

Relation between electric field and electric potential:

$$E = -\frac{\Delta V}{\Delta s}$$

Dielectric constant:

$$\kappa = E_0/E$$

$$C = q/V$$

For a parallel plate capacitor : $C = \epsilon_0 \epsilon A/d$

Capacitors:

$$\text{Energy stored in a capacitor } EPE_{\text{capac}} = \frac{1}{2} CV^2$$

Ohm's law:

$$V = RI \quad \text{For a wire : } R = \rho \frac{L}{A}$$

Electric power:

$$\text{DC generator : } P = I \cdot V = I^2 R = V^2/R$$

$$\text{AC generator : } \bar{P} = I_{\text{rms}} \cdot V_{\text{rms}} = I_{\text{rms}}^2 R = V_{\text{rms}}^2/R$$

Series wiring: Resistors: $R_s = R_1 + R_2 + \dots + R_n$ Capacitors: $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$

Parallel wiring: Resistors: $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$ Capacitors: $C_p = C_1 + C_2 + \dots + C_n$

Magnetic force on a moving charge: $F = qvB \sin \theta$

Radius of circular trajectory of charged particle in a magnetic field: $r = \frac{mv}{qB}$

Magnetic force on current carrying wire of length L: $F = ILB \sin \theta$

Magnetic torque on a N spires coil: $\tau = NIAB \sin \theta$

Magnetic field produced by a long straight wire: $B = \frac{\mu_0 I}{2\pi r}$

Magnetic field produced by a coil (at center): $B = N \frac{\mu_0 I}{2R}$
 $R = \text{radius of the loop, } N = \# \text{ of loops}$

Magnetic field produced by a solenoid: $B = \mu_0 nI$ $n = \text{turns/unit length}$

Ampere's Law: $\oint B_{\parallel} dl = \mu_0 I + \mu_0 \frac{d\Phi_E}{dt}$

Motional emf when v, B, L are mutually perpendicular: $emf = vBL$

$$emf = -N \frac{d\Phi_B}{dt}$$

Farady's law: $N = \# \text{ of loop in coil}$
 $\Phi_B = \text{Magnetic flux} = BA \cos \theta$

Lenz's law:

"The induced emf resulting from a changing magnetic flux has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes the original flux change."

Emf induced in a rotating planar coil (electric generator): $emf = NAB \omega \sin(\omega t)$

Inductance:

$$L = \frac{N\Phi}{I} \quad \text{emf} = -L \frac{dI}{dt}$$

Transformer equation:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

Energy density of electric and magnetic field:

$$u_E = \frac{1}{2} \epsilon_0 E^2 \quad u_B = \frac{1}{2\mu_0} B^2$$

Average Energy density of an EM wave:

$$\bar{u} = \frac{1}{2} \epsilon_0 E_{rms}^2 + \frac{1}{2\mu_0} B_{rms}^2$$

In vacuum $u_E = u_B$ $E = cB$ $\bar{u} = \epsilon_0 E_{rms}^2 = \frac{1}{\mu_0} B_{rms}^2$

Average Intensity of an EM wave:

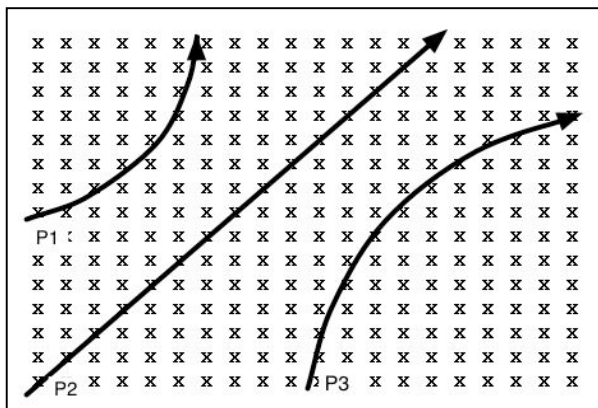
$$S = \frac{P}{A} = c\bar{u} = \frac{1}{2} c \epsilon_0 E_{rms}^2 + \frac{1}{2} \frac{1}{\mu_0} B_{rms}^2 = c \epsilon_0 E_{rms}^2 = \frac{c}{\mu_0} B_{rms}^2$$

