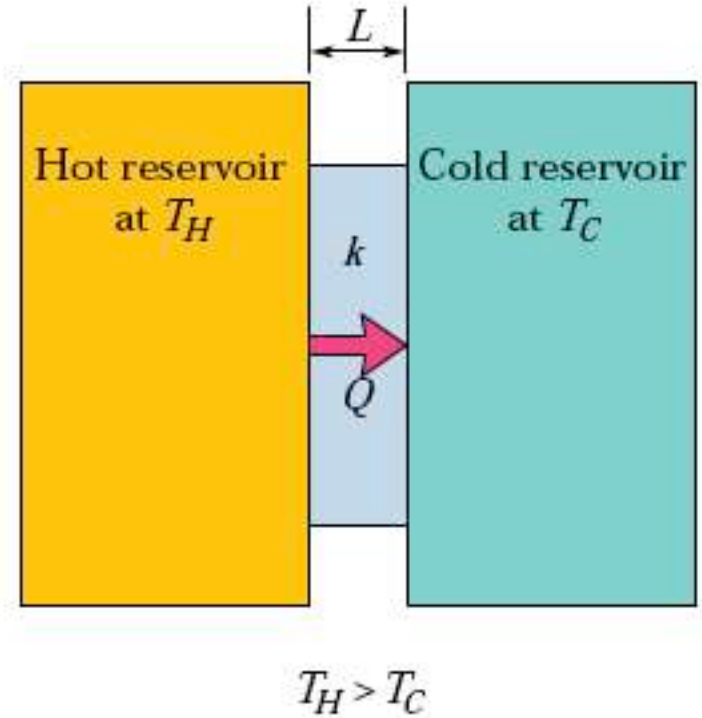
- **Theme Music:** **Mason Williams**
  Classical Gas
- **Cartoon:** **Sydney Harris**

Outline

- Heat flow
  - Fourier's law
  - Insulation in your house
- Modeling Matter:
  The Kinetic Theory of Gases
  - Maxwell's Theoretical Model
  - Bouncing off the wall
- Relating to the Ideal Gas Law
- Making Sense of the Model

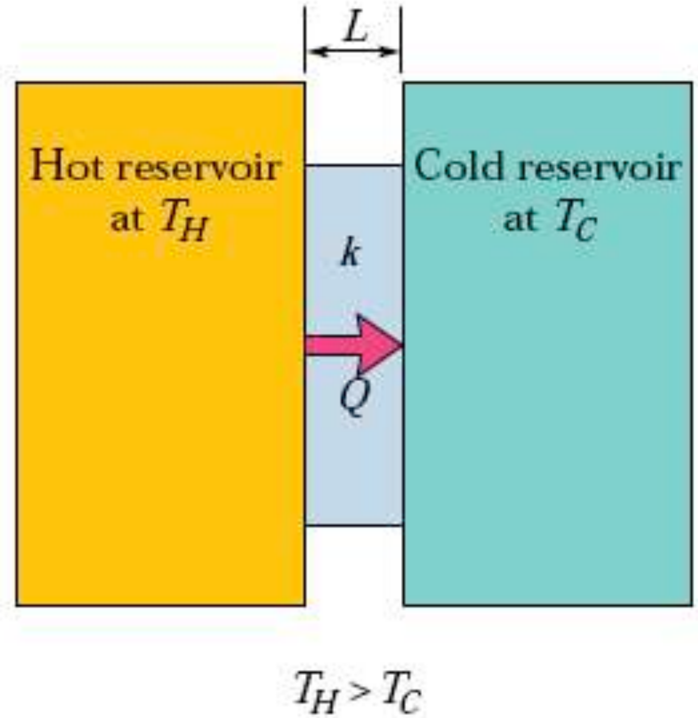
Heat Flow by Conduction

- Simplest case (again)
  - Hot block at T_H
  - Cold block at T_C
  - Connecting block that carries ("conducts") thermal energy from the hot block to the cold.



Creating an equation

- Φ = Flow
  = heat energy/sec
  $[\Phi]$ = Joules/s = Watts
- What drives the flow?
- How does the rate of flow depend on the property of the connecting block?



