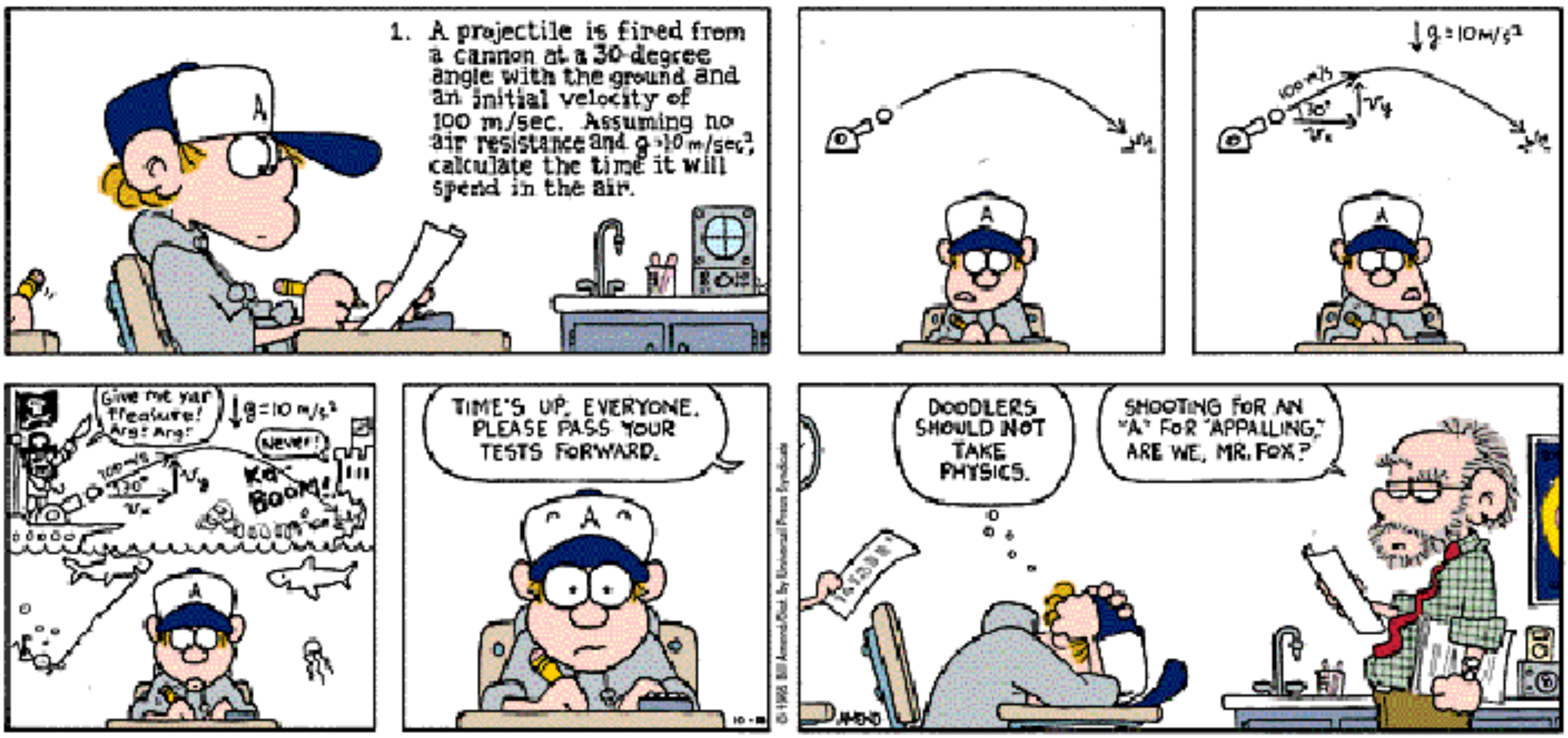
- **Theme Music: Duke Ellington**
  Take the A Train

- **Cartoon: Bill Amend *FoxTrot***



Newton's Laws

- Newton 0:
  - An object responds to the forces it feels at the instant it feels them. There are 4 kinds of forces (so far): 3 touching forces (*N, T, f*) plus the non-touching force of gravity (*W*).
- Newton 1:
  - An object on which all forces are balanced keeps moving with the same velocity (which may = 0).
- Newton 2:
  - An object that is acted upon by other objects changes its velocity so that the acceleration is proportional to the net force and inversely proportional to the object's mass.