Snell's law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Part I: Short Answers (5 points each)

Question 1



Pions are elementary particles which come in three kinds: π^+ , π^- , π^0 . You can consider them as particles of equal mass. The charges of π^+ , π^- are of equal magnitudes and opposite signs (π^+ has positive charge $+q$ and π^- has a negative charge $-q$). The charge of π^0 is zero. These three pions move through a constant magnetic field (flowing into the paper) and follows paths like in the drawing.

- Determine for each of particle of the picture which kind of pion it represents.
- The picture shows that the radius of curvature of P1 is less than that of P3.
Which of the two is moving faster?

A) P1 is positive because it is deviated as predicted by the RHR1
 P2 is neutral because it is not deviated
 P3 is negative because the force is opposite to that predicted by the RHR1

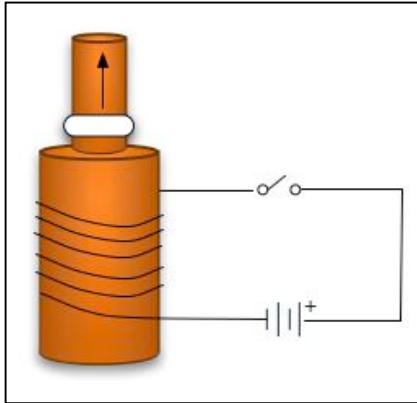
B) $r = \frac{mv}{qB}$ so if $r_1 < r_3$ then $v_1 < v_3$ (given that $|q|$, m , and B are the same)

Question 2

State the SI units and dimensions of each of the following physical quantities:

Quantity	S.I. Unit	Dimension (in terms of [M],[L],[T] and [Q])
Magnetic field	$T = \text{Kg}/\text{C}\cdot\text{s}$	$[\text{M}/\text{QT}]$
Magnetic Force	N	$[\text{ML}/\text{T}^2]$
Induced EMF	$V = \text{J}/\text{C}$	$[\text{ML}^2/\text{QT}^2]$
Magnetic flux	$\text{Wb} = \text{T}\cdot\text{m}^2$	$[\text{ML}^2/\text{QT}]$
Inductance (L)	$H = \text{V}\cdot\text{s}/\text{A}$	$[\text{ML}^2/\text{Q}^2]$

Question 3



A coil is looped around a cylindrical iron core. The coil is connected to a DC generator. A metallic ring is placed around the smaller section side of the core. When the switch of the circuit is closed the ring “jumps” upward.

A) Explain this behavior.

B) What it would happen to the ring if the polarity of the DC generator is reversed?

Hint: remember Lenz’s law.

When the switcher is closed a current start to flow in the coil.

Hence there is a change in the magnetic flux (which initially was zero).

The RHR2 tells us that the magnetic field is oriented with the N pole up.

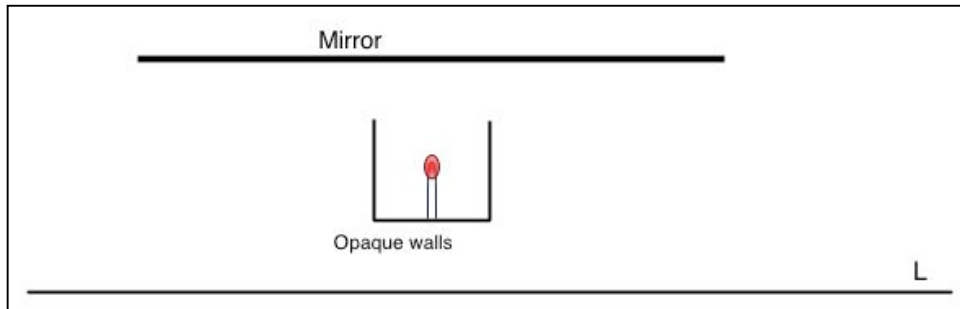
Lenz's law tells us that the change in magnetic flux will induce a current in the ring which will generate a magnetic field that will try to compensate for the change in the magnetic flux.

