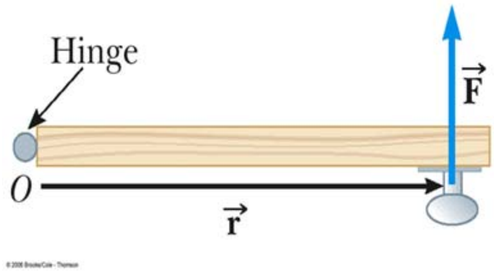
Physic² 121:
Phundament°ls of Phy²ics I

November 15, 2006

Chapter 8

Rotational Equilibrium
and
Rotational Dynamics

Torque



- The door is free to rotate about an axis through O
- There are three factors that determine the effectiveness of the force in opening the door:
  - The *magnitude* of the force
  - The *position* of the application of the force
  - The *angle* at which the force is applied

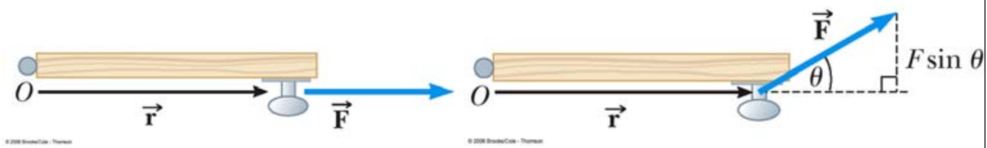
Multiple Torques

- When two or more torques are acting on an object, the torques are added
  - As vectors
- If the net torque is zero, the object's rate of rotation doesn't change

General Definition of Torque

- The applied force is not always perpendicular to the position vector
- The component of the force *perpendicular* to the object will cause it to rotate

General Definition of Torque, cont

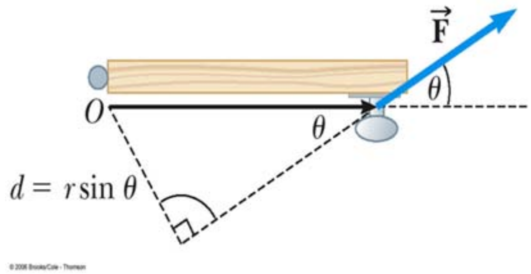


- When the force is parallel to the position vector, no rotation occurs
- When the force is at some angle, the perpendicular component causes the rotation

General Definition of Torque, final

- Taking the angle into account leads to a more general definition of torque:
  - $\tau = r F \sin \theta$
    - F is the force
    - r is the position vector
    - θ is the angle between the force and the position vector

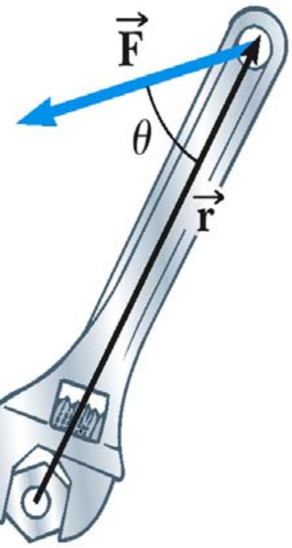
Lever Arm



- The *lever arm*, d, is the perpendicular distance from the axis of rotation to a line drawn along the direction of the force
- d = r sin θ

Right Hand Rule

- Point the fingers in the direction of the position vector
- Curl the fingers toward the force vector
- The thumb points in the direction of the torque



Net Torque

- The net torque is the sum of all the torques produced by all the forces
  - Remember to account for the direction of the tendency for rotation
    - Counterclockwise torques are positive
    - Clockwise torques are negative

Torque and Equilibrium

- First Condition of Equilibrium
  - The net external force must be zero

$$\Sigma \vec{\mathbf{F}} = 0 \ or$$

$$\Sigma \vec{\mathbf{F}}_x = 0 \quad and \quad \Sigma \vec{\mathbf{F}}_y = 0$$

  – This is a necessary, but not sufficient, condition to ensure that an object is in complete mechanical equilibrium