$$\vec{a} = \vec{F}^{net} / m$$

- Newton 3:
  - When two objects interact the forces they exert on each other are equal and opposite.

$$\vec{F}_{A \to B} = -\vec{F}_{B \to A}$$

Classification of Forces

$$\vec{F}_{A\to B} \quad \text{where } F \text{ is either } N, T, f, \text{ or } W$$

- Physical forces are interactions – what two objects do to each other that tends to change each other's velocity.
- Touching forces
  - perpendicular to the surface and pressing in (NORMAL – *N*)
  - hooked to the surface and pulling out (TENSION – *T*)
  - parallel to the touching surfaces and opposing the relative motion of the surfaces (FRICTION – *f*)
- Non-touching forces
  - the earth pulling an object down (GRAVITY – *W*)

$$T = k\Delta s \text{ (spring)}$$

$$f_{A\to B} \le \mu_{AB} N_{A\to B}$$

$$\vec{W}_{E\to A} = m_A \vec{g}$$

The Impulse-Momentum Theorem

- Newton 2 $\vec{a} = \vec{F}^{net} / m$
- Put in definition of *a* $\frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{F}^{net}}{m}$
- Cross Multiply $m\Delta \vec{v} = \vec{F}^{net} \Delta t$
- Define Impulse $\vec{\mathscr{I}}^{net} = \vec{F}^{net} \Delta t$
- Define Momentum $\vec{p} = m\vec{v}$
- Combine to get Impulse-Momentum Theorem

$$\Delta \vec{p} = \vec{I}^{net}$$

Momentum Conservation

- If two objects interact with each other in such a way that the external forces on the pair cancel, then total momentum is conserved.

$$\Delta\left(m_A\vec{v}_A + m_B\vec{v}_B\right) = 0$$

$$m_A\vec{v}_A^i + m_B\vec{v}_B^i = m_A\vec{v}_A^f + m_B\vec{v}_B^f$$

Kinetic Energy and Work

- Consider an object moving along a line feeling a single force, *F*. When is moves a distance Δx (in the direction of or opposite to the force), how much does its speed change?

Definitions:

Kinetic energy = $\frac{1}{2}mv^2$

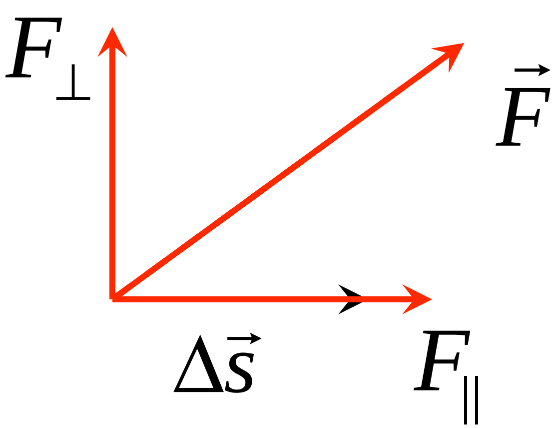
Work done by a force F = $F\ \Delta x$

Result

$$\Delta(\tfrac{1}{2}mv^2) = F^{net}\ \Delta x$$

Work in another direction: The dot product

- Suppose we are moving along a line, but the force we are interested in in pointed in another direction? (How can this happen?)
- Only the part of the force in the direction of the motion counts to change the speed (energy).



$$\text{Work} = F_{||}\,\Delta s = F\cos\theta\,\Delta s \equiv \vec{F}\cdot\Delta\vec{s}$$

