- **Theme Music:**

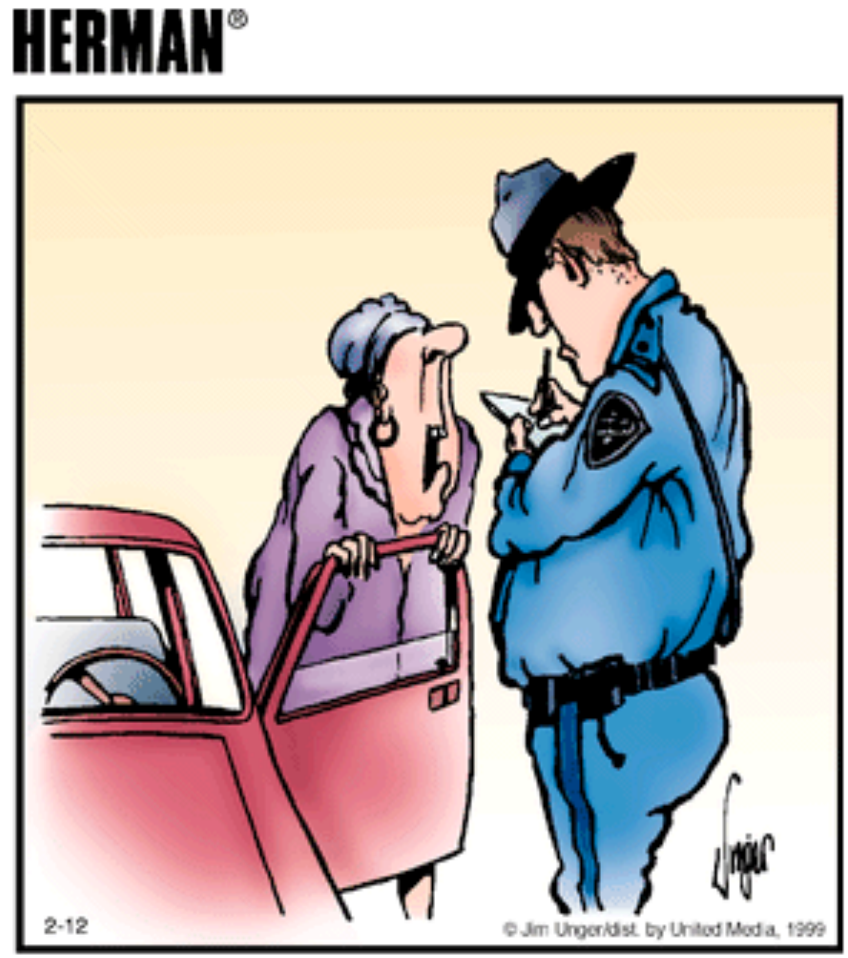
  Coldplay

  High Speed

- **Cartoon:**

  Jim Unger

  Herman



"How could I have been doing 70 miles an hour when I've only been driving for ten minutes?"

Outline

- Recap
  – position
  – velocity
- Acceleration
- With vectors!
- ILD 2: What if something just doesn't make sense? Acceleration at the peak.

What have we learned?
Representations and consistency

- Visualizing where an object is at different times → a position graph
- Visualizing how fast an object is moving at different times → a velocity graph
- Position graph → velocity graph — slopes $v = \dfrac{\Delta x}{\Delta t}$
- Velocity graph → position graph — areas $\Delta x = v\,\Delta t$

Average Acceleration

- We need to keep track not only of the fact that something is moving
  but how that motion is changing.
- Define the average acceleration by

$$< \vec{a} > = \frac{\text{change in velocity}}{\text{time it took to make the change}}$$

$$< \vec{a} > = \frac{\Delta \vec{v}}{\Delta t}$$

Instantaneous acceleration

- Sometimes (often) an object will move so that sometimes it speeds up or slows down at different rates.
- We want to be able to describe this change in motion also.
- If we consider small enough time intervals, the change in velocity will look uniform — for a little while at least.