  – This is a statement of translational equilibrium

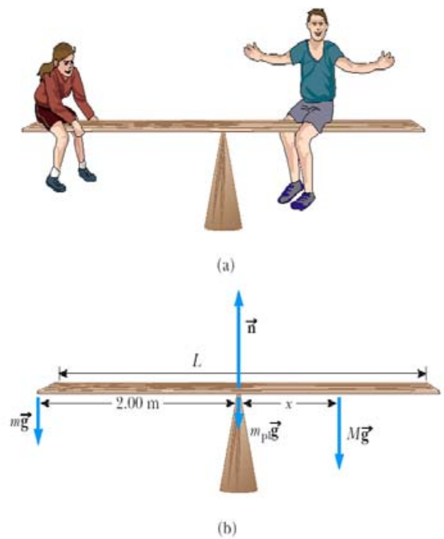
Torque and Equilibrium, cont

- To ensure mechanical equilibrium, you need to ensure rotational equilibrium as well as translational
- The Second Condition of Equilibrium states
  - The net external torque must be zero

$$\Sigma \vec{\tau} = 0$$

Equilibrium Example

- The woman, mass m, sits on the left end of the see-saw
- The man, mass M, sits where the see-saw will be balanced
- Apply the Second Condition of Equilibrium and solve for the unknown distance, x



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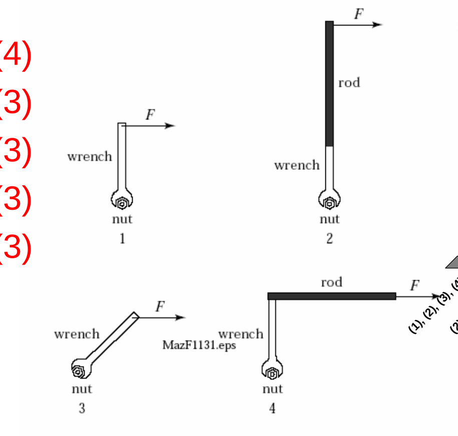
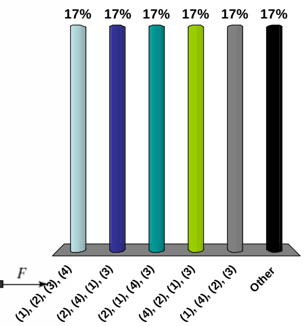
You are using a wrench and trying to loosen a rusty nut. Which of the arrangements shown is most effective for loosening the nut? List in order of descending efficiency.

1. (1), (2), (3), (4)
2. (2), (4), (1), (3)
3. (2), (1), (4), (3)
4. (4), (2), (1), (3)
5. (1), (4), (2), (3)
6. Other





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1	2	3	4	5															

Axis of Rotation

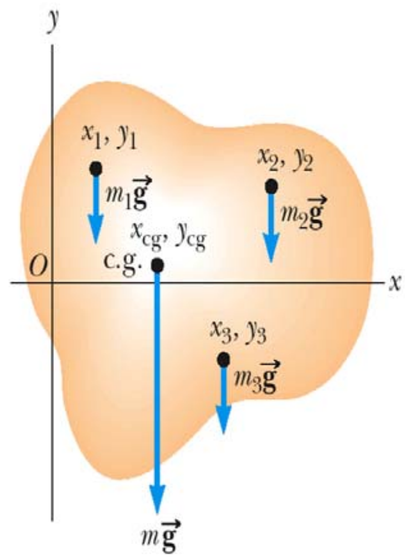
- If the object is in equilibrium, it does not matter where you put the axis of rotation for calculating the net torque
  - The location of the axis of rotation is completely arbitrary
  - Often the nature of the problem will suggest a convenient location for the axis
  - When solving a problem, you *must* specify an axis of rotation
    - Once you have chosen an axis, you must maintain that choice consistently throughout the problem

Center of Gravity

- The force of gravity acting on an object must be considered
- In finding the torque produced by the force of gravity, all of the weight of the object can be considered to be concentrated at a single point

Calculating the Center of Gravity

- The object is divided up into a large number of very small particles of weight (mg)
- Each particle will have a set of coordinates indicating its location (x,y)



Calculating the Center of Gravity, cont.

- We assume the object is free to rotate about its center
- The torque produced by each particle about the axis of rotation is equal to its weight times its lever arm
  - For example, $\tau_1 = m_1 g x_1$

Calculating the Center of Gravity, cont.

