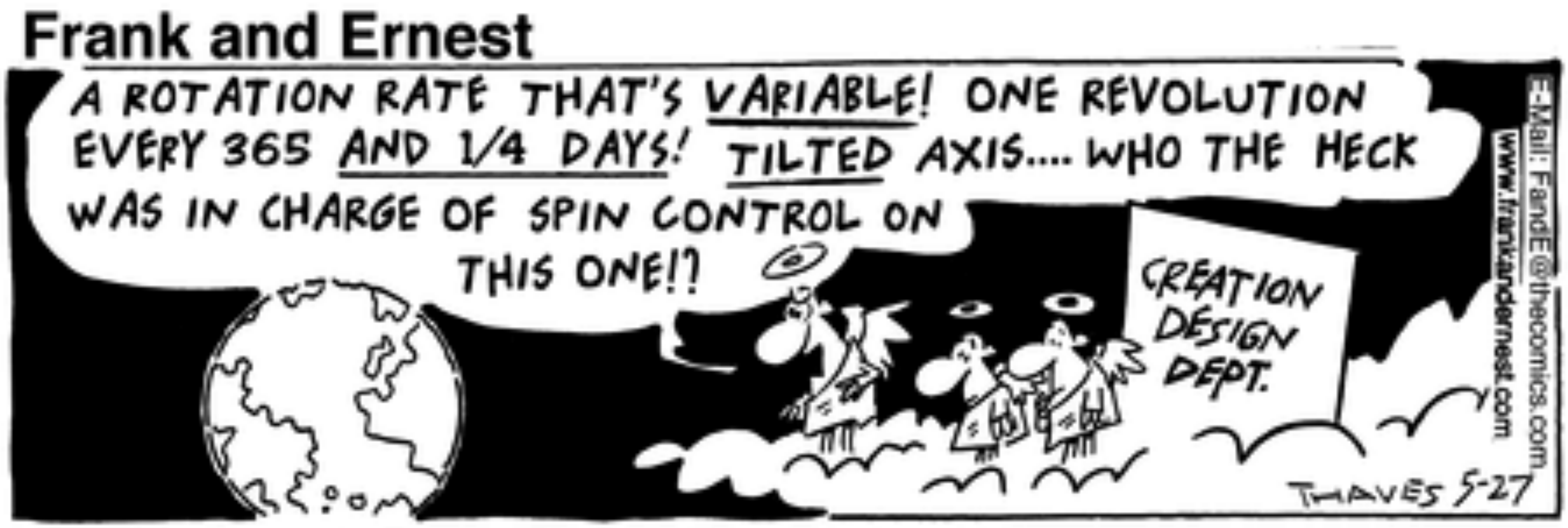
- **<u>Theme Music:</u> The Byrds**

  Turn, Turn, Turn

- **<u>Cartoon:</u> Bob Thaves**

  Frank & Ernest

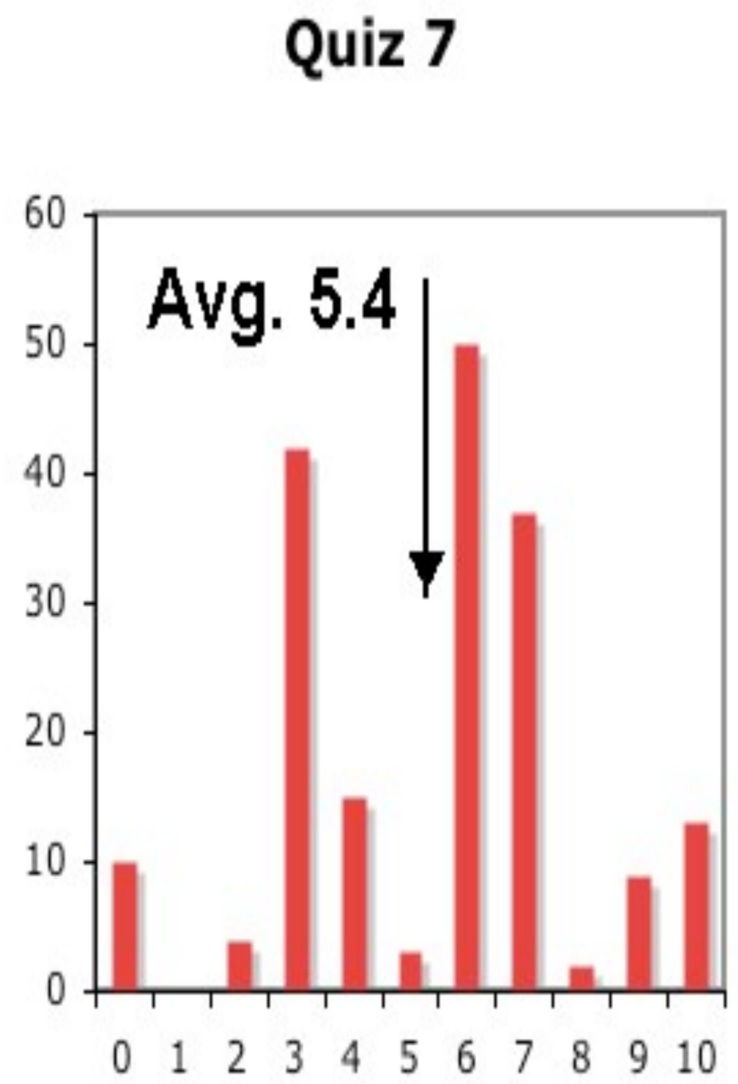


Outline

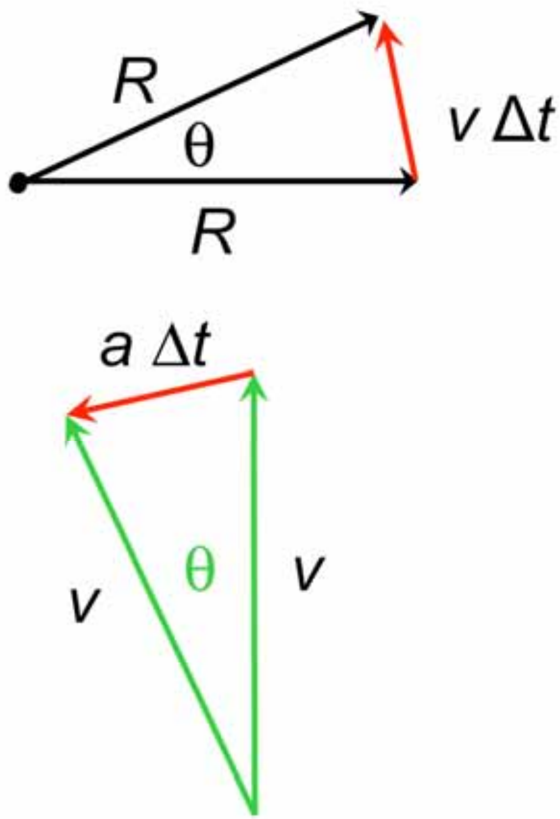
- Go over Quiz 7
- Uniform Circular Motion
- Circular Motion: Polar description
  – Angles
  – Angular velocity
  – Angular acceleration
- Appendix: What if I like calculus better than geometry?

Quiz 7

	7.1	7.2	7.3
a	**31%**	**67%**	0%
b	1%	10%	1%
c	**28%**	1%	1%
d	**77%**	5%	**72%**
e	**19%**	10%	**24%**
f	2%		



Uniform Circular Motion: Equation



Similar triangles imply

$$\frac{v\,\Delta t}{R} = \frac{a\,\Delta t}{v}$$

$$\frac{a}{v} = \frac{v}{R}$$

$$a = \frac{v^2}{R}$$

Uniform Circular Motion: Acceleration vector

$$a = \frac{v^2}{R} \quad \text{pointing in to center}$$

$$\vec{r} = \text{position vector}$$

$$\frac{\vec{r}}{R} = \hat{r} = \text{unit vector in direction of position vector}$$

$$\vec{a} = -\frac{v^2}{R}\hat{r}$$

Uniform Circular Motion: Forces

- Newton 1 says an object with no net force acting on it moves in a straight line with a constant speed.
- So if an object moves in a circle at a constant speed, there must be a net force on it.
  (The velocity is changing direction,
  so there is an acceleration.)
- How much force is needed to cause an object to move in a circle at a constant speed?

