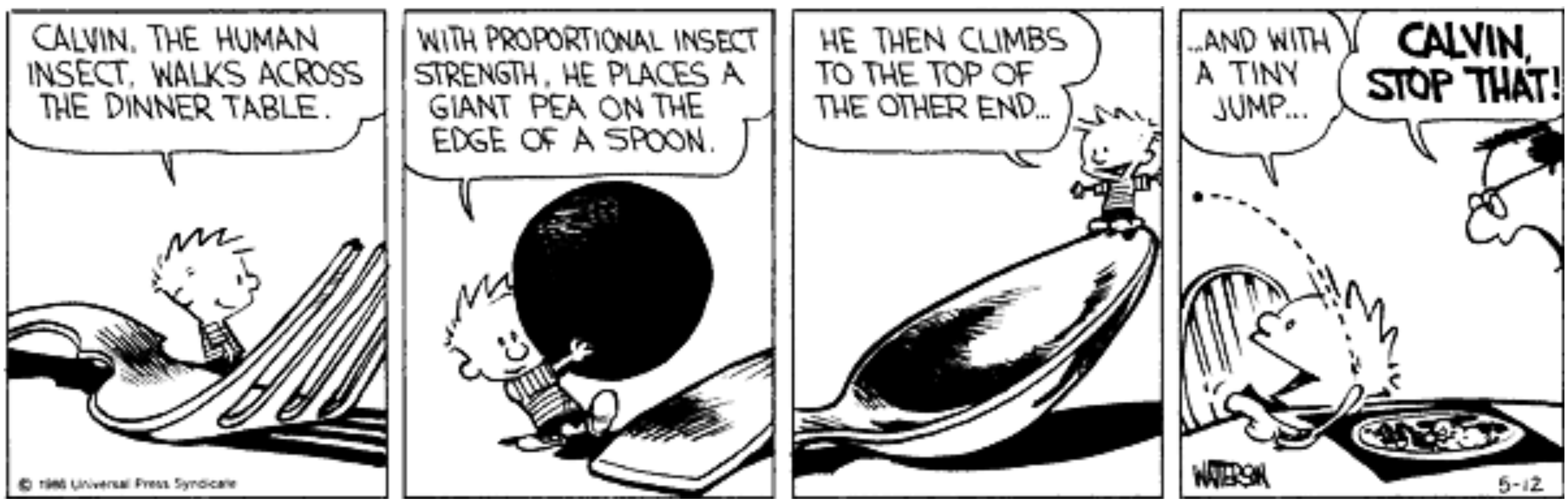
- **Theme Music:** **Rhett Butler**

  Spin

- **Cartoon:** **Bill Watterson**

  Calvin & Hobbes

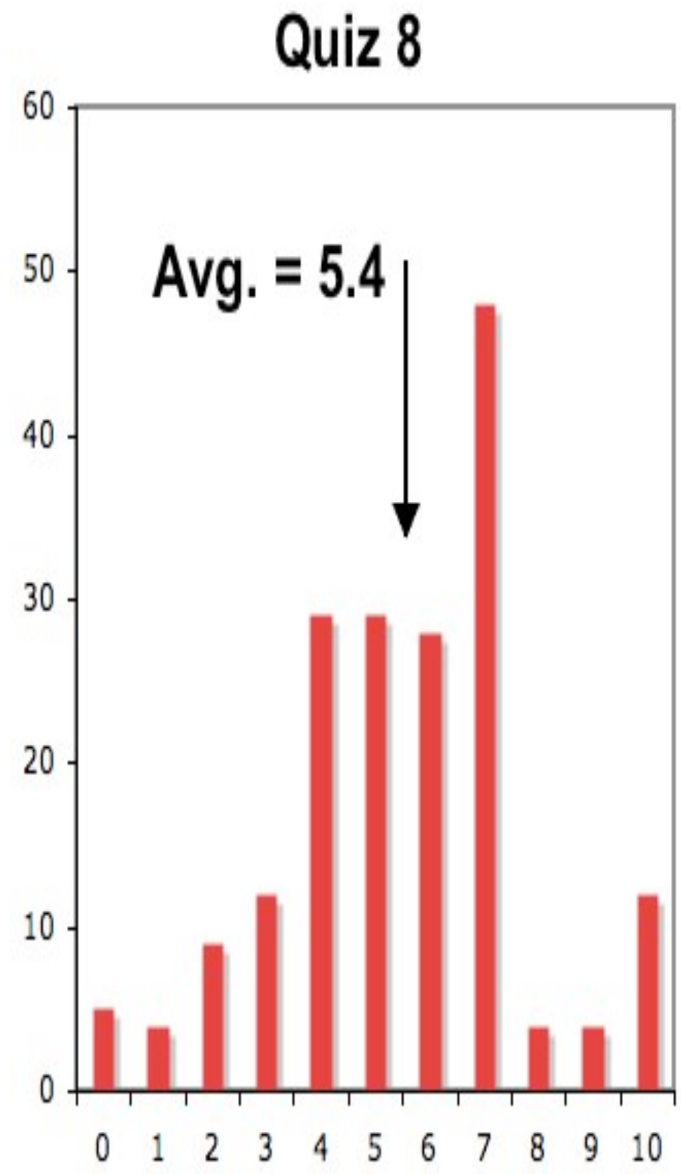


Outline

- Go over Quiz 8
- Extended objects: Center of Mass
- Torque: the balance rule
- Torque as a vector

Quiz 8

	8.1		8.2		8.3
a	0%	1	18%	a=b>c=d	46%
b	61%	2	20%	b>c>a>d	12%
c	27%	3	22%	b>a=c>d	11%
d	8%	4	26%		
e	2%	5	12%		
n	1%	6	1%		

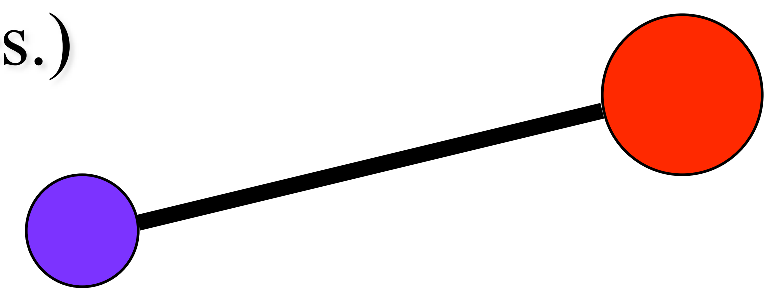


How can we get away with it?

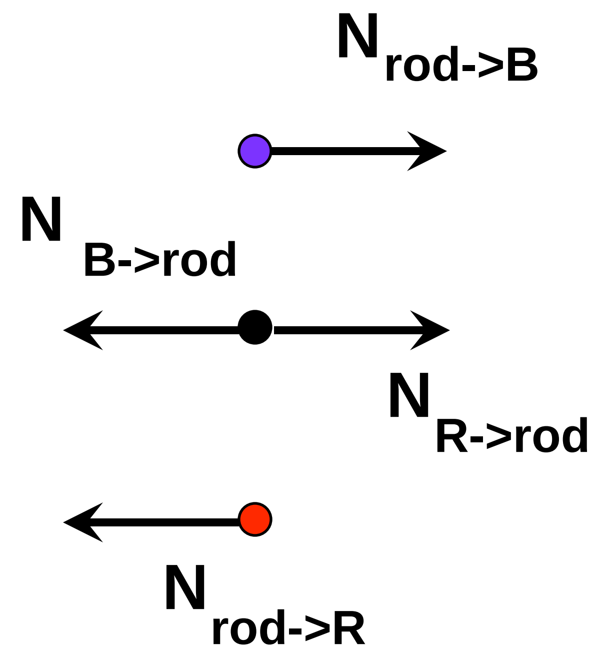
- Up to now we have considered objects that were small enough that we could ignore their size.
- How can we get away with it?
- Consider extended objects.
  Start with the simplest example.

A Simple Extended Object

- In order to understand what's happening, let's take the simplest case we can imagine of an extended object:
  - Two small masses connected by a rigid (nearly massless) rod sliding on a frictionless table.
  - If they were considered as a single object, they should move on a straight line at a constant speed. (No external forces.)