- We wish to locate the point of application of the *single force* whose magnitude is equal to the weight of the object, and whose effect on the rotation is the same as all the individual particles.
- This point is called the *center of gravity* of the object

Coordinates of the Center of Gravity

- The coordinates of the center of gravity can be found from the sum of the torques acting on the individual particles being set equal to the torque produced by the weight of the object

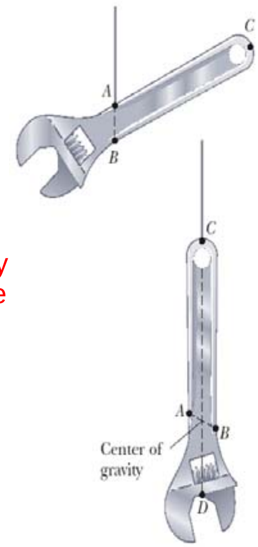
$$x_{cg} = \frac{\Sigma m_i x_i}{\Sigma m_i} \quad and \quad y_{cg} = \frac{\Sigma m_i y_i}{\Sigma m_i}$$

Center of Gravity of a Uniform Object

- The center of gravity of a homogenous, symmetric body must lie on the axis of symmetry.
- Often, the center of gravity of such an object is the *geometric* center of the object.

Experimentally Determining the Center of Gravity

- The wrench is hung freely from two different pivots
- The intersection of the lines indicates the center of gravity
- A rigid object can be balanced by a single force equal in magnitude to its weight as long as the force is acting upward through the object's center of gravity



Notes About Equilibrium

- A zero net torque does not mean the absence of rotational motion
  - An object that rotates at uniform angular velocity can be under the influence of a zero net torque
    - This is analogous to the translational situation where a zero net force does not mean the object is not in motion

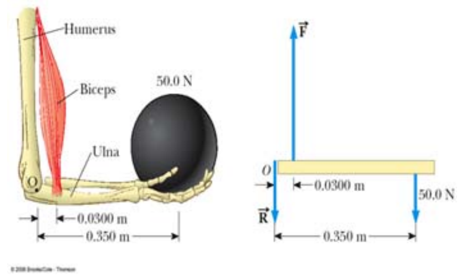
Solving Equilibrium Problems

- Draw a diagram of the system
  - Include coordinates and choose a rotation axis
- Isolate the object being analyzed and draw a free body diagram showing all the external forces acting on the object
  - For systems containing more than one object, draw a separate free body diagram for each object

Problem Solving, cont.

- Apply the Second Condition of Equilibrium
  - This will yield a single equation, often with one unknown which can be solved immediately
- Apply the First Condition of Equilibrium
  - This will give you two more equations
- Solve the resulting simultaneous equations for all of the unknowns
  - Solving by substitution is generally easiest

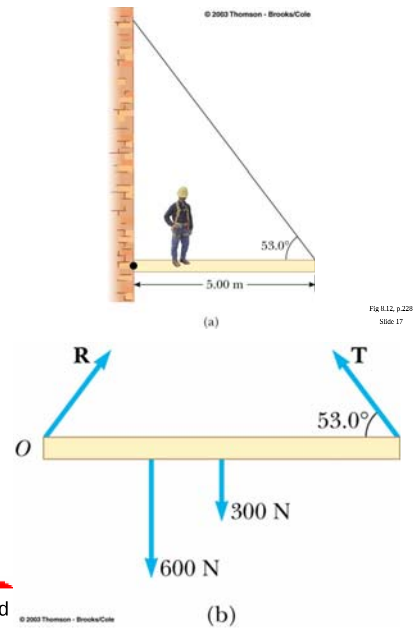
Example of a Free Body Diagram (Forearm)



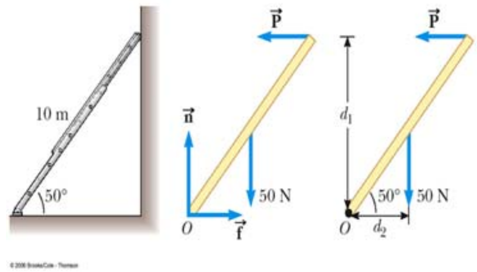
- Isolate the object to be analyzed
- Draw the free body diagram for that object
  - Include all the external forces acting on the object

Example of a Free Body Diagram (Beam)

- The free body diagram includes the directions of the forces
- The weights act through the centers of gravity of their objects



Example of a Free Body Diagram (Ladder)



- The free body diagram shows the normal force and the force of static friction acting on the ladder at the ground
- The last diagram shows the lever arms for the forces