Physic^2 121:
Phundament°ls of Phy^{2}ics I

November 27, 2006

Chapter 8

Rotational Equilibrium

and

Rotational Dynamics

Rotational Kinetic Energy

- An object rotating about some axis with an angular speed, ω, has rotational kinetic energy $\frac{1}{2}I\omega^2$
- Energy concepts can be useful for simplifying the analysis of rotational motion

Total Energy of a System

- Conservation of Mechanical Energy

$$(KE_t + KE_r + PE_g)_i = (KE_t + KE_r + PE_g)_f$$

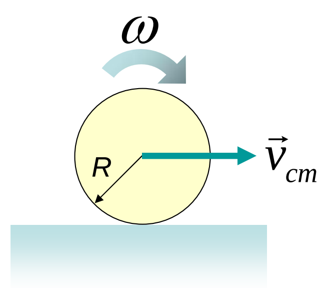
  - Remember, this is for conservative forces, no dissipative forces such as friction can be present
  - Potential energies of any other conservative forces could be added

Work-Energy in a Rotating System

- In the case where there are dissipative forces such as friction, use the generalized Work-Energy Theorem instead of Conservation of Energy
- $W_{nc} = \Delta KE_t + \Delta KE_R + \Delta PE$

Rolling Motion

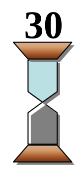
- For an object that rolls without slipping, there is a relationship between the translational motion and the rotational motion:
  - $v_{cm} = R\omega$
  - i.e. the velocity of the center-of-mass is the same as the tangential velocity of the outer edge of the rolling body

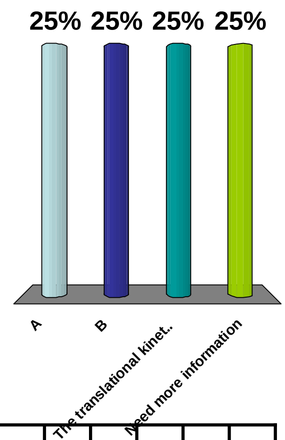


?

Two wheels initially at rest roll the same distance without slipping down identical inclined planes. Wheel B has twice the radius but the same mass as wheel A. All the mass is concentrated in their rims, so that the rotational inertias are $I = mR^2$. Which has more translational kinetic energy when it gets to the bottom?

1. A
2. B
3. The translational kinetic energies are the same
4. Need more information





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1	2	3	4	5

General Problem Solving Hints

- The same basic techniques that were used in linear motion can be applied to rotational motion.
  - Analogies:
    - F becomes τ
    - *m* becomes *I*
    - a becomes α
    - v becomes ω
    - x becomes θ

Angular Momentum

- Similarly to the relationship between force and momentum in a linear system, we can show the relationship between torque and angular momentum
- Angular momentum is defined as
  - L = I ω
    - Vector quantity – right-hand rule determines direction
    - Angular momentum is conserved in a system with no external torques
  - and $$\sum \vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}$$
  - Just like $$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

Angular Momentum, cont

- If the net torque is zero, the angular momentum remains constant
- *Conservation of Angular Momentum* states: The angular momentum of a system is conserved when the net external torque acting on the systems is zero.
  - That is, when

$$\Sigma\tau = 0$$

$$L_i = L_f$$

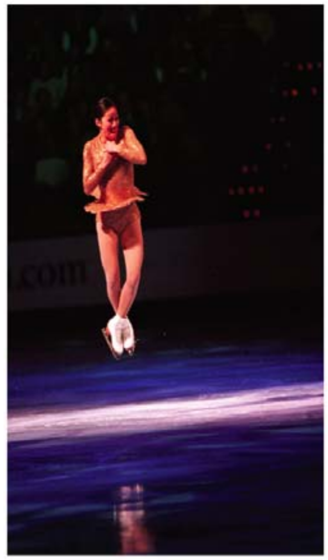
$$or \ \ I_i\omega_i = I_f\omega_f$$

Conservation Rules, Summary

- In an isolated system (no external forces or torques, no non-conservative forces), the following quantities are conserved:
  - Mechanical energy
  - Linear momentum
  - Angular momentum

Conservation of Angular Momentum

- With hands and feet drawn closer to the body, the skater's angular speed increases
  - L is conserved, I decreases, ω increases



Demonstrations