So it will be with the north pole down. But then the magnetic field of the coil and that of the ring will have adjacent north poles and this will lead to repulsion. So the ring will jump.

Inverting the polarity of the DC generator leads to the same effect because both the magnetic field generated by the coild and that generated by the ring will be inverted.

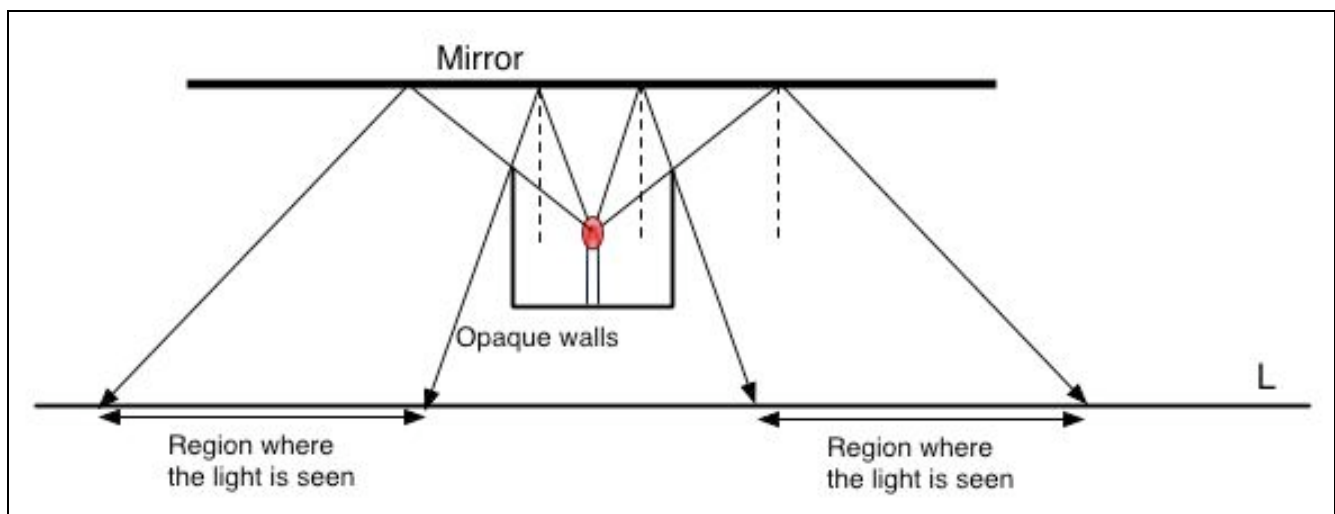
We shall then have S poles adjacent an again repulsion.

Question 4



Where along line L is the light in the bucket visible?

Mark all regions and justify using ray optics.



Part II: Short Exercises (10 points each)

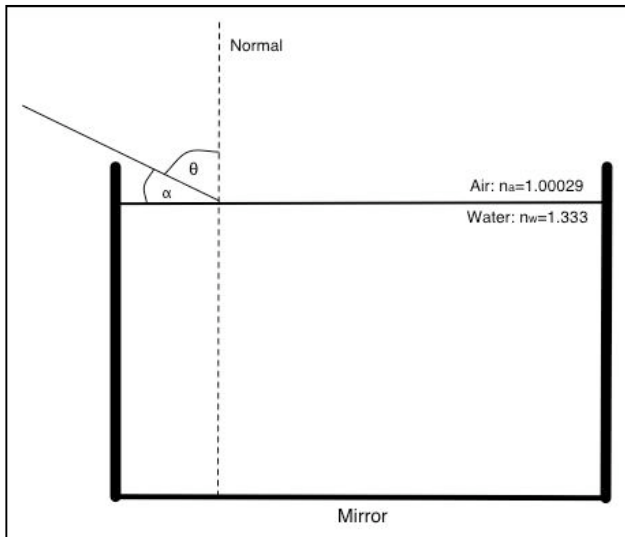
Exercise 1

A constant magnetic field passes through a flat 10 loops coil whose radius is 0.50 m. The magnetic field has an amplitude of 2.5 T and it is inclined at an angle $\theta = 55^\circ$ with respect to the normal to the plane of the coil.

