

## Announcement: WebAssign Bug

- There is an error in the WebAssign solution to problem 8 in HW 8
- The error only affects the first part of the problem
- It will not affect you if the mass of the heavier object is $3 m$
- If the mass of the heavier object is $2 m, 4 m$, or $5 m$, it will mark your answer to the first part as wrong even if it is correct
- The last two parts are not affected.
- I will go in by hand and check your answer to the first part and give credit if it is correct. So please go ahead and do the problem anyway.



## From Last Time

- Angular velocity
- Angular Acceleration
- Angular equations of motion (constant angular acceleration)

$$
\omega=\omega_{i}+\alpha t
$$

$\Delta \theta=\omega_{i} t+\frac{1}{2} \alpha t^{2}$

- Relationship to linear motion quantities
- Subscript "t" refers to tangential motion

$$
\begin{aligned}
& s=r \theta \\
& v_{t}=r \omega \\
& a_{t}=r \alpha
\end{aligned}
$$



## Example Problem (7.9)

- The diameter of the main rotor and tail rotor of a helicopter are 7.60 m and 1.02 m , respectively. The respective rotational speeds are $450 \mathrm{rev} / \mathrm{min}$ and 4,138 rev/min.
- Calculate the speeds of the tips of both rotors.
- Compare to the speed of sound, $343 \mathrm{~m} / \mathrm{s}$.


## Centripetal Acceleration

- An object traveling in a circle, even if it moves with a constant speed, will have an acceleration
- The centripetal acceleration is due to the change in the direction of the velocity


## Centripetal Acceleration, cont.

- Centripetal refers to "center-seeking"
- The direction of the velocity changes
- The acceleration is directed toward the center of the circle of motion

(a)

(b)


## Centripetal Acceleration, final

- The magnitude of the centripetal acceleration is given by

$$
a_{c}=\frac{v^{2}}{r}
$$

- This direction is toward the center of the circle


## Centripetal Acceleration and Angular Velocity

- The angular velocity and the linear velocity are related ( $\mathrm{v}=\omega \mathrm{w}$ )
- The centripetal acceleration can also be related to the angular velocity

$$
a_{c}=\omega^{2} r
$$

## Total Acceleration

- The tangential component of the acceleration is due to changing speed
- The centripetal component of the acceleration is due to changing direction
- Total acceleration can be found from these components

$$
a=\sqrt{a_{t}^{2}+a_{c}^{2}}
$$

## Vector Nature of Angular Quantities

- Angular displacement, velocity and acceleration are all vector quantities
- Direction can be more completely defined by using the right hand rule
- Grasp the axis of rotation with your right hand
- Wrap your fingers in the direction of rotation
- Your thumb points in the direction of $\omega$



## Velocity Directions, Example

- In a, the disk rotates
clockwise, the velocity is into the page
- In b, the disk rotates counterclockwise, the velocity is out of the page

(a)

(b)



## Acceleration Directions

- If the angular acceleration and the angular velocity are in the same direction, the angular speed will increase with time
- If the angular acceleration and the angular velocity are in opposite directions, the angular speed will decrease with time


## Forces Causing Centripetal Acceleration

- Newton's Second Law says that the centripetal acceleration is accompanied by a force
$-F_{C}=m a_{c}$
$-F_{C}$ stands for any force that keeps an object following a circular path
- Tension in a string
- Gravity
- Force of friction



## Banked Curves

- A component of the normal force adds to the frictional force to allow
higher speeds

$$
\tan \theta=\frac{\mathrm{v}^{2}}{\mathrm{rg}}
$$

$$
\text { or } \mathrm{a}_{\mathrm{c}}=\mathrm{g} \tan \theta
$$




