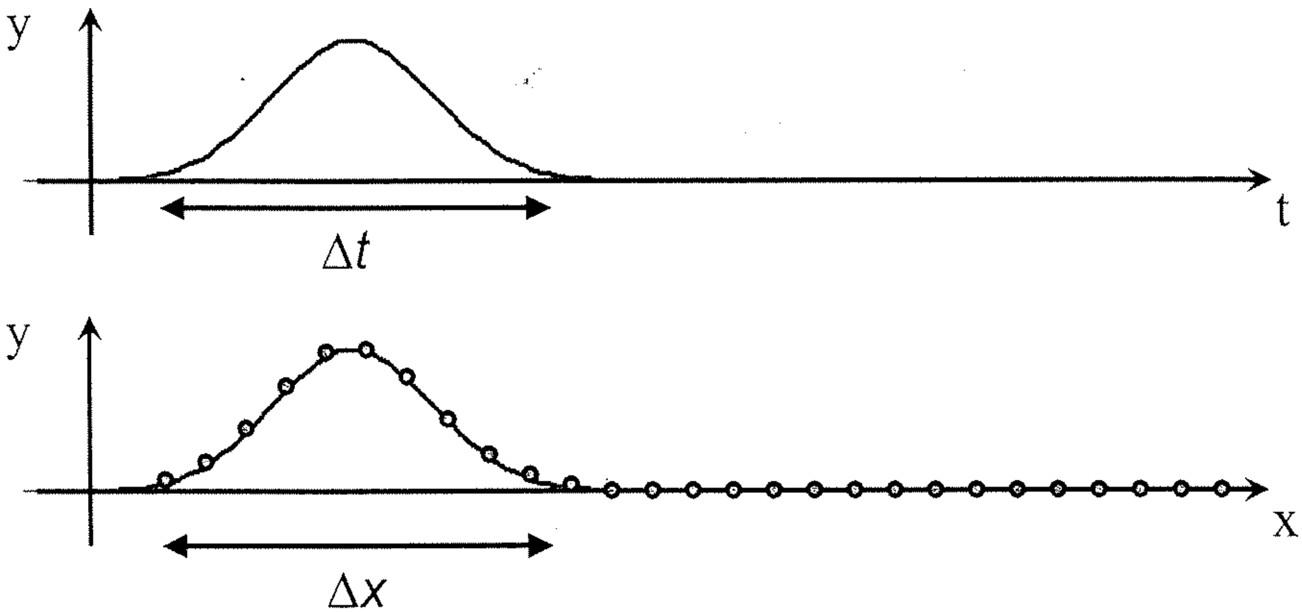
Lecture

2/8/05

What controls the widths of the pulses in time and space?