Uniform Circular Motion: Forces

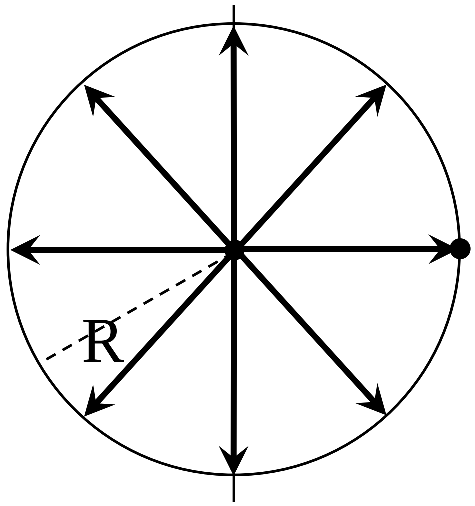
$$\vec{a} = \frac{\vec{F}^{\,net}}{m}$$ always

$$\vec{a} = -\frac{v^2}{R}\hat{r}$$ in order for the object to move in a circle with constant speed.

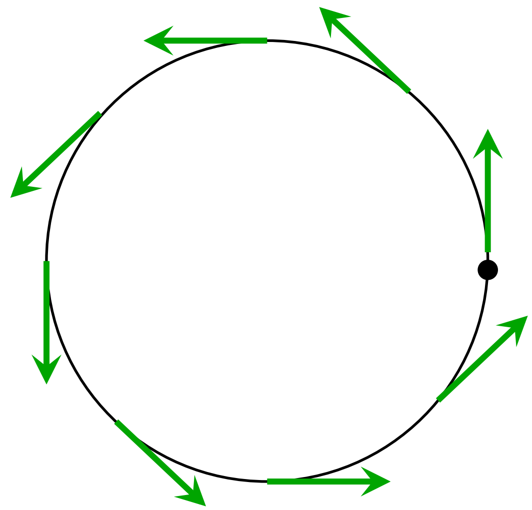
$$\frac{\vec{F}^{\,net}}{m} = -\frac{v^2}{R}\hat{r}$$ Therefore, to do this, we need a net force.

$$\vec{F}^{\,net} = -\frac{mv^2}{R}\hat{r}$$ A(n inward) radial net force is needed to maintain circular motion.