The Heat Flow Equation

$$\Delta T = Z\Phi$$

- We expect the flow to
  - Be less for a longer block (L)
  - Be more for a wider block (A)

$$Z = \rho \frac{L}{A}$$

- ρ = thermal resistivity – a property of the kind of substance the block is made of

A more standard form

- We have written the heat flow equation to have it match the HP equation. It is more standardly written this way:

Heat flow per unit area

$$\phi = \frac{\Phi}{A}$$

$$k = \frac{1}{\rho}$$

Thermal conductance

- The equation then becomes

$$\Delta T = Z\Phi = \frac{\rho L}{A}\Phi = \left(\frac{L}{k}\right)\left(\frac{\Phi}{A}\right)$$

$$\Delta T = R\phi$$

Thermal resistance (R-value)

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Insulating Your Home.

Friday, April 13, 2007, 11:18 PM - Insulation

With the rising cost of hydrocarbon fuel, we should look at ways to save money in the heating and cooling of our home. Insulation is the first thing that comes to mind. When talking about insulation, you hear the term R- value. The R-value is a measure of thermal resistance used in heat transfer problems. R-values can be calculated from thermal conductivity, k, and the thickness of the material, t: R = t/k. Thus, for 100 mm thickness, it is possible to calculate that a fiberglass blanket has a value of 2, whereas aerogel has a value of 5.9. I'm confused. Just think of it this way: the Higher the R - Value the Better it insulates you home from temperature changes.

Some thermal conductances

Material	*k* (W/m-C)	Material	*k* (W/m-C)
Steel	12-45	**Wood**	0.4
Aluminum	200	**Insulation**	0.04
Copper	380	**Air**	0.025

So where does the energy go?

- When we "lose" mechanical energy as a result of non-conservative forces, we know that since total energy is conserved, it must "hide" somewhere. Where?
- We say it "goes into thermal energy." But what is the mechanism for thermal energy? What does it look like?
- Start with the simplest object – a gas.

Modeling the Gas

- One of the most obvious properties of a gas is that it's "springy". (Think of pressing on a bicycle pump.)
- Squeezing on it makes it smaller, but the more you squeeze, the more it presses back.
- How can we imagine how this might come about?

Some Possible Models

- *The "Charmin" Model*: (Newton, Dalton)
  A gas is made up of atoms packed closely together.
  The "squeezability" of the gas comes from
  the "squeezability" of the atoms.

- *The "Spinning Aura" Model*: (Davy, Joule)
  A gas is made up of atoms that have "auras"
  and are spinning. As the density decreases,
  the auras expand to fill space.

- *The "Empty Space" Model*: (D. Bernoulli, Maxwell)
  A gas is made up of very small objects (atoms)
  moving very fast.

Maxwell's Model

- Assume n molecules/m^3 of mass m moving with an average speed v.
- What happens when a molecule hits the wall?

$$\Delta\vec{p}_{mol} = \vec{F}_{wall\rightarrow mol}\Delta t$$

$$\vec{F}_{wall\rightarrow mol} = -\vec{F}_{mol\rightarrow wall}$$

Average force of gas on the wall

= (# of molecules hitting the wall in the time Δt)

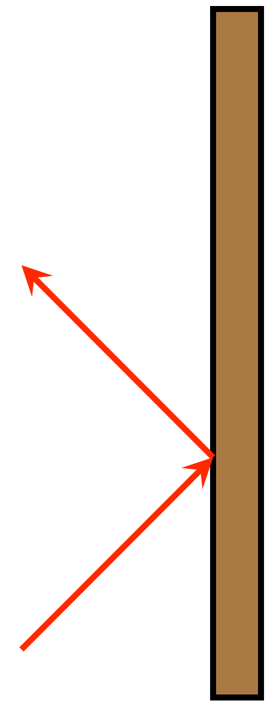
x (force each molecule exerts on the wall)

Only the *x*-component matters.

All we need to figure this out is our three basic equations, and a way to count the number of molecules hitting the wall.

$$F_{wall \to molecule} = m\frac{\Delta v_x}{\Delta t} = -F_{molecule \to wall}$$

