

December 10, 2010

Physics 121

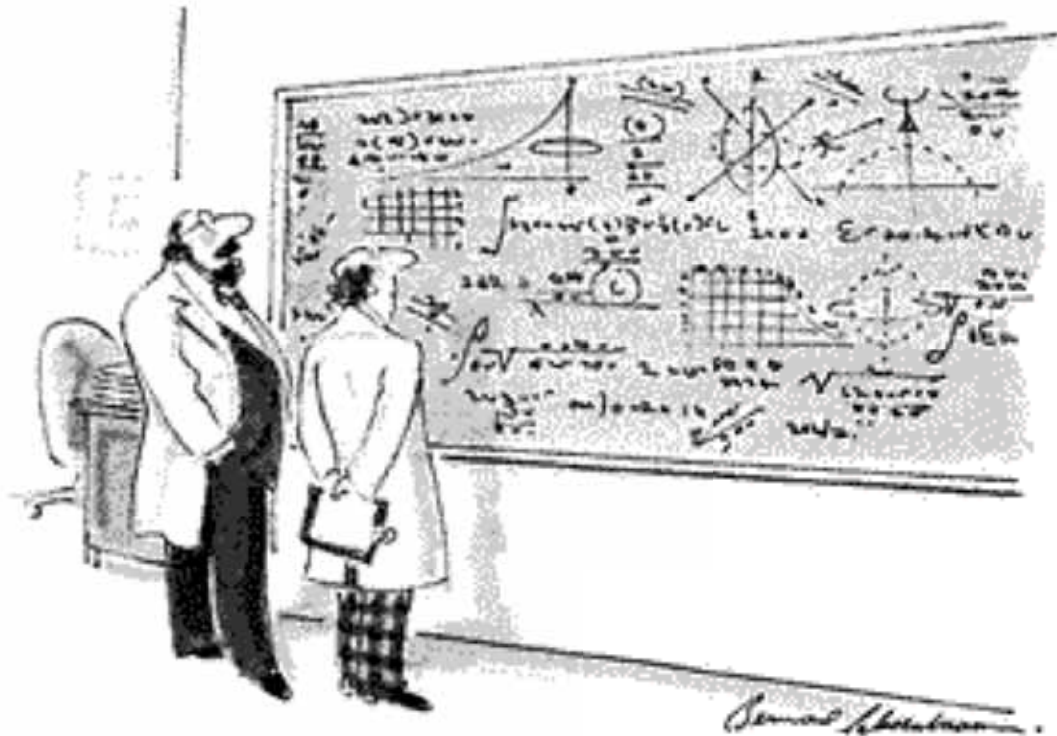
Prof. E. F. Redish

## ■ Theme Music: Count Basie & his Orchestra

### *The Party's Over*

Cartoon:

**Bernard  
Schoenbaum**



*"Oh, if only it were so simple."*

12/10/10

# Outline

- Modeling Matter:
  - The Kinetic Theory of Gases
    - Maxwell's Theoretical Model
    - Bouncing off the wall
- Relating to the Ideal Gas Law
- Making Sense of the Model
- Examples

# Surveys

We have a slight lead over Hamilton's classes (52%-44%) but we are nowhere near 70%!

- Still available until Sunday night!
- Campus evaluation (login at upper right)
  - <https://www.CourseEvalUM.umd.edu>
- On line
  - Post-instruction attitude survey (5 pts)
  - <http://perg-surveys.physics.umd.edu/MPEx2post.php>

# So where does the energy go?

- When we “lose” mechanical energy as a result of non-conservative forces, we know that since total energy is conserved, it must “hide” somewhere. Where?
- We say it “goes into thermal energy.” But what is the mechanism for thermal energy? What does it look like?
- Start with the simplest object – a gas.

