December 8, 2010 Physics 121 Prof. E. F. Redish

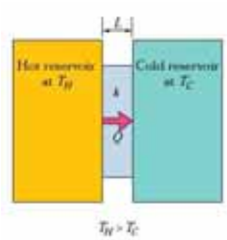
- **Theme Music:** **Mason Williams**
  Classical Gas
- **Cartoon:** **Sydney Harris**

Outline

- Heat flow
  - Fourier's law
  - Insulation in your house
- Modeling Matter:
  The Kinetic Theory of Gases
  - Maxwell's Theoretical Model
  - Bouncing off the wall
- Relating to the Ideal Gas Law
- Making Sense of the Model

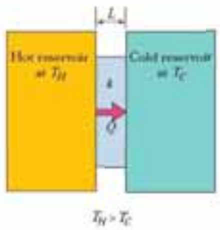
Heat Flow by Conduction

- Simplest case (again)
  - Hot block at T_H
  - Cold block at T_C
  - Connecting block that carries ("conducts") thermal energy from the hot block to the cold.



Creating an equation

- Φ = Flow
  = heat energy/sec
  $[\Phi]$ = Joules/s = Watts
- What drives the flow?
- How does the rate of flow depend on the property of the connecting block?



The Heat Flow Equation

$$\Delta T = Z\Phi$$

- We expect the flow to
  - Be less for a longer block (L)
  - Be more for a wider block (A)

$$Z = \rho\frac{L}{A}$$

- ρ = thermal resistivity – a property of the kind of substance the block is made of

A more standard form

- We have written the heat flow equation to have it match the HP equation. It is more standardly written this way:

Heat flow per unit area → $\phi = \frac{\Phi}{A}$

$k = \frac{1}{\rho}$ ← Thermal conductance

- The equation then becomes

$$\Delta T = Z\Phi = \frac{\rho L}{A}\Phi = \left(\frac{L}{k}\right)\left(\frac{\Phi}{A}\right)$$

$$\Delta T = R\phi$$

R ← Thermal resistance (R-value)

Some thermal conductances

Material	*k* (W/m-C)	Material	*k* (W/m-C)
Steel	12-45	Wood	0.4
Aluminum	200	Insulation	0.04
Copper	380	Air	0.025

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So where does the energy go?

- When we "lose" mechanical energy as a result of non-conservative forces, we know that since total energy is conserved, it must "hide" somewhere. Where?
- We say it "goes into thermal energy." But what is the mechanism for thermal energy? What does it look like?
- Start with the simplest object – a gas.

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Modeling the Gas

- One of the most obvious properties of a gas is that it's "springy". (Think of pressing on a bicycle pump.)
- Squeezing on it makes it smaller, but the more you squeeze, the more it presses back.
- How can we imagine how this might come about?

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Some Possible Models

- *The "Charmin" Model*: (Newton, Dalton)
  A gas is made up of atoms packed closely together. The "squeezability" of the gas comes from the "squeezability" of the atoms.
- *The "Spinning Aura" Model*: (Davy, Joule)
  A gas is made up of atoms that have "auras" and are spinning. As the density decreases, the auras expand to fill space.
- *The "Empty Space" Model*: (D. Bernoulli, Maxwell)
  A gas is made up of very small objects (atoms) moving very fast.

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Maxwell's Model

- Assume n molecules/m^3 of mass m moving with an average speed v.
- What happens when a molecule hits the wall?

$$\Delta\vec{p}_{mol} = \vec{F}_{wall\rightarrow mol}\Delta t$$
$$\vec{F}_{wall\rightarrow mol} = -\vec{F}_{mol\rightarrow wall}$$

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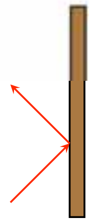
Average force of gas on the wall
= (# of molecules hitting the wall in the time Δt)
x (force each molecule exerts on the wall)

Only the x-component matters.

All we need to figure this out is our three basic equations, and a way to count the number of molecules hitting the wall.



$$F_{wall \to molecule} = m\frac{\Delta v_x}{\Delta t} = -F_{molecule \to wall}$$

$$\langle v_x \rangle = \frac{\Delta x}{\Delta t}$$

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- How many molecules are in the box?

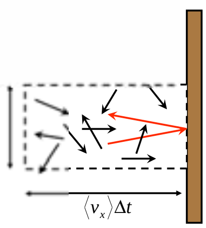
n = number density = # / unit volume

$$N = n \times (\text{Volume}) = nA(\langle v_x \rangle \Delta t)$$

- What's the average momentum change upon collision with a wall?

$$\Delta p_x = 2m\langle v_x \rangle$$

- What fraction of the molecules in the box will hit the wall in the time Δt?



½ (½ going left, ½ going right)

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Technical note

- How does the average x-velocity relate to the average speed of the molecule?

$$\langle v \rangle = \sqrt{\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle} = \sqrt{3\langle v_x^2 \rangle}$$

$$\langle v_x \rangle = \langle v \rangle / \sqrt{3}$$

- From here on out will drop all those averages
  – but we should keep in mind that
  that is what we really mean!

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Putting It All Together

$$F = N\frac{\Delta p}{\Delta t} = \tfrac{1}{2}(nAv_x\Delta t)\left(\frac{2mv_x}{\Delta t}\right) = nmv_x^2 A$$

Interpret

$$F = pA \qquad n = \frac{N}{V} \qquad v_x^2 = \tfrac{1}{3}v^2$$

$$pA = \tfrac{1}{3}\frac{N}{V}mv^2 A$$

$$pV = N(\tfrac{1}{3}mv^2) = N\tfrac{2}{3}\left(\tfrac{1}{2}mv^2\right)$$

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The Behavior of a Dilute Gas

- We have three properties that describe a gas: pressure (*p*), volume (*V*) and temperature (*T*). How do they relate?
- A series of experiments show us:
  - For a given sample of a gas, the combination *pV/T* is a constant if *T* is measured in Kelvin (degrees C starting from absolute zero = -273 C).
  - The constant is proportional to the amount of gas we have.
  - For different gases, the constant is proportional to the chemical combining weight (# of moles).

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The Ideal Gas Law

- The result is written

$$pV = n_{moles}RT$$

- where *R* is a constant independent of the kind of gas you have.
- *R* = 8.31 J/mol-ºK
- This result holds for any dilute gas. (It has corrections if the gas gets too dense.)

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Interpreting the Ideal Gas Law

- To relate this to our model, note that since the number of molecules in one mole is the same (Avogadro's number)

$$N = n_{moles} N_A$$

where $N_A = 6.02 \times 10^{23}$.

- This allows us to make the connection to our molecular model.

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Put the equations together

$$pV = N\tfrac{2}{3}(\tfrac{1}{2}mv^2) \qquad pV = nRT$$

Make the N parts look alike.

$$n = N / N_A$$

$$pV = N\left(\frac{R}{N_A}\right)T$$

Define: $k_B = \left(\frac{R}{N_A}\right)$ so $pV = Nk_BT$

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Interpreting

- The "physicist's form" of the ideal gas law lets us interpret where the p comes from and what T means.
- p arises from molecules hitting the wall and transferring momentum to it;
- T corresponds to the KE of one molecule (up to a constant factor).

$$p = Nmv_x^2 \qquad k_BT = \tfrac{2}{3}\left(\tfrac{1}{2}mv^2\right)$$

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