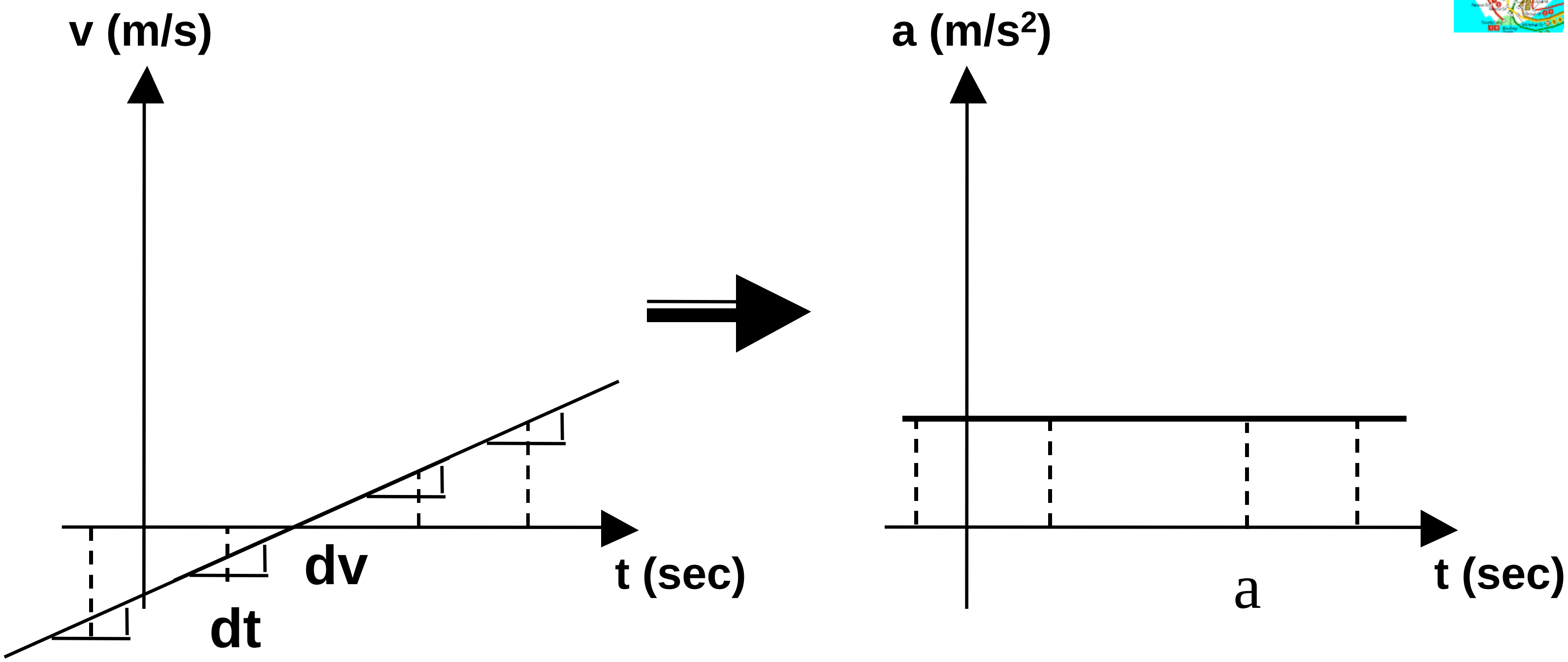
Instantaneous acceleration

- If we consider a small enough time interval so that the object is (approximately) in uniformly accelerated motion during that time interval, we can define the "acceleration at the instant at the center of the time interval" by

$$\vec{a}(t) = \frac{d\vec{v}}{dt}$$

$$\vec{a}(t) = \frac{\vec{v}(t + \Delta t/2) - \vec{v}(t - \Delta t/2)}{\Delta t}$$