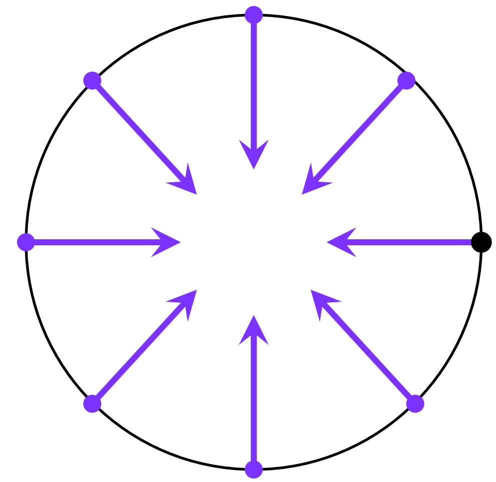
Uniform Circular Motion



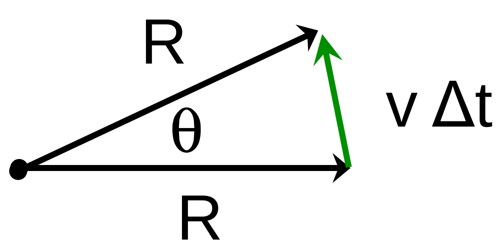
Position

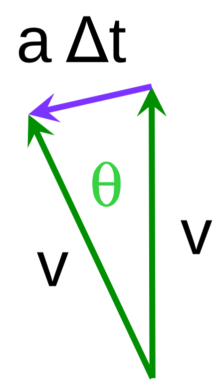


Velocity



Acceleration

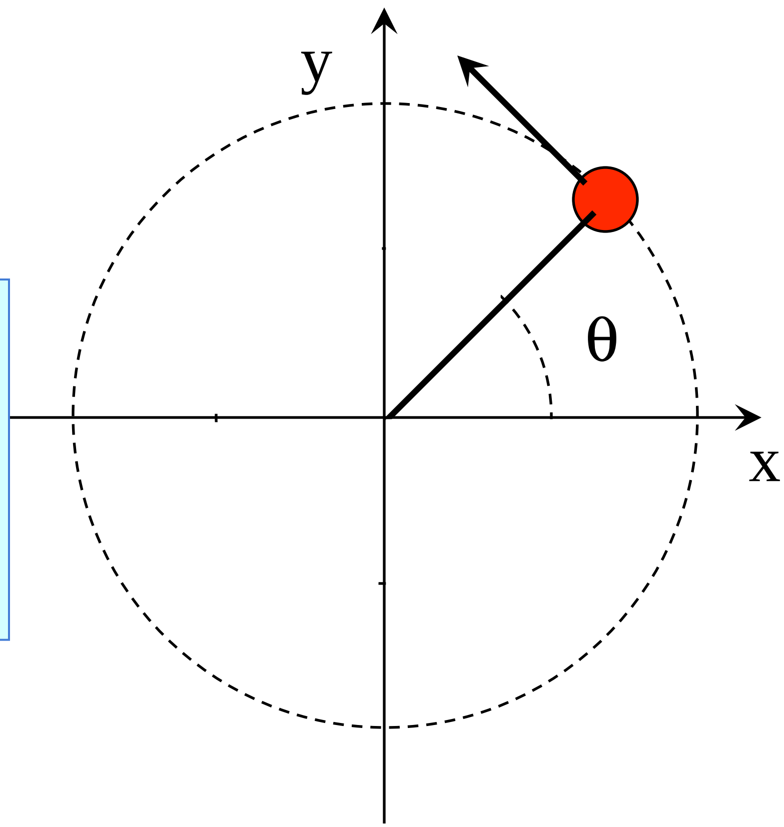




$$\frac{v\,\Delta t}{R} = \frac{a\,\Delta t}{v} \qquad \frac{a}{v} = \frac{v}{R} \qquad a = \frac{v^2}{R}$$

Rotational Kinematics: Polar Description of Motion

- Describing the angular position of an object.
  - Angle (radians) θ
  - Angular velocity ω
  - Angular acceleration α



$$\theta \text{ (in radians)} = \frac{2\pi}{360}\theta \text{ (in degrees)}$$

$$\langle \omega \rangle = \frac{\Delta\theta}{\Delta t} \qquad \langle \alpha \rangle = \frac{\Delta\omega}{\Delta t}$$

Uniform motion: $\Delta\theta = \omega_0 \, \Delta t$

Uniform Circular Motion

- In uniform circular motion, the speed is constant. This means the angle grows at a constant rate.

$$\langle\omega\rangle = \omega_0 = \frac{\Delta\theta}{\Delta t}$$

$$\Delta\theta = \omega_0 \, \Delta t$$

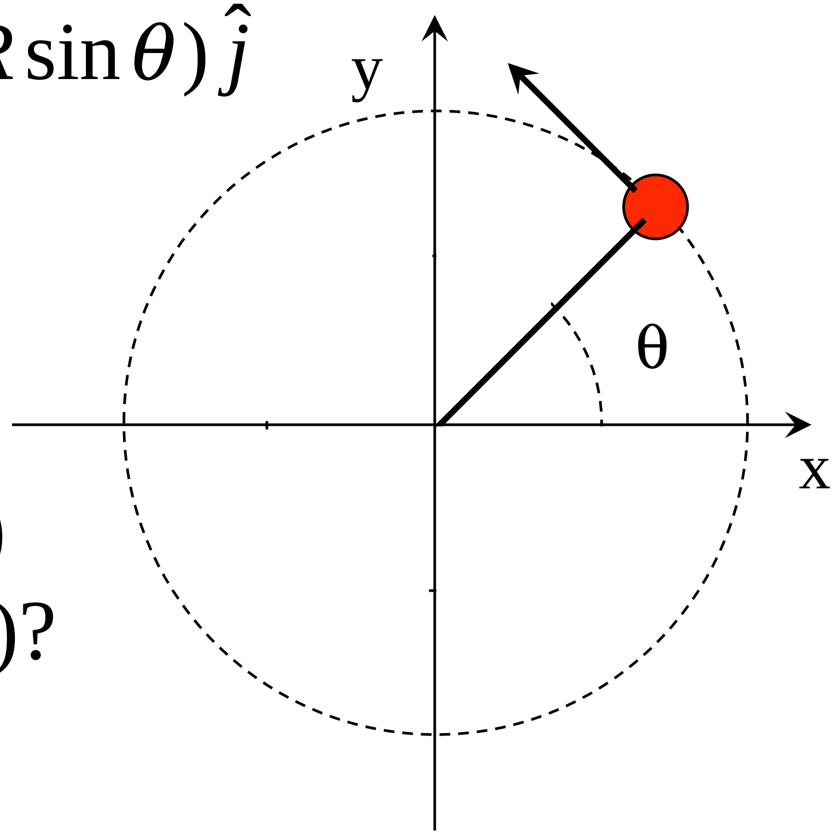
$$\theta - \theta_o = \omega_0 (t - t_0)$$

