Physic^2 121:
Phundament°ls of Phy^{2}ics I

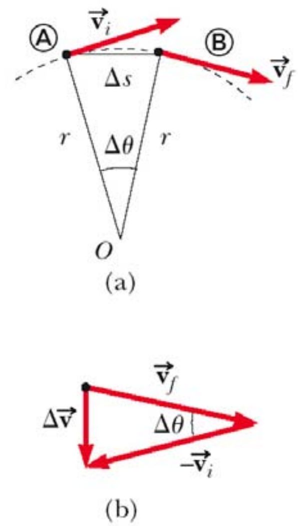
November 6, 2006

Chapter 7

Rotational Motion

Centripetal Acceleration, cont.

- Centripetal refers to "center-seeking"
- The direction of the velocity changes
- The acceleration is directed toward the center of the circle of motion



Centripetal Acceleration

- The magnitude of the centripetal acceleration is given by

$$a_c = \frac{v^2}{r}$$

  - This direction is toward the center of the circle
- The angular velocity and the linear velocity are related ($v = \omega r$)
- The centripetal acceleration can also be related to the angular velocity

$$a_C = \omega^2 r$$

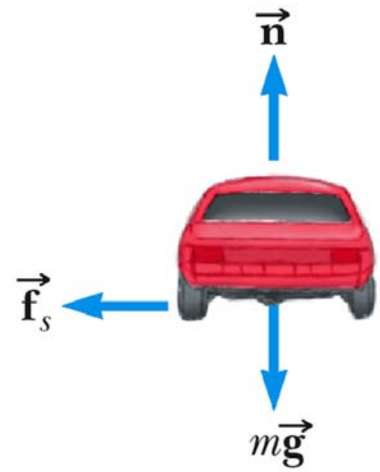
Forces Causing Centripetal Acceleration

- Newton's Second Law says that the centripetal acceleration is accompanied by a force
  - $F_C = ma_C$
  - F_C stands for any force that keeps an object following a circular path
    - Tension in a string
    - Gravity
    - Force of friction

Level Curves

- Friction is the force that produces the centripetal acceleration
- Can find the frictional force, μ, or v

$$v = \sqrt{\mu r g}$$

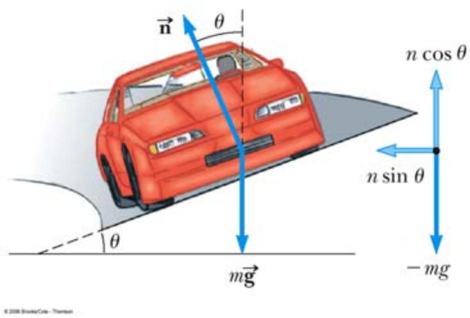


Banked Curves

- A component of the normal force adds to the frictional force to allow higher speeds

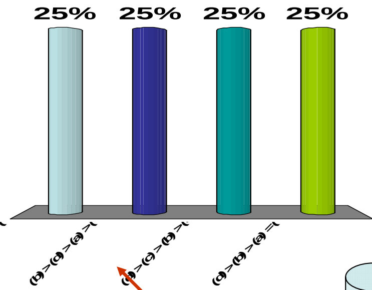
$$\tan\theta = \frac{v^2}{rg}$$

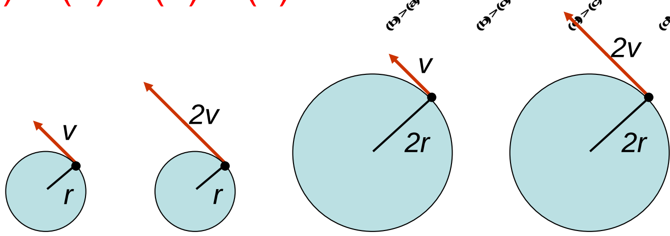
$$or \;\; a_c = g\tan\theta$$



Rank in order, from largest to smallest, the centripetal accelerations of the particles shown below

1. (b) > (a) > (d) > (c)
2. (b) > (d) > (a) > (c)
3. (d) > (c) > (b) > (a)
4. (d) > (b) > (a) = (c)





(a) (b) (c) (d)

0 of 5

1	2	3	4	5															

Newton's Law of Universal Gravitation

- Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of the masses and inversely proportional to the square of the distance between them.

