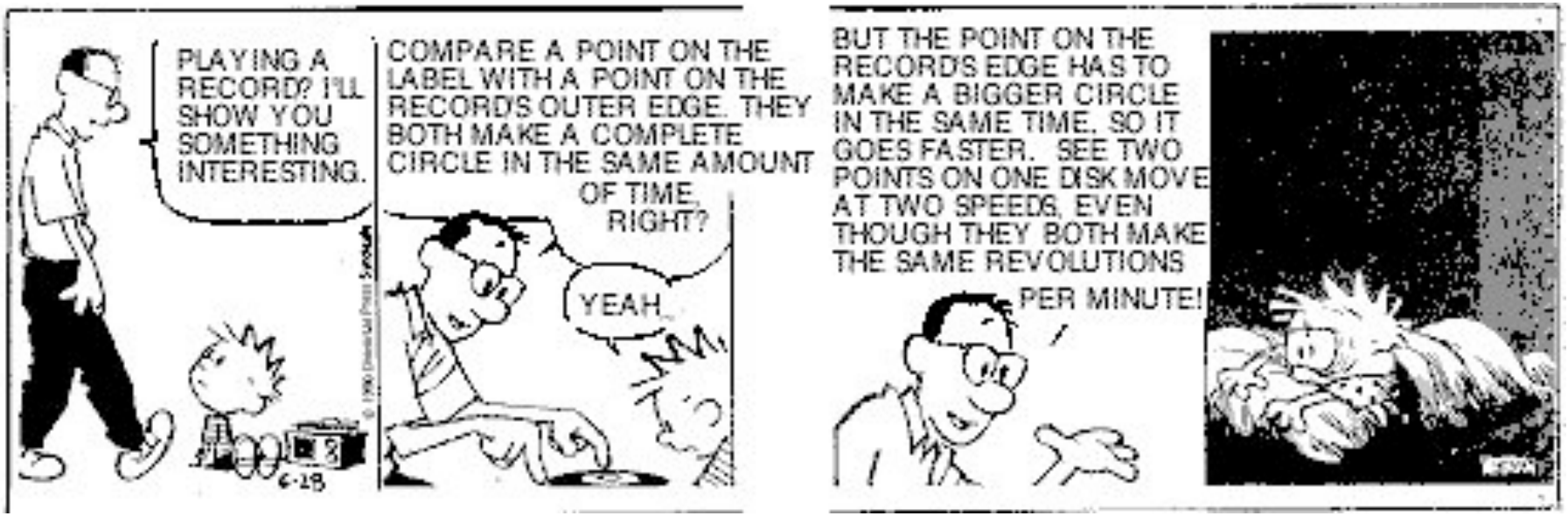
- **Theme Music: by Joni Mitchell***

Circle Game

- **Cartoon: Bill Watterson**

Calvin & Hobbes

***playing the original cover by Tom Rush**



Outline

- Quiz 7: Energy
- ILD 5: Circular motion
- Circular Motion
  - position
  - velocity
  - acceleration
  - equations
  - force

Circular Motion

- We've focused so far on motion along a line (1D motions) or 2D where the motions along two perpendicular directions are each independent 1D motions.
- Let's consider an example (the simplest one) in which the only change in a velocity is its direction.
- Uniform circular motion
  = motion in a circle at a constant speed.