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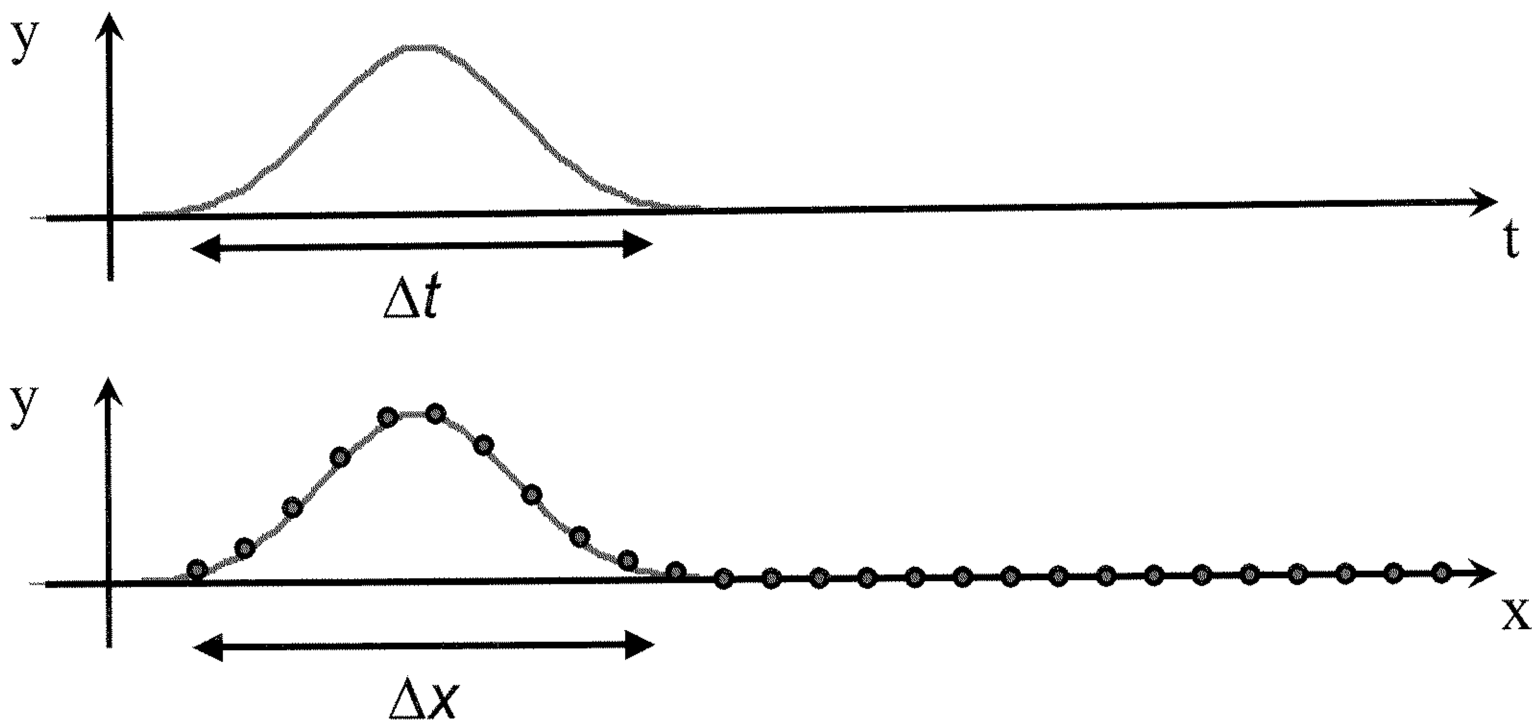

Speed of a pulse on an elastic string

- Imagine the string as a series of tiny masses (beads), each connected to its neighboring masses by tiny springs. Suppose the string has mass m and length l and is under a tension τ.
- Our observations suggest that the velocity of a pulse on the string might depend on those parameters, but on no others. For example, the velocity of a pulse does not seem to depend on the amplitude of the pulse.
- Might we possibly be able to figure out how the pulse velocity depends on those parameters by using dimensional analysis? Let's try.

$$[v] = L/T$$

$$[\tau] = ML/T^2 \quad \text{(force = mass times acceleration)}$$

$$\downarrow$$

$$[\tau l/m] = L^2/T^2 = [v^2]$$

This suggests that the pulse velocity $v = \sqrt{\tau l/m}$ or $\sqrt{\tau/\mu}$, where μ is the (linear) mass density of the string.

There's a rule of thumb in physics that any relationship that neat and simple is likely to be correct. And so it is here!

Foothold Ideas

- *Key concept*: One must distinguish between the motion of the bits of matter that compose the medium and the motion of the wave pattern.
- *Matter speed*: The speed of the medium bits of matter depends on both the size and shape of the wave pattern and on the pattern propagation speed.
- *Pattern speed*: The pattern propagation speed depends on the properties of the medium but does **not** depend on the pattern shape.
- *Mechanism*: The forces that cause the propagation of the pattern originate in the forces between adjacent bits of matter that compose the medium.
- *Superposition*: If patterns from different sources overlap in the medium, the net displacement of each bit of matter in the medium is the sum (taking into account sign) of the displacements due to each of the overlapping patterns. (The underlying source of this is the linearity of the applicable relationships, e.g., doubling a force doubles the corresponding acceleration.)

How many of the following list of equations do you think might describe some type of mechanical wave in a medium?

$y = A \sin(kx-\omega t)$

$y = A \sin(kx+\omega t)$

$y = A \sin(kx)\cos(\omega t)$

$y = A \sin(kx)\sin(\omega t)$

$y = A \cos(kx-\omega t)$

$y = A \cos(kx+\omega t)$

$y = A \cos(kx)\sin(\omega t)$

$y = A \cos(kx)\cos(\omega t)$

1. one
2. two
3. three
4. four
5. five
6. six
7. seven
8. eight
9. all of them
10. none of them

Now that you've done the easy part, explain what kind of wave your picks describe.