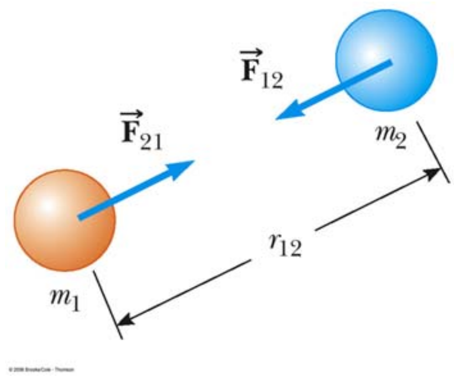
$$F = G\frac{m_1 m_2}{r^2}$$

Universal Gravitation, 2

- G is the constant of universal gravitational
- $G = 6.673 \times 10^{-11}$ N m² /kg²
- This is an example of an *inverse square law*

Universal Gravitation, 3

- The force that mass 1 exerts on mass 2 is equal and opposite to the force mass 2 exerts on mass 1
- The forces form a Newton's third law action-reaction



Applications of Universal Gravitation

- Acceleration due to gravity
- g will vary with altitude

$$g = G\frac{M_E}{r^2}$$

TABLE 7.1

Free-Fall Acceleration *g* at Various Altitudes

Altitude (km)[a]	*g* (m/s²)
1 000	7.33
2 000	5.68
3 000	4.53
4 000	3.70
5 000	3.08
6 000	2.60
7 000	2.23
8 000	1.93
9 000	1.69
10 000	1.49
50 000	0.13

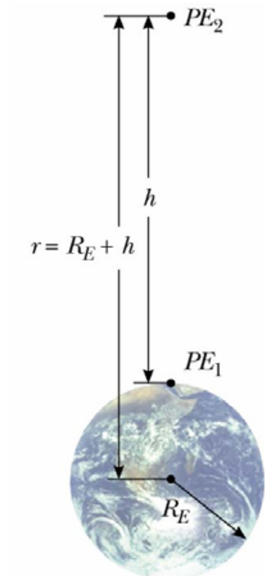
[a]All figures are distances above Earth's surface.

Gravitational Potential Energy

- PE = mgy is valid only near the earth's surface
- For objects high above the earth's surface, an alternate expression is needed

$$PE = -G\frac{M_E m}{r}$$

  - Zero reference level is infinitely far from the earth
  - Otherwise, PE < 0 (negative)



Escape Speed

- The escape speed is the speed needed for an object to soar off into space and not return
- Initial Energy:

$$E_i = KE + PE$$
$$= \tfrac{1}{2}mv^2 - G\frac{M_E m}{R_E}$$

- Really far from the earth ($r \to \infty$), PE → 0. To "escape", object needs to get infinitely far away. To just barely escape, it will slow down to zero at $r = \infty$, so KE = 0. This means total energy = 0:

$$0 = \tfrac{1}{2}mv^2 - G\frac{M_E m}{R_E}$$
$$\tfrac{1}{2}mv^2 = G\frac{M_E m}{R_E}$$
$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

- For the earth, v_{esc} is about 11.2 km/s
- Note, v is independent of the mass of the object

Kepler's Laws

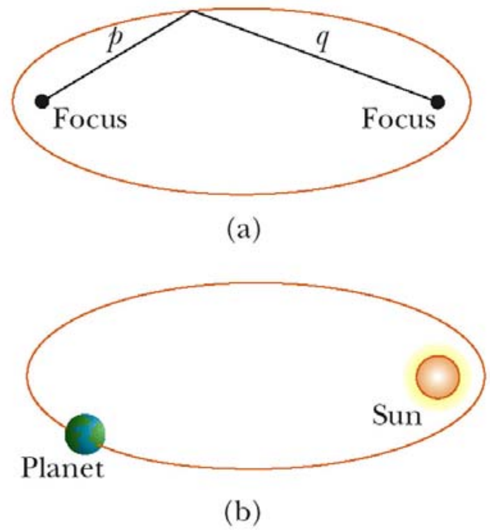
- All planets move in elliptical orbits with the Sun at one of the focal points.
- A line drawn from the Sun to any planet sweeps out equal areas in equal time intervals.
- The square of the orbital period of any planet is proportional to cube of the average distance from the Sun to the planet.
  - $T^2 \propto r^3$

Kepler's Laws, cont.

- Based on observations made by Brahe
- Newton later demonstrated that these laws were consequences of the gravitational force between any two objects together with Newton's laws of motion

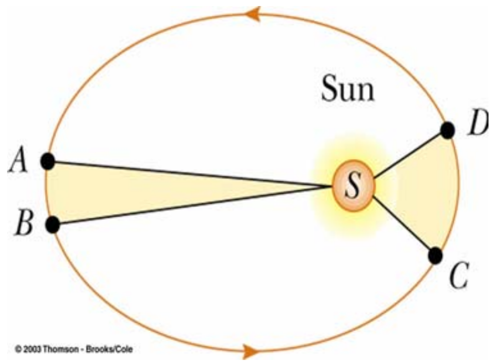
Kepler's First Law

- All planets move in elliptical orbits with the Sun at one focus.
  - Any object bound to another by an inverse square law will move in an elliptical path
  - Second focus is empty



Kepler's Second Law

- A line drawn from the Sun to any planet will sweep out equal areas in equal times
  - Area from A to B and C to D are the same



Kepler's Third Law

- The square of the orbital period of any planet is proportional to cube of the average distance from the Sun to the planet.

$$T^2 = Kr^3$$

  - For orbit around the Sun, $K = K_S = 2.97 \times 10^{-19}\ s^2/m^3$
  - K is independent of the mass of the planet