# Physic² 121:
# Phundament°ls of Phy²ics I

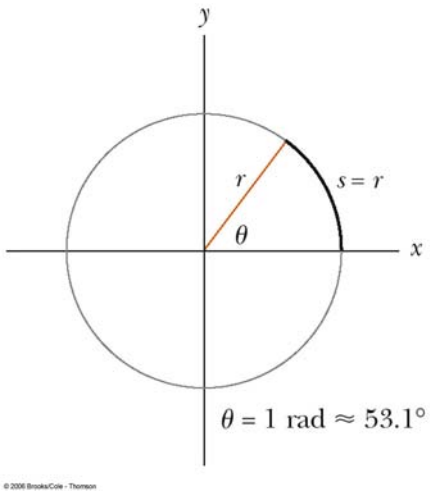
November 1, 2006

# Chapter 7

## Rotational Motion

# The Radian

- The radian is a unit of angular measure
- The radian can be defined as the arc length *s* along a circle divided by the radius *r*
- $$\theta = \frac{s}{r}$$



# More About Radians

- Comparing degrees and radians

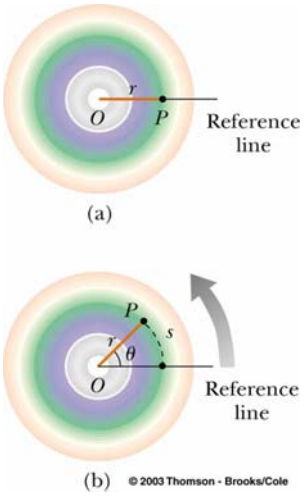
$$1\,\text{rad} = \frac{360^\circ}{2\pi} = 57.3^\circ$$

- Converting from degrees to radians

$$\theta\,[\text{rad}] = \frac{\pi}{180^\circ}\,\theta\,[\text{degrees}]$$

# Angular Displacement

- Axis of rotation is the center of the disk
- Need a fixed reference line
- During time t, the reference line moves through angle θ



# Rigid Body

- Every point on the object undergoes circular motion about the point O
- All parts of the object of the body rotate through the same angle during the same time
- The object is considered to be a **rigid body**
  - This means that each part of the body is fixed in position relative to all other parts of the body

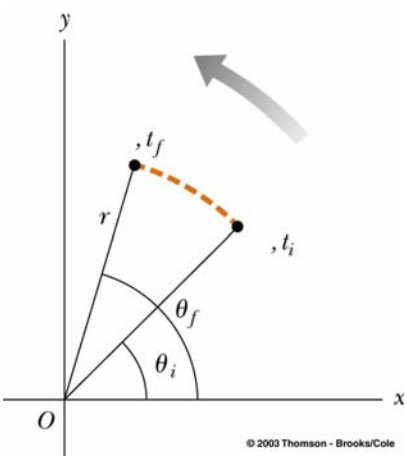
# Angular Displacement, cont.

- The *angular displacement* is defined as the angle the object rotates through during some time interval
- $$\Delta\theta = \theta_f - \theta_i$$
- The unit of angular displacement is the radian
- Each point on the object undergoes the same angular displacement

# Average Angular Speed

- The average angular speed, ω, of a rotating rigid object is the ratio of the angular displacement to the time interval

$$\omega_{av} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$



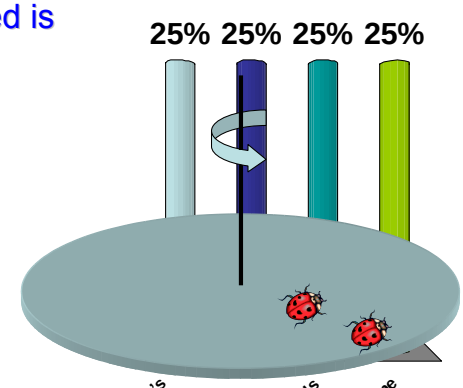
# Angular Speed, cont.

- The *instantaneous* angular speed is defined as the limit of the average speed as the time interval approaches zero
- Units of angular speed are radians/sec
  - rad/s
- Speed will be positive if θ is increasing (counterclockwise)
- Speed will be negative if θ is decreasing (clockwise)

?

A lady bug sits at the outer edge of a merry-go-round, and a gentleman bug sits halfway between her and the axis of rotation. The merry-go-round makes a complete revolution once each second. The gentleman bug's angular speed is

1. Half the lady bug's
2. The same as the lady bug's
3. Twice the lady bug's
4. Impossible to determine

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## Average Angular Acceleration

- The average angular acceleration $\alpha$ of an object is defined as the ratio of the change in the angular speed to the time it takes for the object to undergo the change:

$$\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

# Angular Acceleration, cont

- Units of angular acceleration are rad/s²
- Positive angular accelerations are in the counterclockwise direction and negative accelerations are in the clockwise direction
- When a rigid object rotates about a fixed axis, every portion of the object has the same angular speed and the same angular acceleration

# Angular Acceleration, final

- The sign of the acceleration does not have to be the same as the sign of the angular speed
- The instantaneous angular acceleration is defined as the limit of the average acceleration as the time interval approaches zero

# Analogies for Linear and Rotational Motion

| Linear Motion with $a$ Constant (Variables: $x$ and $v$) | Rotational Motion about a Fixed Axis with $\alpha$ Constant (Variables: $\theta$ and $\omega$) | |
|---|---|---|
| $v = v_i + at$ | $\omega = \omega_i + \alpha t$ | [7.7] |
| $\Delta x = v_i t + \frac{1}{2} a t^2$ | $\Delta\theta = \omega_i t + \frac{1}{2} \alpha t^2$ | [7.8] |
| $v^2 = v_i^2 + 2a\Delta x$ | $\omega^2 = \omega_i^2 + 2\alpha\Delta\theta$ | [7.9] |

## Relationship Between Angular and Linear Quantities

- Displacements

$$s = \theta r$$

- Speeds

$$v_t = \omega r$$

- Accelerations

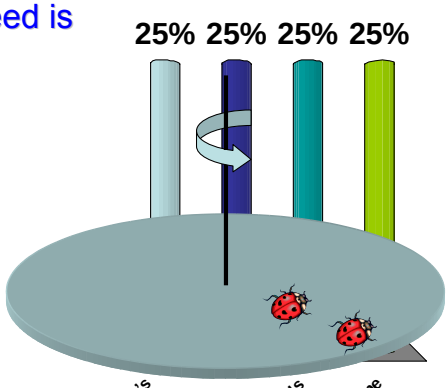
$$a_t = \alpha r$$

- Every point on the rotating object has the same angular motion
- Every point on the rotating object does *not* have the same linear motion
- *s* is a displacement with units of distance
- The subscript *"t"* refers to *tangential*

?

A lady bug sits at the outer edge of a merry-go-round, and a gentleman bug sits halfway between her and the axis of rotation. The merry-go-round makes a complete revolution once each second. The gentleman bug's tangential speed is

1. Half the lady bug's
2. The same as the lady bug's
3. Twice the lady bug's
4. Impossible to determine

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