# Maxwell's Model

- Assume  $n$  molecules/m<sup>3</sup> of mass  $m$  moving with an average speed  $v$ .
- What happens when a molecule hits the wall?

$$\Delta \vec{p}_{mol} = \vec{F}_{wall \rightarrow mol} \Delta t$$

$$\vec{F}_{wall \rightarrow mol} = -\vec{F}_{mol \rightarrow wall}$$

Average force of gas on the wall

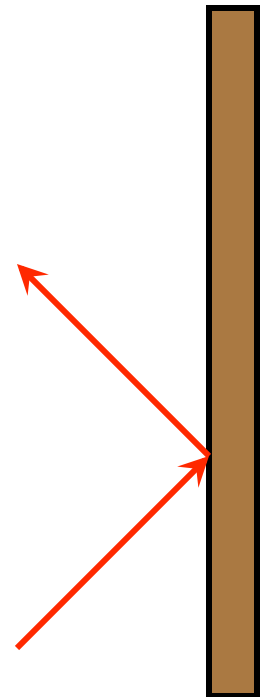
= (# of molecules hitting the wall in the time  $\Delta t$ )  
x (force each molecule exerts on the wall)

Only the  $x$ -component matters.

All we need to figure this out is our three basic equations, and a way to count the number of molecules hitting the wall.

$$F_{\text{wall} \rightarrow \text{molecule}} = m \frac{\Delta v_x}{\Delta t} = -F_{\text{molecule} \rightarrow \text{wall}}$$

$$\langle v_x \rangle = \frac{\Delta x}{\Delta t}$$



- How many molecules are in the box?

$n$  = number density  
= # / unit volume

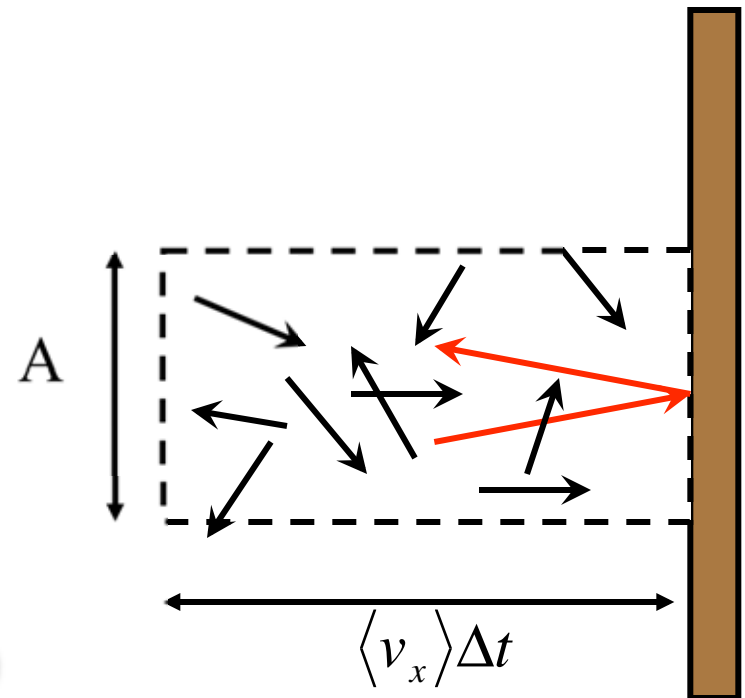
$$N = n \times (\text{Volume}) = nA(\langle v_x \rangle \Delta t)$$

- What's the average momentum change upon collision with a wall?

$$\Delta p_x = 2m\langle v_x \rangle$$

- What fraction of the molecules in the box will hit the wall in the time  $\Delta t$ ?

$\frac{1}{2}$  ( $\frac{1}{2}$  going left,  $\frac{1}{2}$  going right)



# Technical note

- How does the average  $x$ -velocity relate to the average speed of the molecule?

$$\langle v \rangle = \sqrt{\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle} = \sqrt{3\langle v_x^2 \rangle}$$

$$\langle v_x \rangle = \langle v \rangle / \sqrt{3}$$

- From here on out will drop all those averages – but we should keep in mind that that is what we really mean!