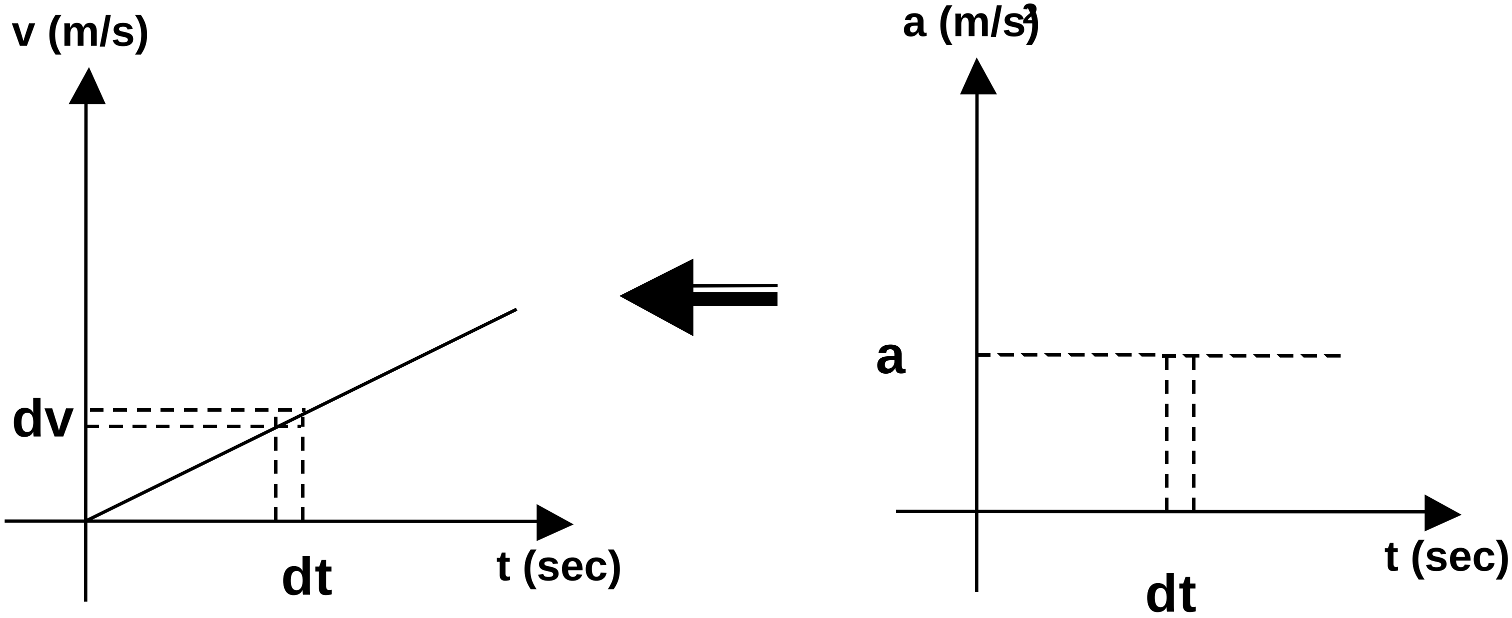
Velocity to acceleration



$$a(t) = \frac{dv}{dt}$$

$$a(t) = \frac{v(t + \Delta t/2) - v(t - \Delta t/2)}{\Delta t}$$

Acceleration to velocity



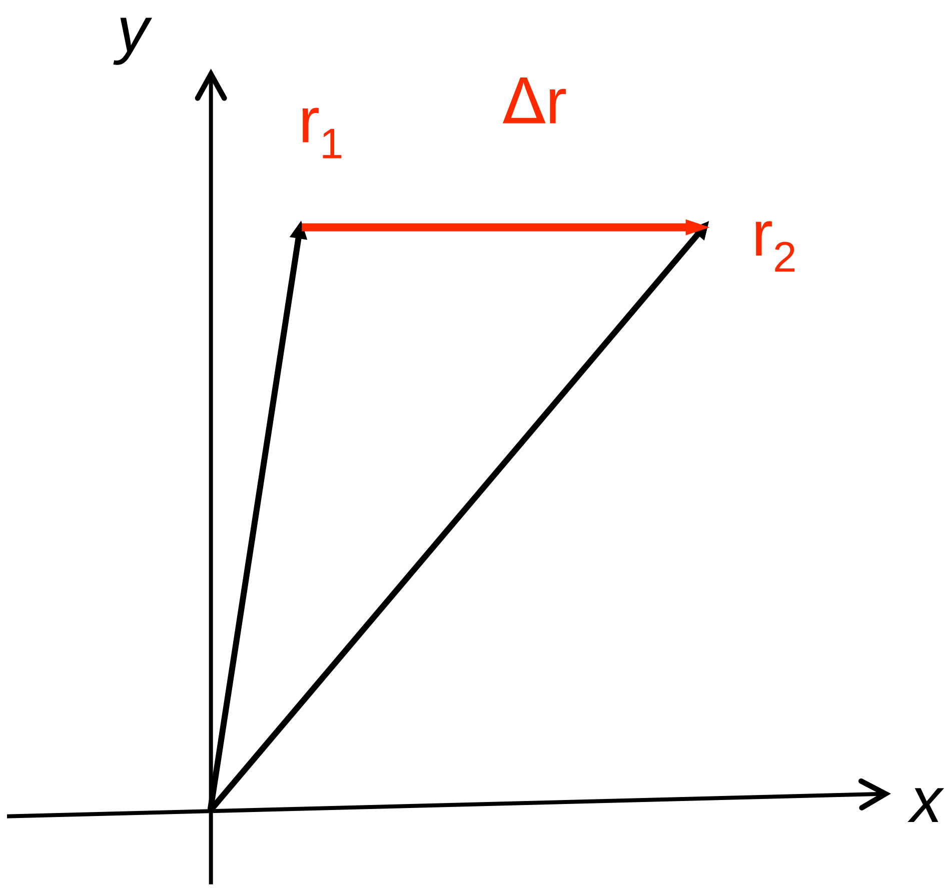
$$dv = a(t)\, dt$$

change in velocity over a small time interval

sum ("Δ") in the changes in velocity over many small time intervals.