Analysis



$$m_R \vec{a}_R = \vec{N}_{rod \to R}$$

$$m_B \vec{a}_B = \vec{N}_{rod \to B}$$

$$m_{rod} \vec{a}_{rod} = \vec{N}_{R \to rod} + \vec{N}_{B \to rod}$$

$$m_R \vec{a}_R + m_B \vec{a}_B + m_{rod} \vec{a}_{rod} = \cancel{\vec{N}_{rod \to R}} + \cancel{\vec{N}_{R \to rod}} + \cancel{\vec{N}_{B \to rod}} + \cancel{\vec{N}_{rod \to B}}$$

$$m_R \vec{a}_R + m_B \vec{a}_B = 0$$

The Center of Mass

$$m_R\vec{a}_R + m_B\vec{a}_B = 0$$

$$m_R \frac{\Delta\vec{v}_R}{\Delta t} + m_B \frac{\Delta\vec{v}_B}{\Delta t} = 0$$

$$\frac{\Delta(m_R\vec{v}_R + m_B\vec{v}_B)}{\Delta t} = 0$$

$$m_R\vec{v}_R + m_B\vec{v}_B = \text{constant}$$

$$m_R\vec{v}_R + m_B\vec{v}_B = m_R \frac{\Delta\vec{r}_R}{\Delta t} + m_B \frac{\Delta\vec{r}_B}{\Delta t}$$

$$m_R\vec{v}_R + m_B\vec{v}_B = M \frac{\Delta\vec{R}}{\Delta t}$$

$$\vec{R} = \frac{m_R\vec{r}_R + m_B\vec{r}_B}{m_R + m_B}$$

Properties of the CM

- The center of mass (CM) of an object is the average position of its component masses, weighted by the fraction of the mass the component contains.

$$\vec{R} = \frac{m_1}{M}\vec{r}_1 + \frac{m_2}{M}\vec{r}_2 + \frac{m_3}{M}\vec{r}_3 + \ldots = \sum_{i=1}^{n} \frac{m_i}{M}\vec{r}_i$$

- The center of mass of an object moves as if it were a point mass with only the external forces on the components acting on it.

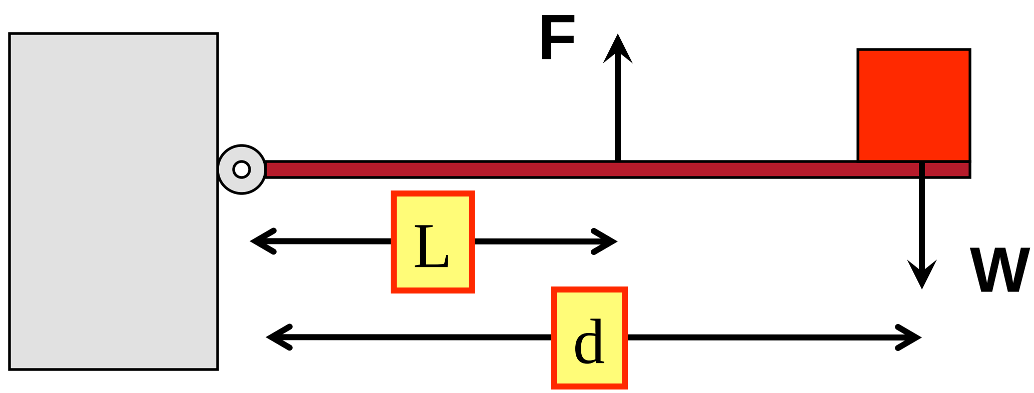
Newton's Laws for an Extended Object

- For an extended object, the Newton's laws we have described so far apply to the center of mass (CM).
- Even if the CM is at rest – or moving at a constant velocity – the object can still rotate around the CM,
- Motion can be created for extended objects even if all forces are balanced if the balanced forces are applied at different points.