Calculating dot products

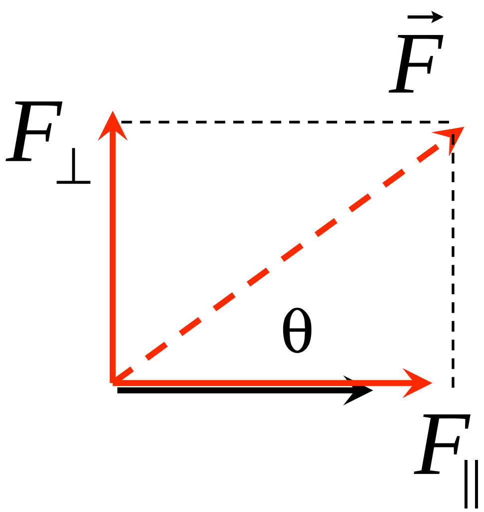
$$F_{\parallel}\,\Delta s \equiv \vec{F}\cdot\Delta\vec{s}$$

$$\vec{F}\cdot\Delta\vec{s} = F\cos\theta\,\Delta s$$

In general, for any two vectors that have an angle θ between them, the dot product is defined to be

$$\vec{a}\cdot\vec{b} = ab\cos\theta$$

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y$$



The dot product is a scalar.
Its value does not depend on the coordinate system we select.

Potential Energy

- For some forces (gravity, springs) the amount of work done only depends of the change in position.
- Such forces are called conservative.
- For these forces the work done by them is written

$$\vec{F} \cdot \Delta\vec{r} = -\Delta U$$

- *U* is called a *potential energy*.

Potential Energy: Gravity

- Since the work term also looks like a change, when there are no other forces, we can bring it to the left and get a *conservation law*.

$$\Delta(\tfrac{1}{2}mv^2 + mgh) = 0$$

$$U_g = mgh$$

- We interpret the quantity *mgh* as a new kind of energy — *gravitational potential energy*.

Potential Energy: Springs

- Since the work term also looks like a change, when there are no other forces, we can bring it to the left and get a *conservation law*.

$$\Delta\left(\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2\right) = 0$$

$$U_s = \tfrac{1}{2}kx^2$$

- We interpret the quantity $\frac{1}{2}kx^2$ as a new kind of energy — *spring potential energy*.

Non-conservative forces/situations

- Friction / drag
  - Three kinds of forces drain ME: friction (indep. of v), viscosity (prop. to v), drag (prop. to v^2)
- Breaking / crushing
  - Normal forces are typically springy and conservative.
  - If an object is deformed too much, the structure can change (break) and drain ME.
- Chemical reactions
  - Chemical structure is another place energy can be stored. It can create or drain ME.

Using the Work-Energy Theorem

- The work-energy theorem is most useful when you want to find how a position and velocity are related but you don't need to know "when" (anything about times).
- To use the WETh you need to
  - choose your objects
  - decide what forces act on them (FBD!)
  - use PEs for conservative forces
  - calculate work for non-conservative forces.

A~~n~~ Mechanical Energy Conservation Theorem

- Suppose our system has both gravity and spring forces
  - The only force that changes the object's speed is gravity.
  - Other forces (normal forces) can change direction.
  - Friction must be negligible.
- Using this can be tricky. Typically,
  - Springs act in one part of the problem, gravity in another. One must focus on the physics to decide what is appropriate.

$$\Delta(\tfrac{1}{2}mv^2 + mgh + \tfrac{1}{2}kx^2) = 0$$

$$\tfrac{1}{2}mv_i^2 + mgh_i + \tfrac{1}{2}kx_i^2 = \tfrac{1}{2}mv_f^2 + mgh_f + \tfrac{1}{2}kx_f^2$$

Dimensions and Units of Energy

- $[1/2\ mv^2] = M\text{-}(L/T)^2 = ML^2/T^2$
- $1\ kg\text{-}m^2/s^2 = 1\ N\text{-}m = 1$ Joule
- Other units of energy are common (and will be discussed later)
  - Calorie
  - eV (electron Volt)
  - erg ($=1\ g\text{-}cm^2/s^2$)

Power

- An interesting question about work and energy is the rate at which energy is changed or work is done. This is called *power*.

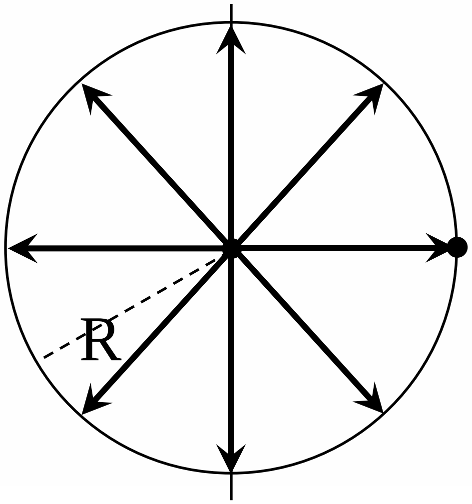
$$\text{Power} = \frac{\text{Energy change}}{\text{time to make the change}}$$

$$= \frac{\Delta W}{\Delta t} = \vec{F}^{net} \cdot \frac{\Delta \vec{r}}{\Delta t} = \vec{F}^{net} \cdot \vec{v} \quad \text{(for mechanical work)}$$