$$v = \sum dv = \int a(t)\, dt$$

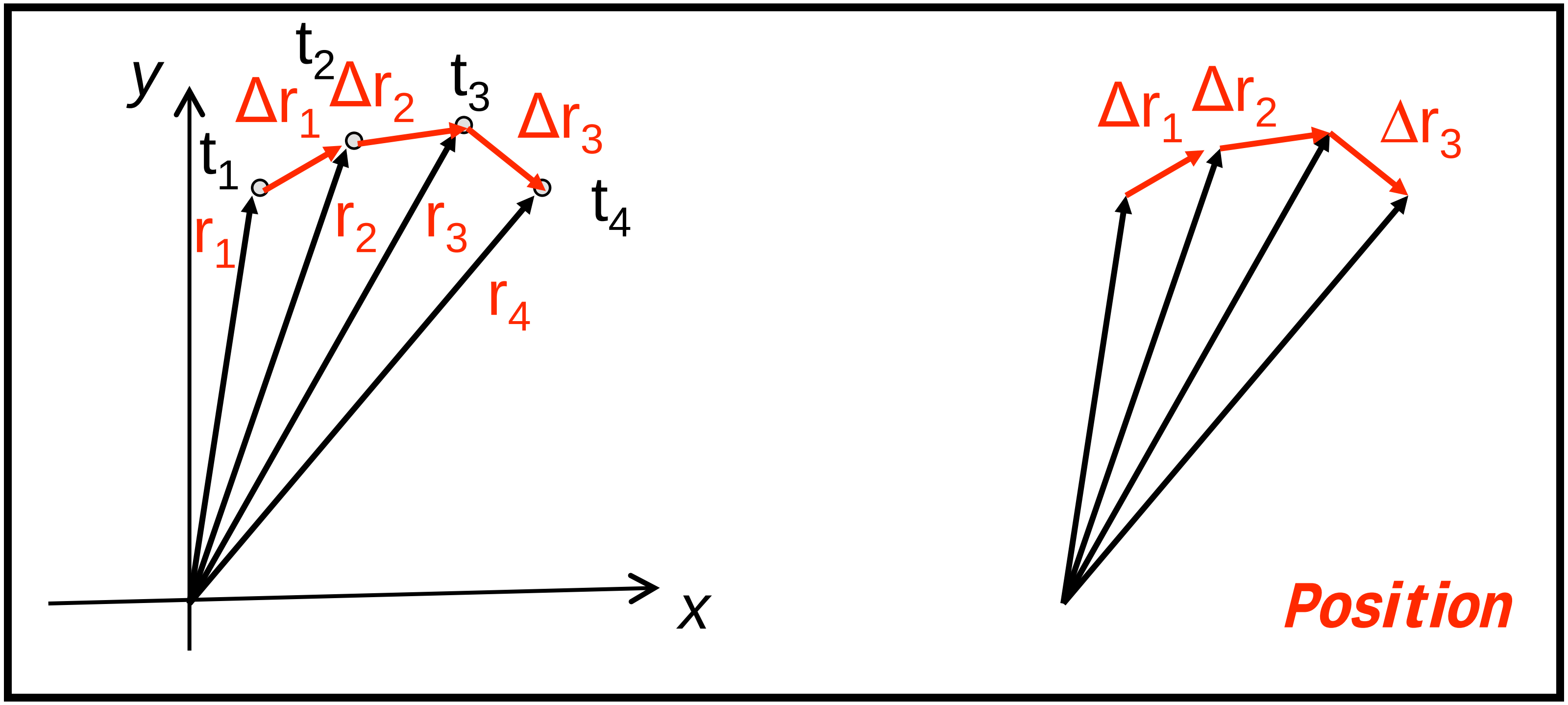
Working with Vectors