# Putting It All Together

$$F = N \frac{\Delta p}{\Delta t} = \frac{1}{2} (nA v_x \Delta t) \left( \frac{2m v_x}{\Delta t} \right) = n m v_x^2 A$$

Interpret

$$F = pA \qquad n = \frac{N}{V} \qquad v_x^2 = \frac{1}{3} v^2$$

$$pA = \frac{1}{3} \frac{N}{V} m v^2 A$$

$$pV = N \left( \frac{1}{3} m v^2 \right) = N \frac{2}{3} \left( \frac{1}{2} m v^2 \right)$$

# The Behavior of a Dilute Gas

- We have three properties that describe a gas: pressure ( $p$ ), volume ( $V$ ) and temperature ( $T$ ). How do they relate?
- A series of experiments show us:
  - For a given sample of a gas, the combination  $pV/T$  is a constant if  $T$  is measured in Kelvin (degrees C starting from absolute zero = -273 C).
  - The constant is proportional to the amount of gas we have.
  - For different gases, the constant is proportional to the chemical combining weight (# of moles).

# The Ideal Gas Law

- The result is written

$$pV = n_{\text{moles}}RT$$

- where  $R$  is a constant independent of the kind of gas you have.
- $R = 8.31 \text{ J/mol-}^\circ\text{K}$
- This result holds for any dilute gas.  
(It has corrections if the gas gets too dense.)

# Interpreting the Ideal Gas Law

- To relate this to our model, note that since the number of molecules in one mole is the same (Avogadro's number)

$$N = n_{\text{moles}} N_A$$

where  $N_A = 6.02 \times 10^{26}$  /kg-mole

- This allows us to make the connection to our molecular model.

# Put the equations together

$$pV = N \frac{2}{3} \left( \frac{1}{2} m v^2 \right) \qquad pV = nRT$$

Make the  $N$  parts look alike.

$$n = N / N_A$$

$$pV = N \left( \frac{R}{N_A} \right) T$$

$$\text{Define: } k_B = \left( \frac{R}{N_A} \right) \text{ so } pV = N k_B T$$

# Interpreting



- The “physicist’s form” of the ideal gas law lets us interpret where the  $p$  comes from and what  $T$  means.
- $p$  arises from molecules hitting the wall and transferring momentum to it;
- $T$  corresponds to the KE of one molecule (up to a constant factor).