The balance rule

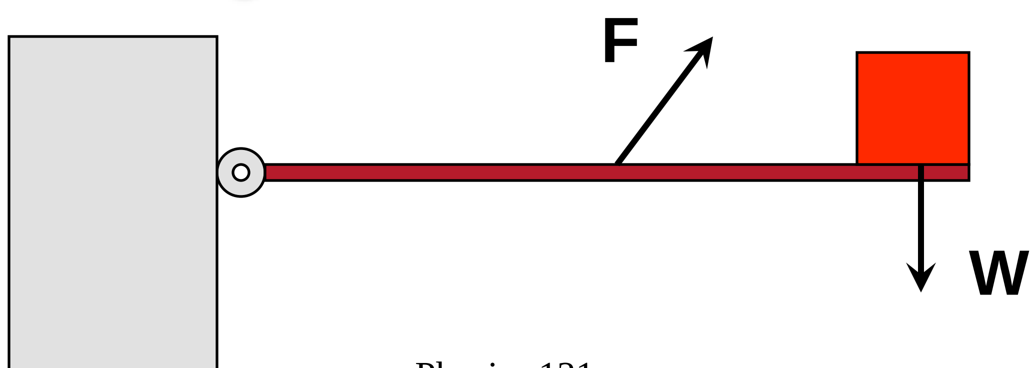
- Each force tries to turn the bar in a particular direction. When the forces and distances satisfy the balance rule, it stays balanced.

$$FL = Wd$$



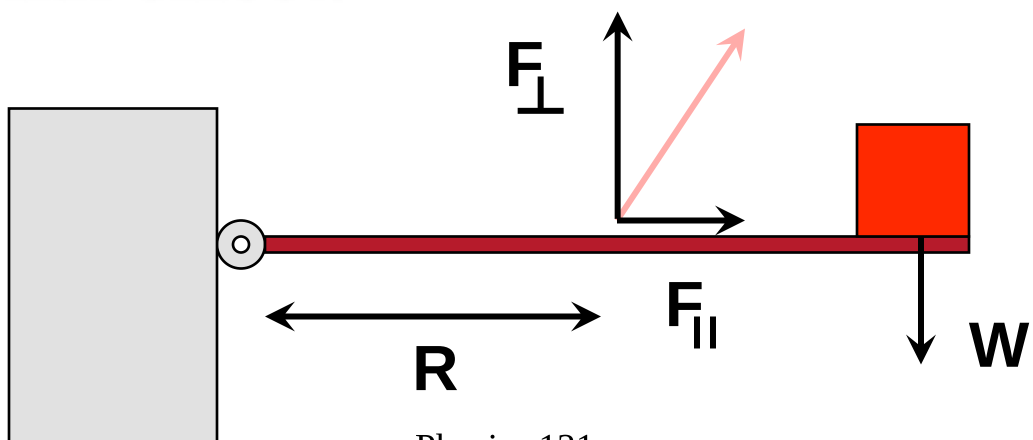
Rotational Effect of Forces: Relevant Factors

- Experimentally, the effect is proportional to the distance from the center.
- The angle at which the force is applied clearly makes a difference
  with perpendicular being most effective,
  at another angle less.



Rotational Effect of Forces: Torque

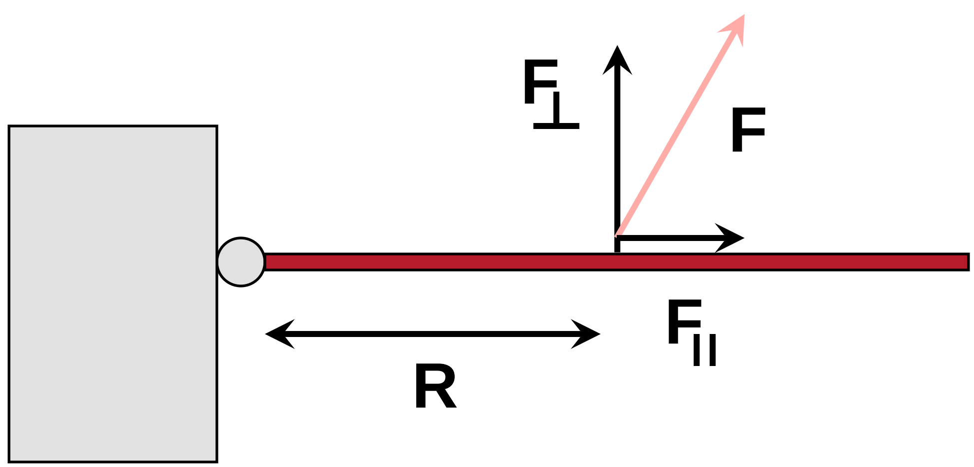
- We can figure out a measure of effectiveness by doing a component decomposition of the force vector:
- Only the perpendicular component has a rotational effect.



Definition of Torque

- "Torque" measures the effectiveness of the rotational tendency produced by a force.
- In order for an object not to rotate the torques tending to rotate it opposite ways must balance.