- If the magnetic field decreases to zero in 0.50 s, what is the magnitude of the average emf induced in the loop?
- If the magnetic field is held constant at its initial value of 2.5 T, what is the magnitude of the rate $\Delta A / \Delta t$ at which the area of the coil should change so that the average emf has the same magnitude as that obtained in a)?

$a) \quad \text{Emf} = N \frac{\Delta B}{\Delta t} = N \frac{(B_f - B_i)}{\Delta t} A \cos \theta = 10 \frac{(0 - 2.5 \text{ T})}{0.50 \text{ s}} \pi (0.50 \text{ m})^2 \cos 55^\circ = 22.5 \text{ V}$
$b) \quad \text{Emf} = N \frac{\Delta B}{\Delta t} = N \frac{(A_f - A_i)}{\Delta t} B \cos \theta \quad \left \frac{\Delta A}{\Delta t} \right = \frac{\text{Emf}}{NB \cos \theta} = \frac{22.5 \text{ V}}{10 \cdot 2.5 \text{ T} \cdot \cos 55^\circ} = 1.57 \frac{\text{m}^2}{\text{s}}$

Exercise 2



A rectangular fish tank has a bottom made with a mirror. It is filled with water ($n=1.333$). A laser beam is shined at an angle $\alpha=30^\circ$ from the horizontal. Consider the index of refraction of air equal to one.

- Determine the angle of incidence θ_1 and that of refraction θ_2 . Draw in the graph the path of the laser beam.
- Determine the angle of incidence θ_3 and reflection θ_4 of the beam on the mirror at the bottom.
- With what angle α' w.r.t. the horizontal it will emerge from the

surface of the water back into the air.

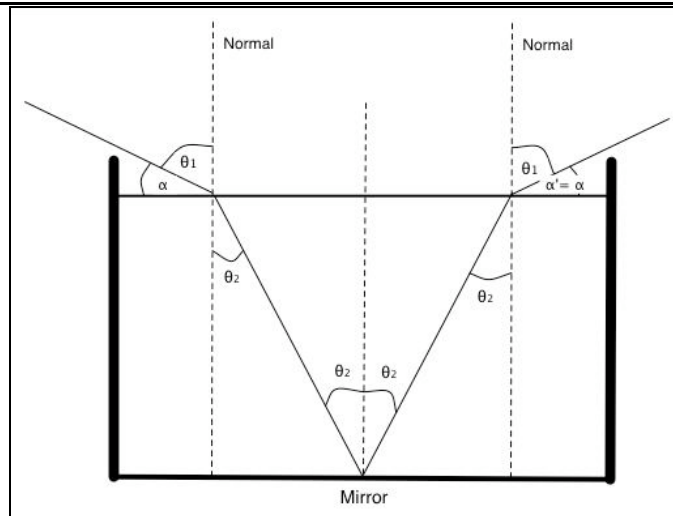
$$a) \theta_1 = 90^\circ - 30^\circ = 60^\circ$$

$$\text{by Snell's law : } \theta_2 = \sin^{-1} \left[\frac{n_1}{n_2} \sin \theta_1 \right] = \frac{1.000}{1.333} \sin 60^\circ = \sin^{-1} 0.65 = 40.5^\circ$$