$$\langle v_x \rangle = \frac{\Delta x}{\Delta t}$$



n = number density
= # / unit volume

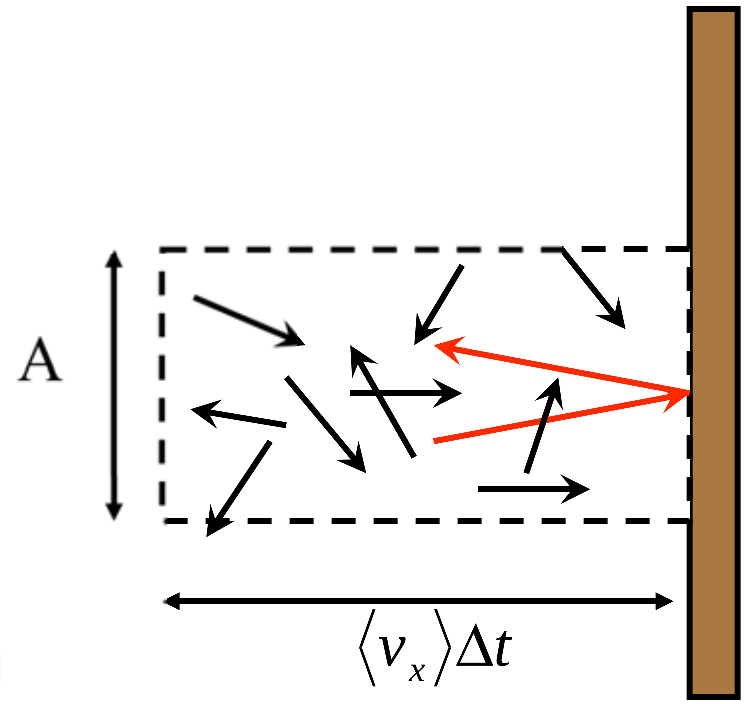
- How many molecules are in the box?

$$N = n \times (\text{Volume}) = nA\left(\langle v_x \rangle \Delta t\right)$$

- What's the average momentum change upon collision with a wall?

$$\Delta p_x = 2m\langle v_x \rangle$$

- What fraction of the molecules in the box will hit the wall in the time Δt?



½ (½ going left, ½ going right)

Technical note

- How does the average x-velocity relate to the average speed of the molecule?

$$\langle v \rangle = \sqrt{\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle} = \sqrt{3\langle v_x^2 \rangle}$$

$$\langle v_x \rangle = \langle v \rangle / \sqrt{3}$$

- From here on out will drop all those averages – but we should keep in mind that that is what we really mean!

Putting It All Together

$$F = N\frac{\Delta p}{\Delta t} = \tfrac{1}{2}(nAv_x\Delta t)\left(\frac{2mv_x}{\Delta t}\right) = nmv_x^2A$$

Interpret

$$F = pA \qquad n = \frac{N}{V} \qquad v_x^2 = \tfrac{1}{3}v^2$$

$$pA = \tfrac{1}{3}\frac{N}{V}mv^2A$$

$$pV = N(\tfrac{1}{3}mv^2) = N\tfrac{2}{3}\left(\tfrac{1}{2}mv^2\right)$$

The Behavior of a Dilute Gas

- We have three properties that describe a gas: pressure (p), volume (V) and temperature (T). How do they relate?
- A series of experiments show us:
  - For a given sample of a gas, the combination pV/T is a constant if T is measured in Kelvin (degrees C starting from absolute zero = -273 C).
  - The constant is proportional to the amount of gas we have.
  - For different gases, the constant is proportional to the chemical combining weight (# of moles).

The Ideal Gas Law

- The result is written

$$pV = n_{moles}RT$$

- where R is a constant independent of the kind of gas you have.
- $R = 8.31$ J/mol-^{o}K
- This result holds for any dilute gas. (It has corrections if the gas gets too dense.)

Interpreting the Ideal Gas Law

- To relate this to our model, note that since the number of molecules in one mole is the same (Avogadro's number)

$$N = n_{moles} N_A$$

where $N_A = 6.02 \times 10^{23}$.

- This allows us to make the connection to our molecular model.

Put the equations together

$$pV = N\tfrac{2}{3}\left(\tfrac{1}{2}mv^2\right) \qquad\qquad pV = nRT$$

Make the N parts look alike.

$$n = N / N_A$$

$$pV = N\left(\frac{R}{N_A}\right)T$$

Define: $k_B = \left(\dfrac{R}{N_A}\right)$ so $pV = Nk_BT$

Interpreting

- The "physicist's form" of the ideal gas law lets us interpret where the p comes from and what *T* means.
- *p* arises from molecules hitting the wall and transferring momentum to it;
- *T* corresponds to the KE of <u>one</u> molecule (up to a constant factor).

$$p = Nmv_x^2 \qquad k_B T = \tfrac{2}{3}\left(\tfrac{1}{2}mv^2\right)$$