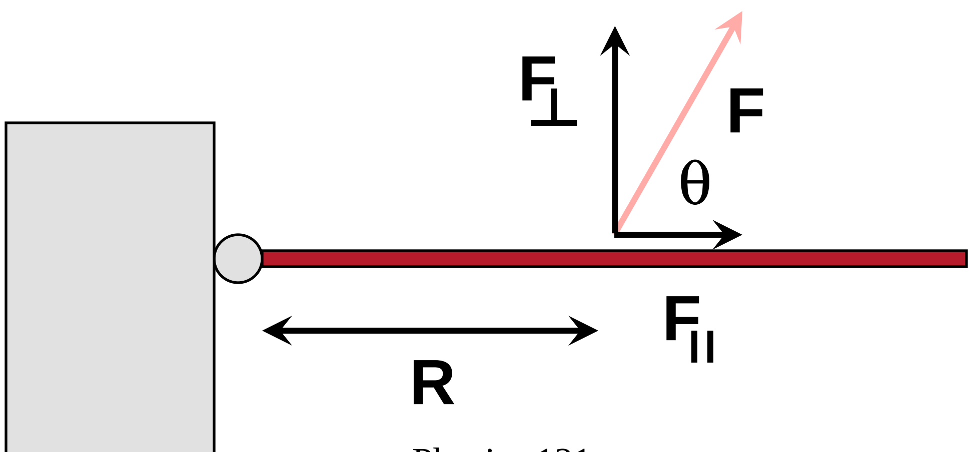
$$\tau = F_{\perp} R = FR\sin\theta$$



Definition of Torque

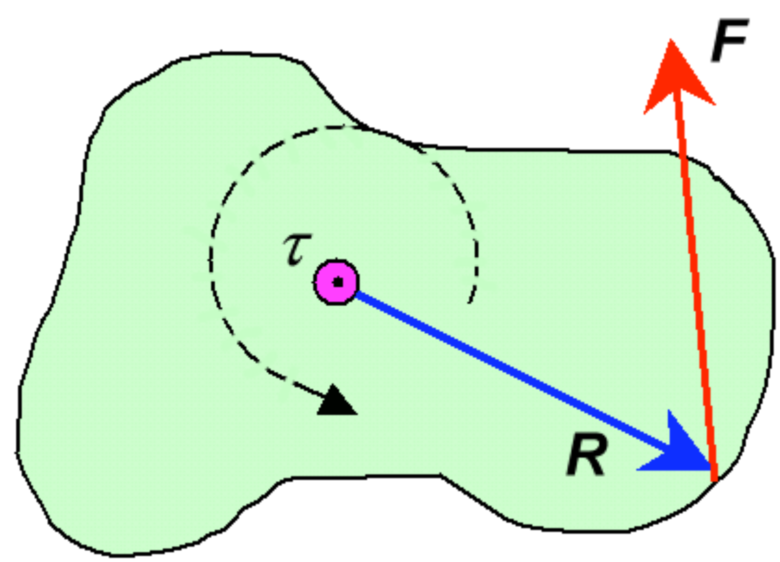
- Define a quantity that measures the effectiveness of the rotational tendency produced by a force.

$$\tau = F_{\perp} R = FR \sin\theta$$



Torque as a vector

- Torque involves a position vector.
  - This vector is drawn from a reference point to the point at which the force is applied.
- The torque is defined to point in the direction of the axis through the reference point that it tries to rotate the object about.



$$\vec{\tau} = \vec{R} \times \vec{F}$$

The Cross Product

- The cross product is a way to multiply two vectors to get a vector – one that is perpendicular to both the vectors you are multiplying.
- (Remember that the dot product is a way to multiply two vectors to get a scalar – a number without any direction.)

The Principles of Balance

- For an object to be stationary
  - if it's CM doesn't move, the sum of all forces acting on it must balance.

$$\vec{F}^{net} = 0 \quad \rightarrow \quad F^{up} = F^{down}$$

$$F^{left} = F^{right}$$

  - if it doesn't rotate about a particular point, the sum of all torques around that point must balance

$$\tau^{\text{net}} = 0 \rightarrow \tau^{\text{clockwise}} = \tau^{\text{counter-clockwise}}$$