ILD 5

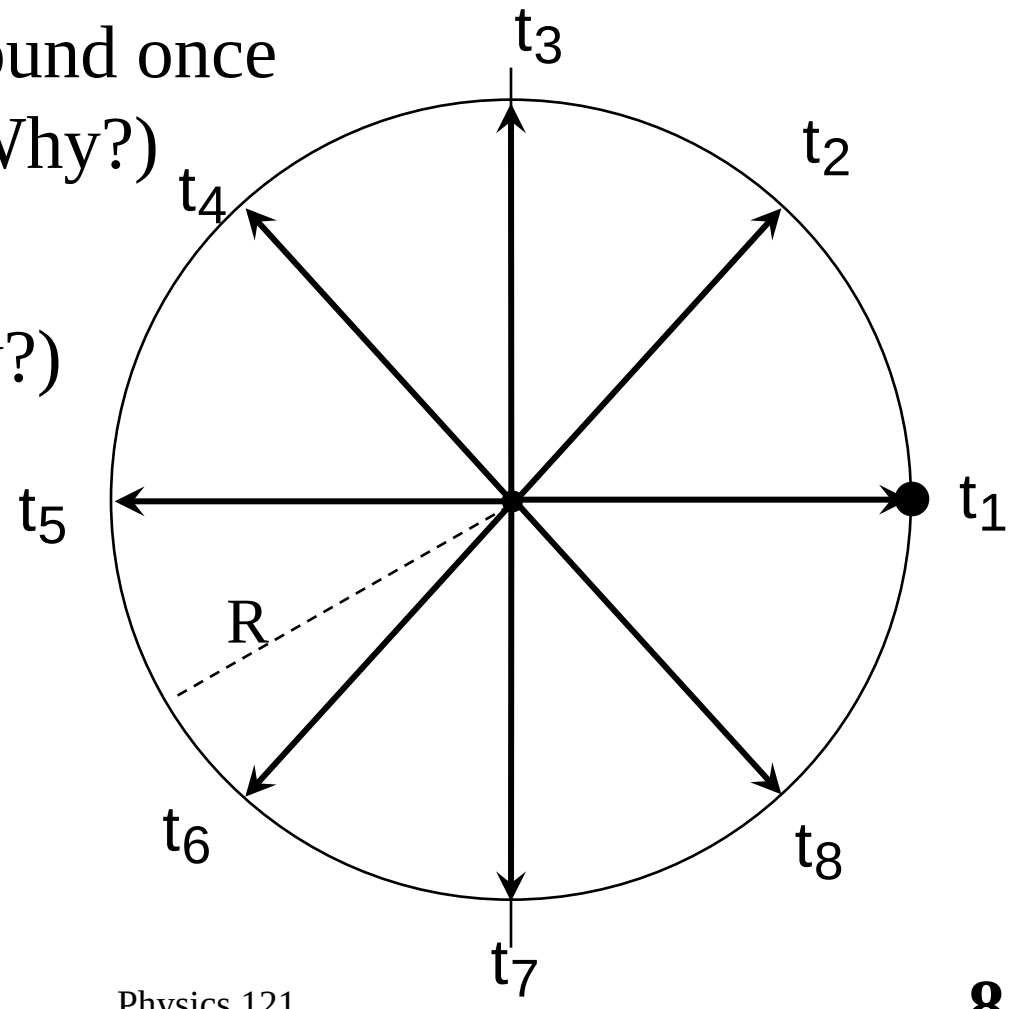
Circular Motion:
Checking for coherence
and reconciling