$$b) \theta_3 = \theta_2 = 40.5^\circ$$

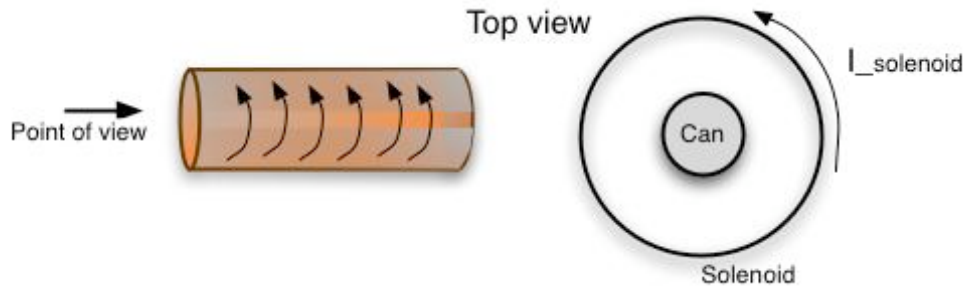
$$\text{by reflection law : } \theta_4 = \theta_3 = \theta_2 = 40.5^\circ$$

$$c) \theta_5 = \theta_4 = \theta_2 = 40.5^\circ \quad \theta_1 = \theta_5 = 60^\circ \quad \alpha' = 30^\circ$$



Part III: Problems (20 points each)

Problem 1



An aluminum can of radius 3 cm is inside a 100-turn and 10 cm long solenoid of wire. The electrical current inside the solenoid goes from 0 A to 100 A in a time interval of 2×10^{-5} seconds.

- What is the magnitude and direction (in or out of the paper in the picture) of the magnetic field inside the solenoid when the current is 100 A?
- What is the magnitude (the absolute value) of the induced EMF around the circumference of the can? Hint: consider the can as a 1 loop coil.
- If the resistance encountered by the current moving around the circumference of the can is $R=0.1 \, \Omega$, what is the current flowing on the surface of the can?
- What is the magnitude and direction of the magnetic force acting on the induced current in the can? Will the can be smashed or blown up? Hint: consider the force on a current with length equal to the circumference of the can.

$$a) \quad B = \mu_0 n I = \mu_0 \frac{N}{L} I = 4\pi \times 10^{-7} \frac{T \cdot m}{A} \times \frac{100}{0.1 \, m} \times 100 A = 4\pi \times 10^{-2} \, T$$

Using the RHR2 one can see that the magnetic field points out of the paper.

$$b) \quad |Emf| = N \frac{\Delta B}{\Delta t} = \frac{\Delta B}{\Delta t} A_{can} = \frac{(4\pi \times 10^{-2} \, T - 0 \, T)}{2 \times 10^{-5} \, s} \times (3 \times 10^{-2} \, m)^2 = 18\pi^2 \times 10^{-1} \, V = 17.8 \, V$$

$$c) \quad I = |Emf| / R = 17.8 \, V / 0.1 \, \Omega = 178 \, A$$

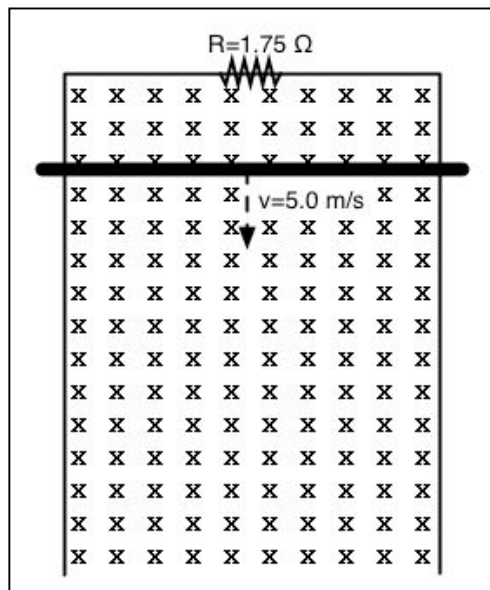
d) The induced current in the can will flow in the opposite direction of the solenoid current (by Lenz's law)

So the magnetic force will be pointing inward (RHR1) and smash the can. The magnitude of the force is

$F = IBl$ where l is the circumference of the can.

$$F = IBl = IB2\pi r = (4\pi \times 10^{-2} \, T)(178 \, A)2\pi(3 \times 10^{-2} \, m) = 4.2 \, N$$

Problem 2



A conducting rod of mass $M = 0.50 \text{ Kg}$ slides down between two vertical copper tracks at a constant speed of $v = 5.0 \text{ m/s}$ perpendicular to a magnetic field B . The resistance of the rod and the tracks is negligible. The rod maintains contact with the tracks at all times and has a length $L = 1.5 \text{ m}$. A resistance $R = 1.75 \Omega$ is attached between the tops of the tracks.