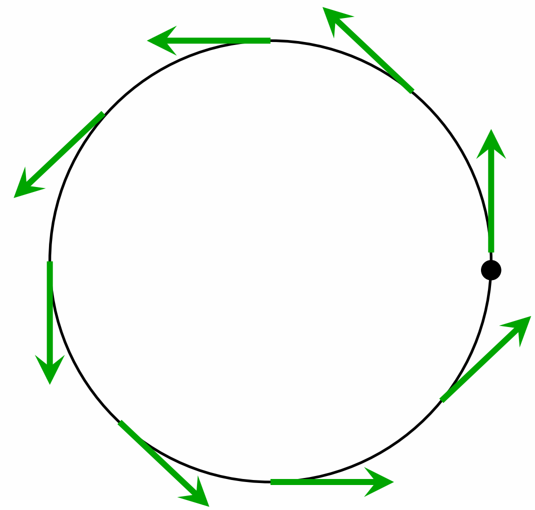
- Unit of power

1 Joule/sec = 1 Watt

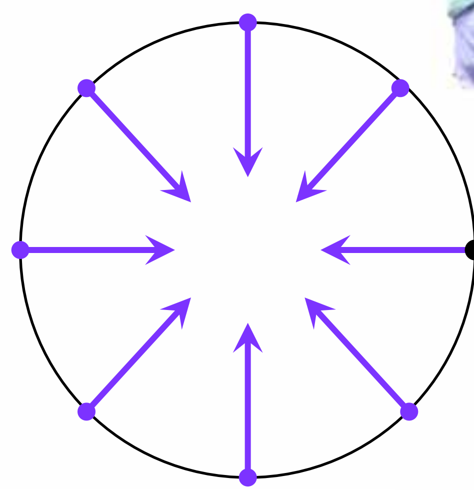
Uniform Circular Motion



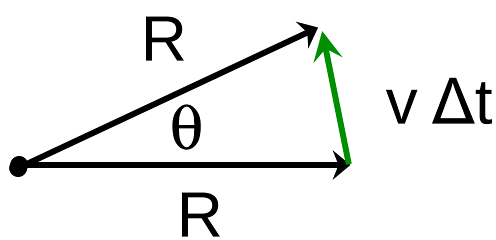
Position

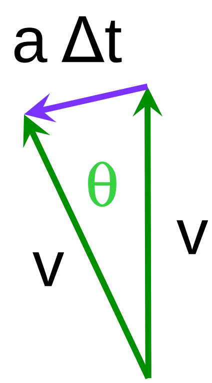


Velocity



Acceleration





$$\frac{v\,\Delta t}{R} = \frac{a\,\Delta t}{v} \qquad \frac{a}{v} = \frac{v}{R} \qquad a = \frac{v^2}{R}$$

Uniform Circular Motion: Forces

$$\vec{a} = \frac{\vec{F}^{\,net}}{m}$$

always

$$\vec{a} = -\frac{v^2}{R}\hat{r}$$

in order for the object to move in a circle with constant speed.

$$\frac{\vec{F}^{\,net}}{m} = -\frac{v^2}{R}\hat{r}$$

Therefore, to do this, we need a net force.

$$\vec{F}^{\,net} = -\frac{mv^2}{R}\hat{r}$$

Rotational Kinematics: Polar Description of Motion

- Describing the angular position of an object.
  - Angle (radians) θ
  - Angular velocity ω
  - Angular acceleration α

$$\theta \text{ (in radians)} = \frac{2\pi}{360}\theta \text{ (in degrees)}$$

$$\langle\omega\rangle = \frac{\Delta\theta}{\Delta t} \qquad \langle\alpha\rangle = \frac{\Delta\omega}{\Delta t}$$

Uniform motion: $\Delta\theta = \omega_0 \, \Delta t$