Uniform Circular Motion: Position

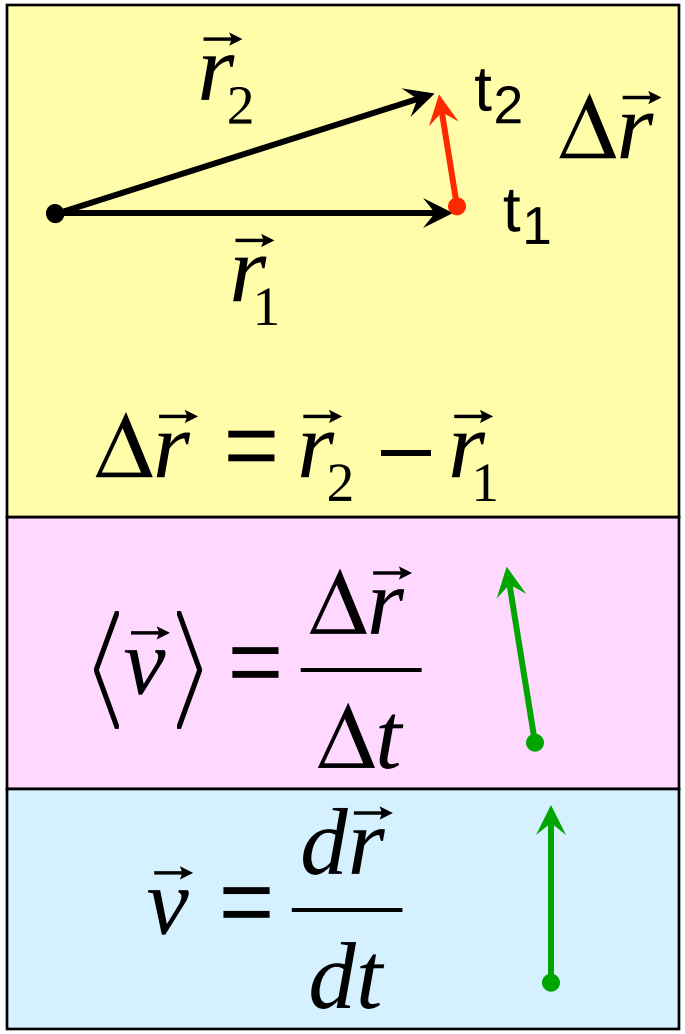
Period = time to go around once

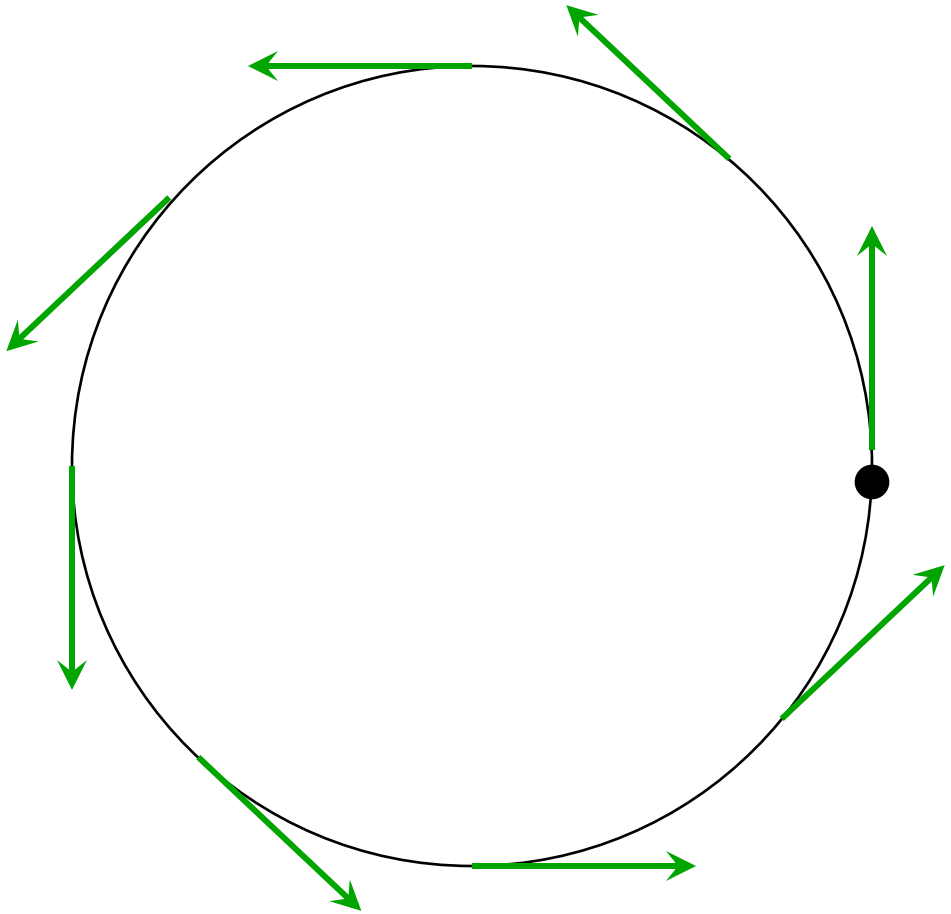
$T = 4\,(t_3 - t_1)$ (Why?)

Speed = $2\pi R / T$ (Why?)