- What is the magnitude of the magnetic field?
- If the magnetic field flows into the paper in the picture is the current in the rod clockwise or anti-clockwise?
- Find the change in the gravitational potential energy in a time of 0.30 s .
- Find the energy dissipated in the resistor in 0.30 s .

$$a) \text{ If } v = \text{const} \quad F_{\text{net}} = 0 \quad F_{\text{magn}} \quad W = 0 \quad IBL = Mg \quad B = \frac{Mg}{IL}$$

$$I = ? \quad I = \text{Emf} / R = \frac{vBL}{R} \quad B = \frac{Mg}{L} \frac{R}{vBL} \quad B^2 = \frac{RMg}{vL^2}$$

$$B = \sqrt{\frac{RMg}{vL^2}} = \sqrt{\frac{1.75 \Omega \cdot 0.5 \text{ Kg} \cdot 9.8 \text{ m/s}^2}{5 \text{ m/s} \cdot (1.5 \text{ m})^2}} = 0.873 \text{ T}$$

b) anti-clockwise by RHR1 and due to the fact that if the circuit is open below then the current will flow up in the branch with the resistance.

NOTE : in the original picture the circuit was erroneously closed below.

In this case the current would have flowed clockwise (not in the branch with resistance).

$$c) \quad GPE = Mgh = Mgvt = 7.35 \text{ J}$$

$$d) \quad P = I^2 R = \left(\frac{vBL}{R} \right)^2 R = \frac{(vBL)^2}{R} = \frac{(5 \text{ m/s} \cdot 0.873 \text{ T} \cdot 1.5 \text{ m})^2}{1.75 \Omega} = 24.3 \text{ W}$$

$$E_{\text{dissip}} = Pt = 7.35 \text{ J}$$

Problem 3

An electromagnetic beam is emitted from a spaceship. The beam transmits an average power of $1.50 \times 10^4 \text{ W}$ and has a cross-sectional area of 150 m^2 .

- What is the average value of the intensity of the beam?
- What are the rms values of the electric and magnetic fields?
- If an electron ($q = -1.6 \times 10^{-19} \text{ C}$) is placed in the path of the beam what is the maximum electric force it will experience?
- If the electron is moving with speed $v = 10^4 \text{ m/s}$ orthogonal to magnetic field, what is the magnitude of the maximum magnetic force it will experience?

$$\begin{aligned}
 a) \quad \bar{S} &= P/A = \frac{1.50 \times 10^4 \text{ W}}{150 \text{ m}^2} = 100 \frac{\text{W}}{\text{m}^2} \\
 b) \quad \bar{S} &= c \epsilon_0 E_{rms}^2 \quad E_{rms} = \sqrt{\frac{\bar{S}}{c \epsilon_0}} = \sqrt{\frac{100 \text{ W/m}^2}{3 \times 10^8 \text{ m/s} \times 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2}} = 194 \text{ N/C} \\
 c) \quad F_{el, \max} &= qE_0 = qE_{rms} \sqrt{2} = 1.6 \times 10^{-19} \text{ C} \times 194 \text{ N/C} \times \sqrt{2} = 439 \times 10^{-19} \text{ N} \\
 d) \quad F_{mag, \max} &= qvB_0 = qvB_{rms} \sqrt{2} \\
 B_{rms} &= E_{rms}/c \quad F_{mag, \max} = qvB_0 = qvB_{rms} \sqrt{2} = q \frac{v}{c} E_{rms} \sqrt{2} = \\
 F_{mag, \max} &= 1.6 \times 10^{-19} \text{ C} \times \frac{10^4 \text{ m/s}}{3 \times 10^8 \text{ m/s}} \times 194 \text{ N/C} \times \sqrt{2} = 1.5 \times 10^{-21} \text{ N}
 \end{aligned}$$