$$\theta = \theta_0 + \omega_0 (t - t_0)$$

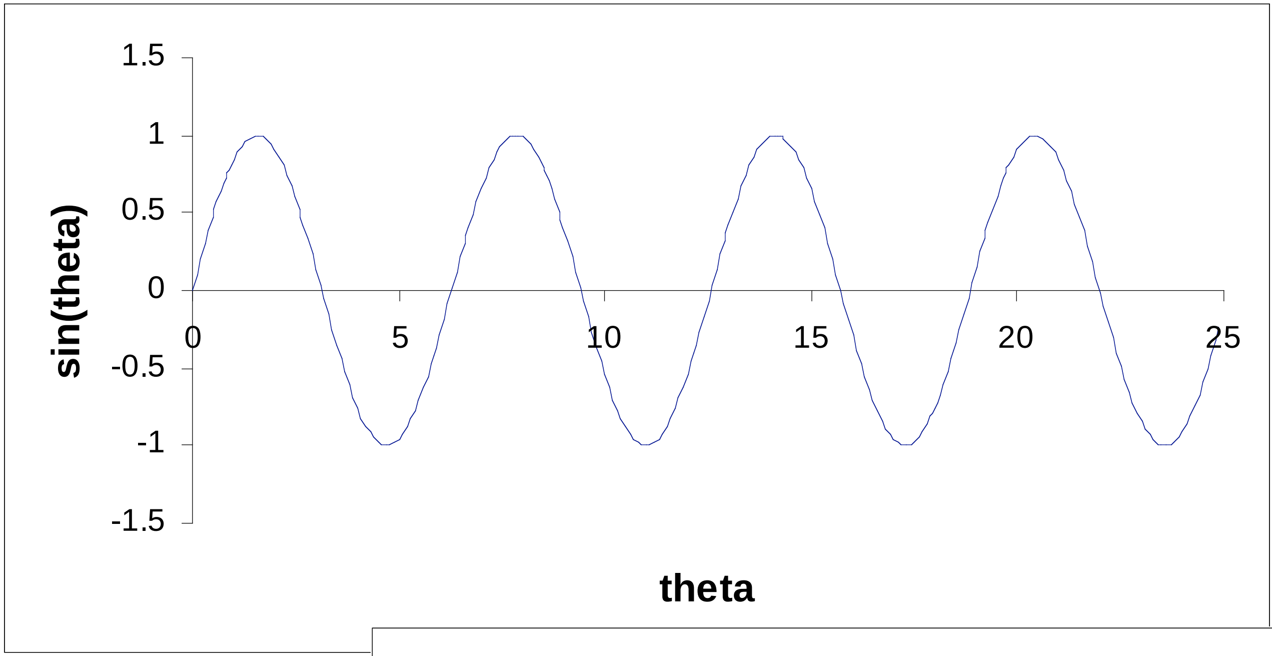
Trigonometry for big angles

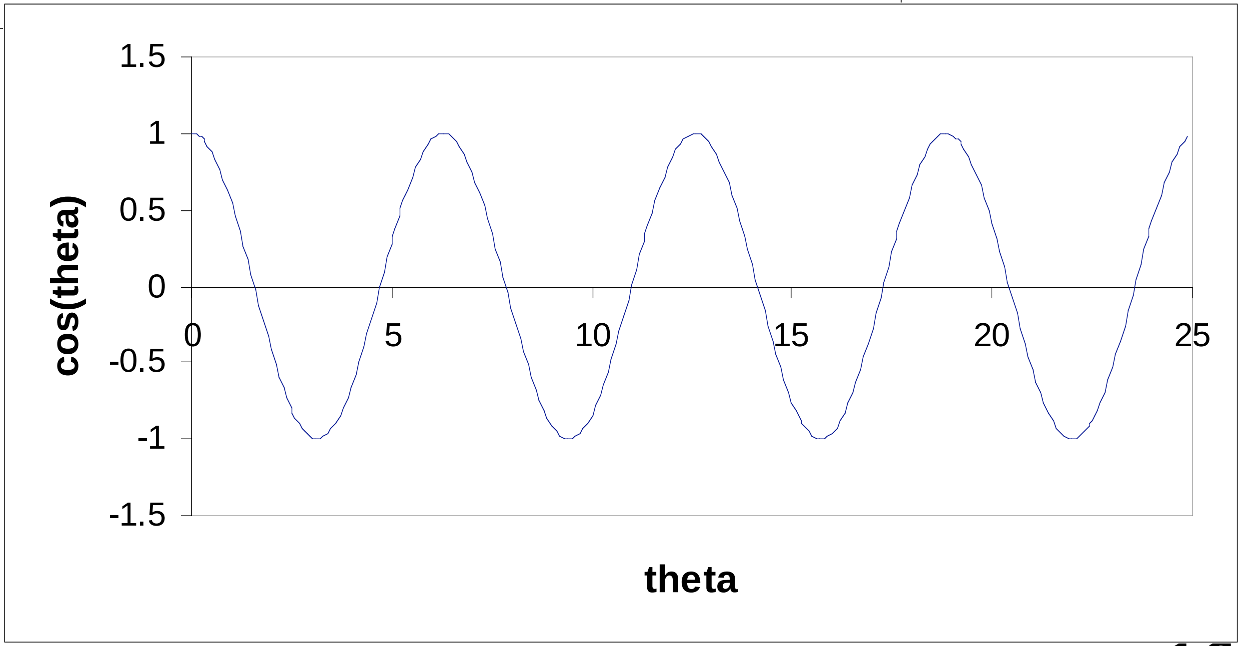
$$\vec{r} = x\hat{i} + y\hat{j} = (R\cos\theta)\hat{i} + (R\sin\theta)\hat{j}$$

$$\theta = \theta_0 + \omega_0(t - t_0)$$