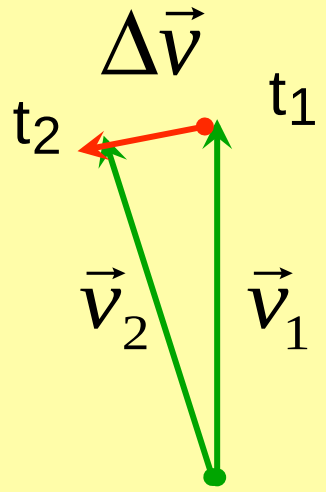
Uniform Circular Motion: Velocity





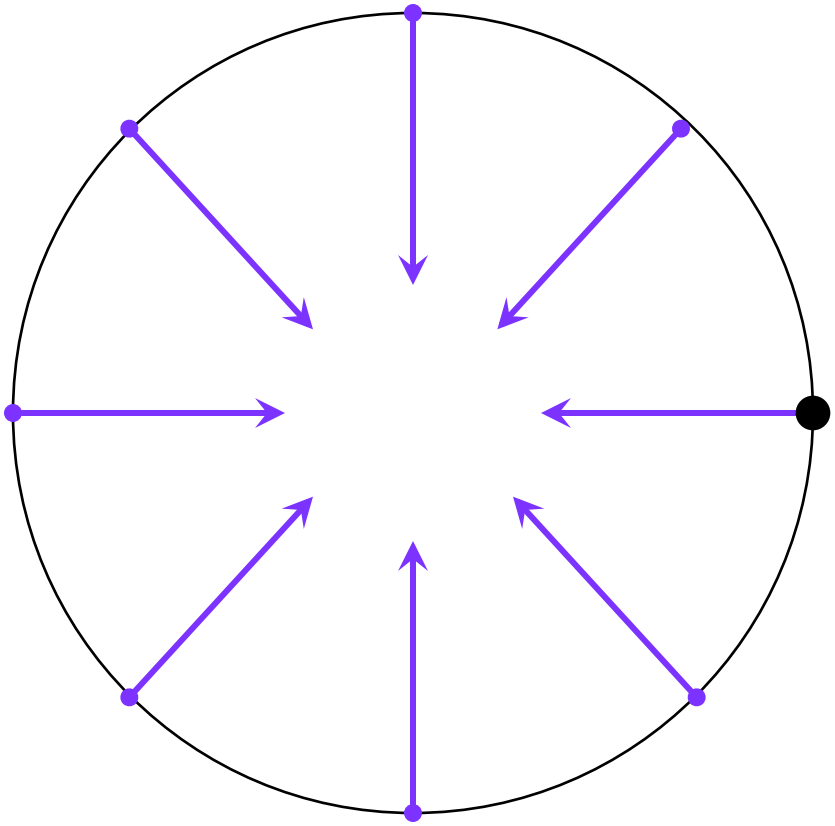
Uniform Circular Motion: Acceleration

$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$$

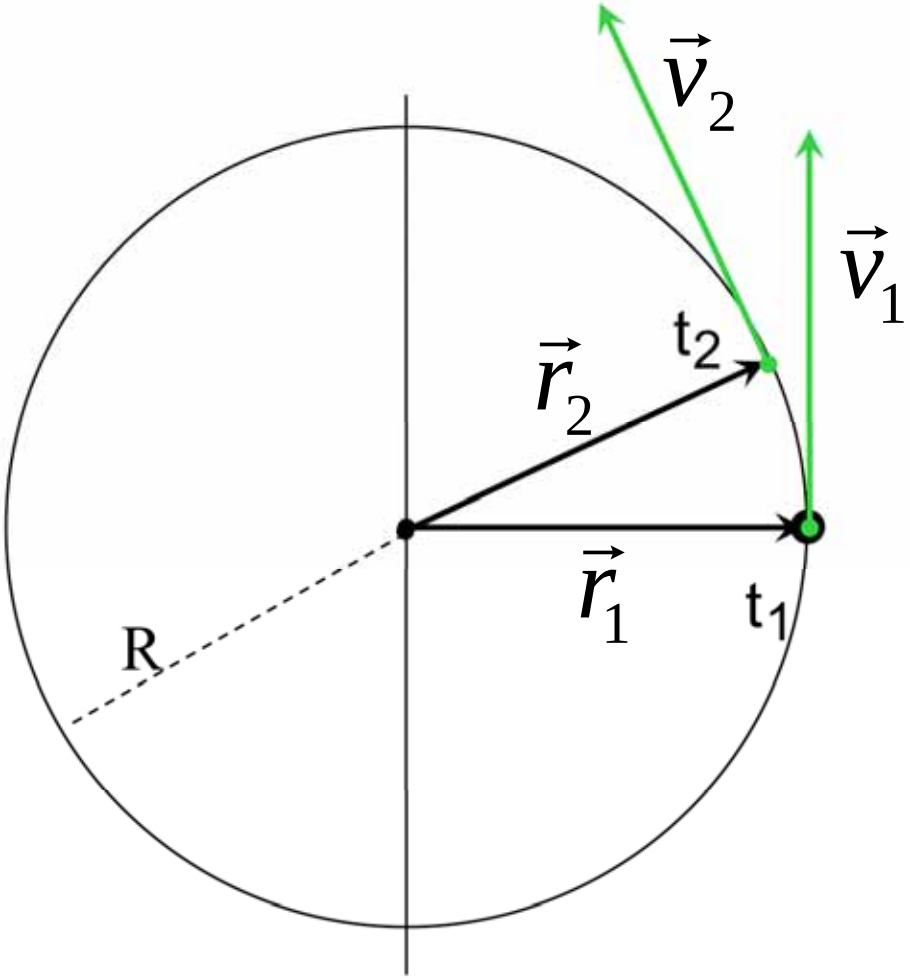


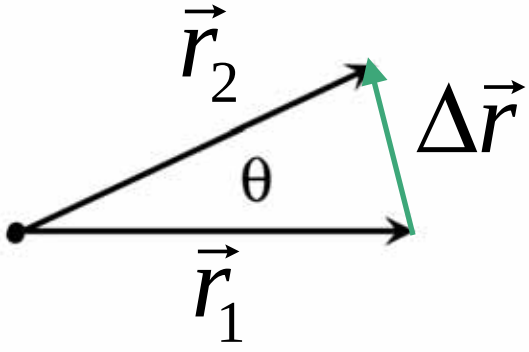
$$\langle\vec{a}\rangle = \frac{\Delta\vec{v}}{\Delta t}$$

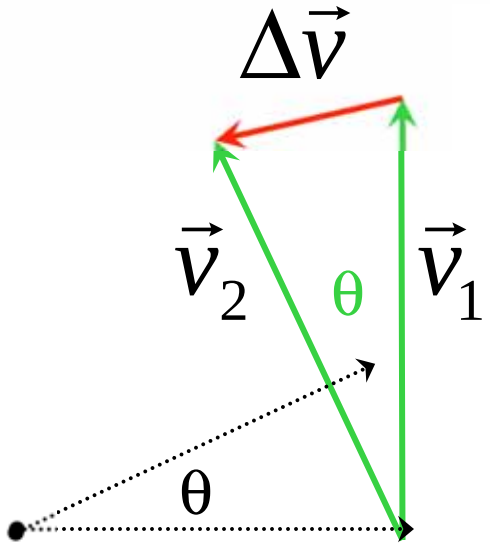
$$\vec{a} = \frac{d\vec{v}}{dt}$$



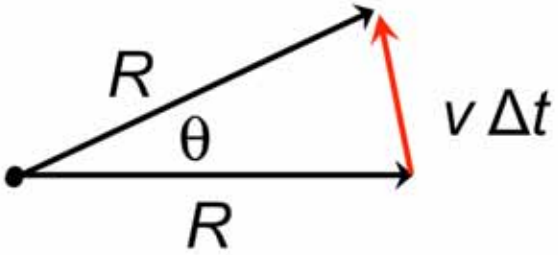
Uniform Circular Motion: Geometry

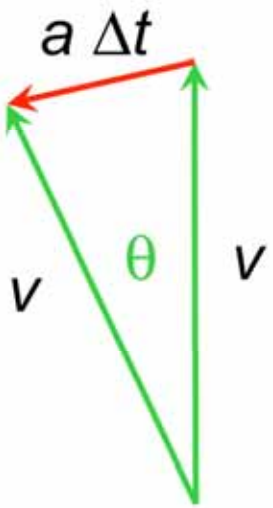






Uniform Circular Motion: Equation





Similar triangles imply

$$\frac{v\,\Delta t}{R} = \frac{a\,\Delta t}{v}$$

$$\frac{a}{v} = \frac{v}{R}$$

$$a = \frac{v^2}{R}$$

Uniform Circular Motion: Acceleration vector

$a = \frac{v^2}{R}$ pointing in to center

$\vec{r}$ = position vector

$\frac{\vec{r}}{R} = \hat{r}$ = unit vector in direction of position vector

$\vec{a} = -\frac{v^2}{R}\hat{r}$

Uniform Circular Motion: Forces

- Newton 1 says an object with no net force acting on it moves in a straight line with a constant speed.
- So if an object moves in a circle at a constant speed, there must be a net force on it.
  (The velocity is changing direction,
  so there is an acceleration.)
- How much force is needed to cause an object to move in a circle at a constant speed?

Uniform Circular Motion: Forces

$$\vec{a} = \frac{\vec{F}^{net}}{m}$$ always

$$\vec{a} = -\frac{v^2}{R}\hat{r}$$ in order for the object to move in a circle with constant speed.

$$\frac{\vec{F}^{net}}{m} = -\frac{v^2}{R}\hat{r}$$ Therefore, to do this, we need a net force.

$$\vec{F}^{net} = -\frac{mv^2}{R}\hat{r}$$ A(n inward) radial net force is needed to maintain circular motion.