November 10, 2010 Physics 121 Prof. E. F. Redish

- **Theme Music: Rhett Butler**
  Spin
- **Cartoon: Bill Watterson**
  Calvin & Hobbes

11/10/10 Physics 121 1

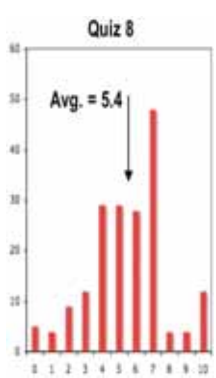
Outline

- Go over Quiz 8
- Extended objects: Center of Mass
- Torque: the balance rule
- Torque as a vector

11/10/10 Physics 121 2

Quiz 8

	8.1		8.2		8.3
a	0%	1	18%	a=b>c=d	46%
b	61%	2	20%	b>c>a>d	12%
c	27%	3	22%	b>a=c>d	11%
d	8%	4	26%		
e	2%	5	12%		
n	1%	6	1%		



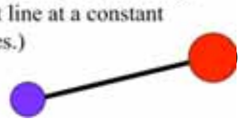
11/10/10 Physics 121 3

How can we get away with it?

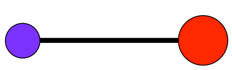
- Up to now we have considered objects that were small enough that we could ignore their size.
- How can we get away with it?
- Consider extended objects. Start with the simplest example.

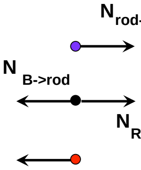
A Simple Extended Object

- In order to understand what's happening, let's take the simplest case we can imagine of an extended object:
  - Two small masses connected by a rigid (nearly massless) rod sliding on a frictionless table.
  - If they were considered as a single object, they should move on a straight line at a constant speed. (No external forces.)



Analysis





$$m_R\vec{a}_R = \vec{N}_{rod\to R}$$
$$m_B\vec{a}_B = \vec{N}_{rod\to B}$$
$$m_{rod}\vec{a}_{rod} = \vec{N}_{R\to rod} + \vec{N}_{B\to rod}$$
$$m_R\vec{a}_R + m_B\vec{a}_B + m_{rod}\vec{a}_{rod} = \cancel{\vec{N}_{rod\to R}} + \cancel{\vec{N}_{R\to rod}} + \cancel{\vec{N}_{B\to rod}} + \cancel{\vec{N}_{rod\to B}}$$
$$m_R\vec{a}_R + m_B\vec{a}_B = 0$$

The Center of Mass

$$m_R\vec{a}_R + m_B\vec{a}_B = 0$$

$$m_R\frac{\Delta\vec{v}_R}{\Delta t} + m_B\frac{\Delta\vec{v}_B}{\Delta t} = 0$$

$$\frac{\Delta(m_R\vec{v}_R + m_B\vec{v}_B)}{\Delta t} = 0$$

$$m_R\vec{v}_R + m_B\vec{v}_B = \text{constant}$$

$$m_R\vec{v}_R + m_B\vec{v}_B = m_R\frac{\Delta\vec{r}_R}{\Delta t} + m_B\frac{\Delta\vec{r}_B}{\Delta t}$$

$$m_R\vec{v}_R + m_B\vec{v}_B = M\frac{\Delta\vec{R}}{\Delta t}$$

$$\vec{R} = \frac{m_R\vec{r}_R + m_B\vec{r}_B}{m_R + m_B}$$

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Properties of the CM

- The center of mass (CM) of an object is the average position of its component masses, weighted by the fraction of the mass the component contains.

$$\vec{R} = \frac{m_1}{M}\vec{r}_1 + \frac{m_2}{M}\vec{r}_2 + \frac{m_3}{M}\vec{r}_3 + \ldots = \sum_{i=1}^{n}\frac{m_i}{M}\vec{r}_i$$

- The center of mass of an object moves as if it were a point mass with only the external forces on the components acting on it.

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Newton's Laws for an Extended Object

- For an extended object, the Newton's laws we have described so far apply to the center of mass (CM).
- Even if the CM is at rest – or moving at a constant velocity – the object can still rotate around the CM,
- Motion can be created for extended objects even if all forces are balanced if the balanced forces are applied at different points.

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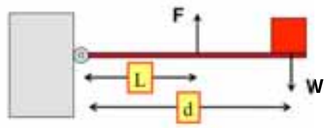
The balance rule

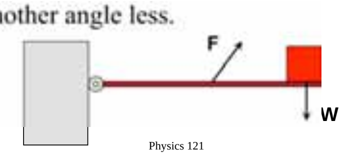
- Each force tries to turn the bar in a particular direction. When the forces and distances satisfy the balance rule, it stays balanced.

$$FL = Wd$$



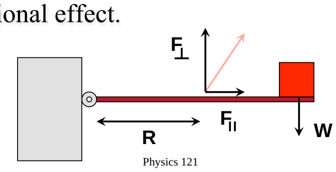
Rotational Effect of Forces: Relevant Factors

- Experimentally, the effect is proportional to the distance from the center.
- The angle at which the force is applied clearly makes a difference with perpendicular being most effective, at another angle less.



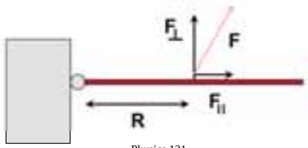
Rotational Effect of Forces: Torque

- We can figure out a measure of effectiveness by doing a component decomposition of the force vector:
- Only the perpendicular component has a rotational effect.



Definition of Torque

- "Torque" measures the effectiveness of the rotational tendency produced by a force.
- In order for an object not to rotate the torques tending to rotate it opposite ways must balance. $\tau = F_{\perp}R = FR\sin\theta$

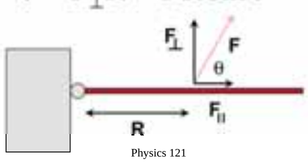


11/10/10 Physics 121 13

Definition of Torque

- Define a quantity that measures the effectiveness of the rotational tendency produced by a force.

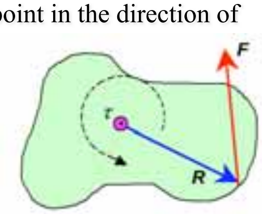
$$\tau = F_{\perp}R = FR\sin\theta$$



11/10/10 Physics 121 14

Torque as a vector

- Torque involves a position vector.
  - This vector is drawn from a reference point to the point at which the force is applied.
- The torque is defined to point in the direction of the axis through the reference point that it tries to rotate the object about.



$$\vec{\tau} = \vec{R} \times \vec{F}$$

11/10/10 Physics 121 15

The Cross Product

- The cross product is a way to multiply two vectors to get a vector – one that is perpendicular to both the vectors you are multiplying.
- (Remember that the dot product is a way to multiply two vectors to get a scalar – a number without any direction.)

11/10/10 Physics 121 **16**

The Principles of Balance

- For an object to be stationary
  - if it's CM doesn't move, the sum of all forces acting on it must balance.

$$\vec{F}^{net} = 0 \quad \rightarrow \quad F^{up} = F^{down}$$
$$F^{left} = F^{right}$$

  - if it doesn't rotate about a particular point, the sum of all torques around that point must balance

$$\tau^{net} = 0 \rightarrow \tau^{clockwise} = \tau^{counter\text{-}clockwise}$$

11/10/10 Physics 121 **22**