What happens as *t* (and θ) gets large (bigger than 2π)?


sin(theta)
1.5
1
0.5
0
-0.5
-1
-1.5
0
5
10
15
20
25
theta


cos(theta)
1.5
1
0.5
0
-0.5
-1
-1.5
0
5
10
15
20
25
theta

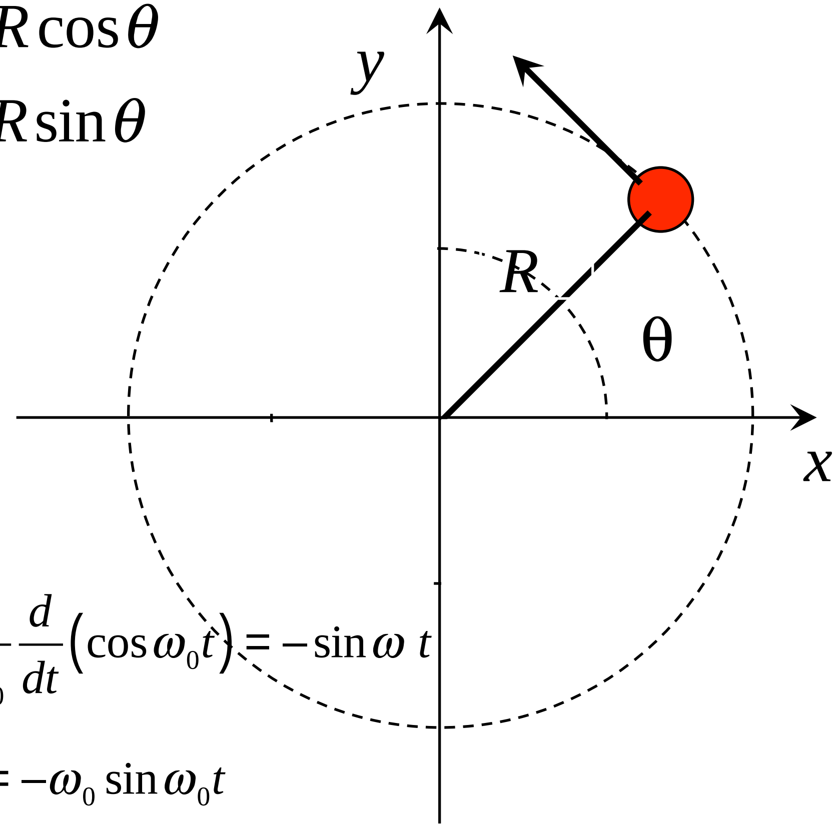
Appendix: Rotational kinematics using calculus - 1