$$p = Nm v_x^2$$

$$k_B T = \frac{2}{3} \left( \frac{1}{2} m v^2 \right)$$

# The Ideal Gas Law

Chemist's  
form

$$pV = n_{\text{moles}}RT$$

$$n_{\text{moles}} = \frac{N}{N_A}$$

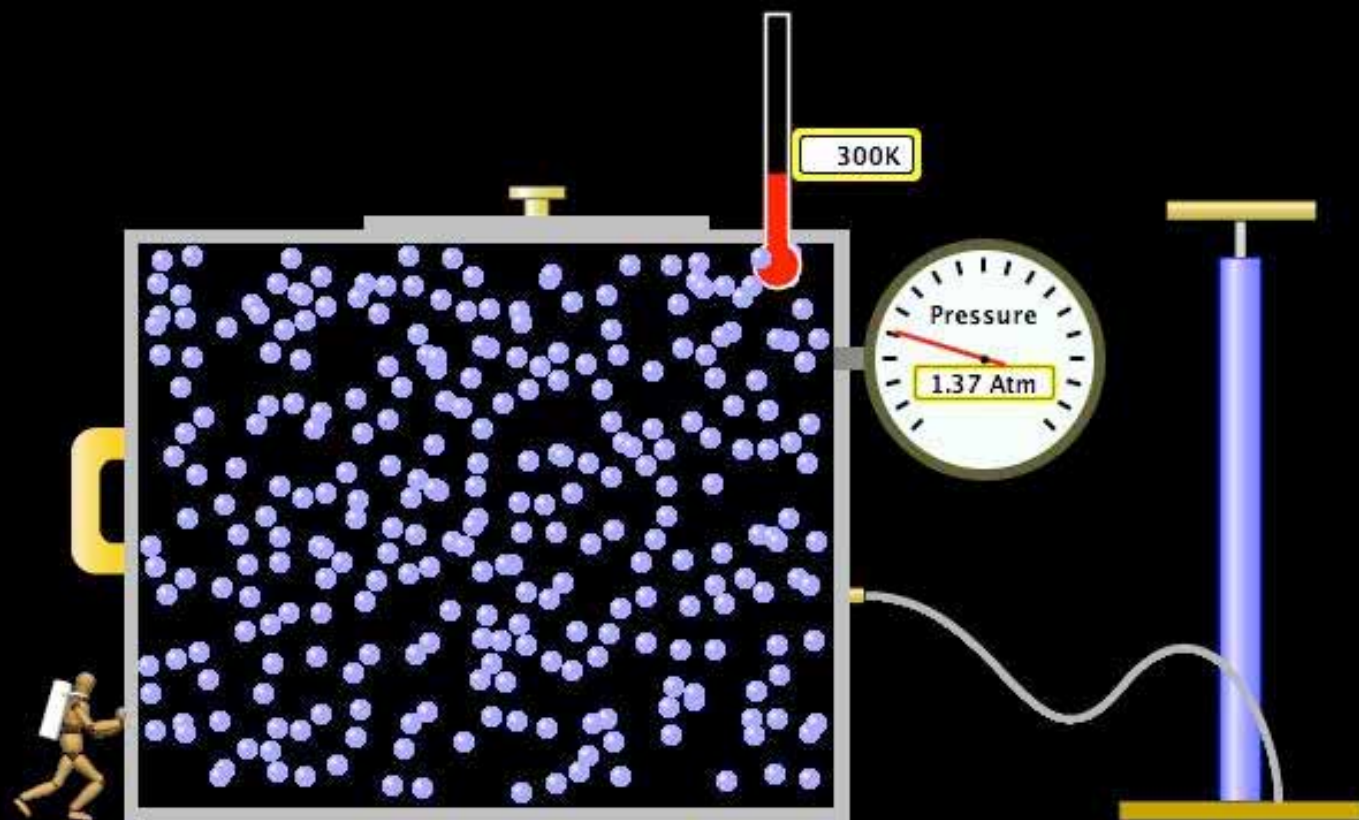
$$R = k_B N_A$$

Physicist's  
form

$$pV = Nk_B T$$

$$p = nmv_x^2$$

$$\frac{3}{2} k_B T = \frac{1}{2} mv^2$$



Heat Control

— Add  
0  
— Remove

Gas in Pump

☒ Heavy Species  
☐ Light Species



Constant Parameter

☐ Volume ☐ Pressure  
☐ Temperature ☒ None

Gas in Chamber

Heavy Species 279  
Light Species 0

Gravity

0 ————— Lots

Tools &amp; Options

&lt;&lt; Hide Tools

- ☐ Layer tool
- ☐ Ruler
- ☐ Species information
- ☐ Stopwatch
- ☐ Energy histograms
- ☐ Center of mass markers

Advanced Options &gt;&gt;

Reset

Help!





# Question



- If the molecules in a gas are all moving freely except when they collide with each other (rarely), why don't they fall to the ground?
- Consider a FBD for a gas molecule.

An interesting textbook problem:  
How would you solve this?



- From the engineering version of our (un-used) textbook:
  - On a hot (35 C) day, you perspire 1.0 kg of water during your workout. What volume is occupied by the evaporated water?

# Recap: Kinetic Theory 1

- Our model of matter as made up of lots of little moving particles, lets us resolve some apparent inconsistencies.
- Newton's laws tell us that motion continues forever unless something unbalanced tries to stop it, yet we observe motion always dies away.
- Our model lets us “hide” the energy of motion that has “died away” at the macro level in the internal motion.

# Recap: Kinetic Theory 2

- The model unifies the idea of heat and temperature with our ideas of motion.
- The model opens the possibility of using the hidden energy stored in matter as a result of its (non-0) temperature.
- This leads to heat engines, refrigerators, and the first industrial revolution.

# For the final!

- Final exam: Friday 12/17, 8-12 AM, here.\*
- Review slides will soon be posted on the Lecture Slides page (12/17 date).
- Office hours in the CC T 2-4, Th 12-2.
- Q&A session here W 2:00-3:30.

\*Unless you have arranged to take it 1-5 in room 1303.