$$\frac{d}{d\theta}\sin\theta = \cos\theta$$

$$\frac{d}{d\theta}\cos\theta = -\sin\theta$$

$$x = R\cos\theta$$

$$y = R\sin\theta$$



Uniform motion: $\theta = \omega_0 t$ with $\omega_0 =$ constant

$$\frac{d}{d(\omega_0 t)} = \frac{1}{\omega_0}\frac{d}{dt}$$

$$\frac{d}{d\theta}\sin\theta = \frac{1}{\omega_0}\frac{d}{dt}\left(\sin\omega_0 t\right) = \cos\omega\ t$$

$$\frac{d}{d\theta}\cos\theta = \frac{1}{\omega_0}\frac{d}{dt}\left(\cos\omega_0 t\right) = -\sin\omega\ t$$

$$\frac{d}{dt}\left(\sin\omega_0 t\right) = \omega_0\cos\omega_0 t$$

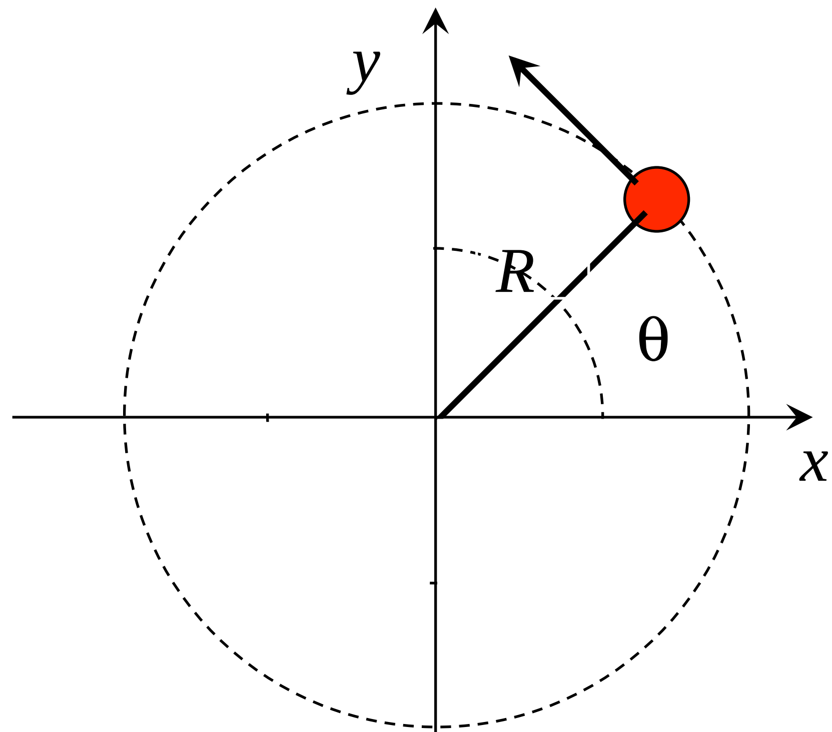
$$\frac{d}{dt}\left(\cos\omega_0 t\right) = -\omega_0\sin\omega_0 t$$

Appendix: Rotational kinematics using calculus - 2

$$x = R\cos\theta$$
$$y = R\sin\theta$$

$$v_x = \frac{dx}{dt} = R\frac{d}{dt}\cos\theta = -\omega_0 R\sin\theta = -\omega_0 y$$
$$v_y = \frac{dy}{dt} = R\frac{d}{dt}\sin\theta = \omega_0 R\cos\theta = \omega_0 x$$

$$a_x = \frac{dv_x}{dt} = -\omega_0 R\frac{d}{dt}\sin\theta = -\omega_0^2 R\cos\theta = -\omega_0^2 x$$
$$a_y = \frac{dv_y}{dt} = \omega_0 R\frac{d}{dt}\cos\theta = -\omega_0^2 R\sin\theta = -\omega_0^2 y$$



Now figure out what all that means!