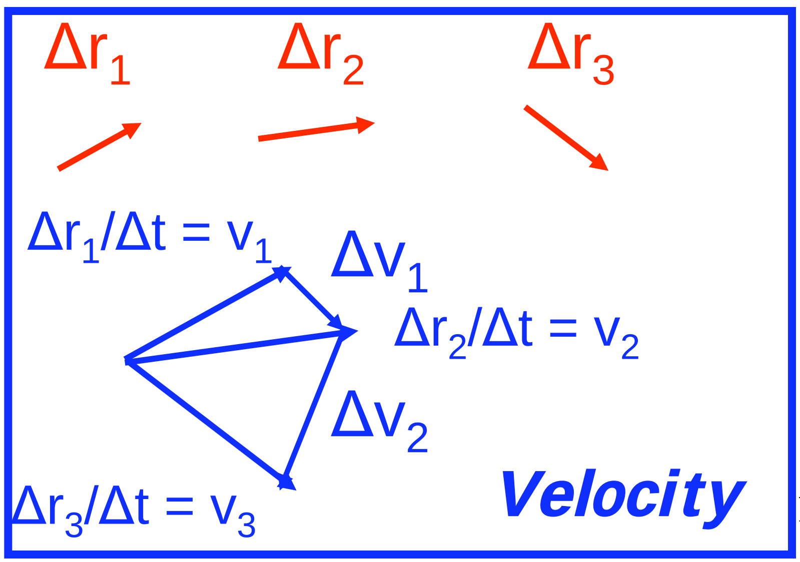


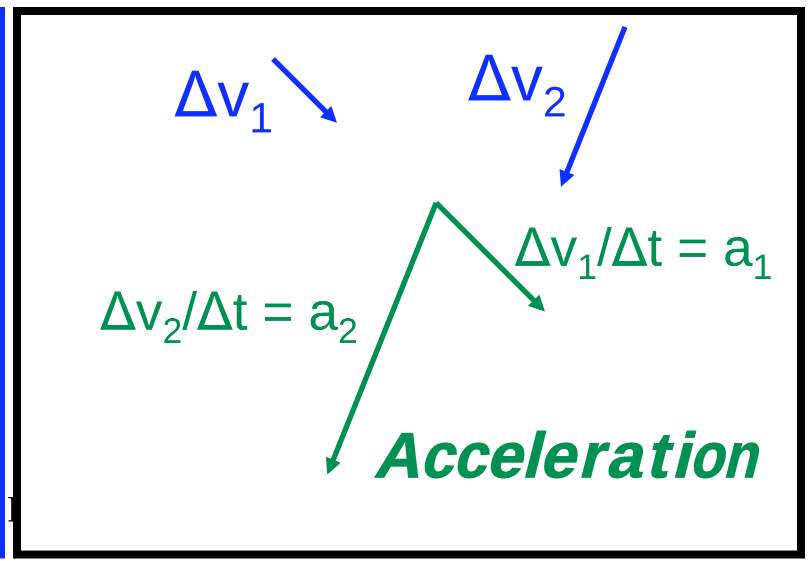
$$\vec{r}_2 = \vec{r}_1 + \Delta\vec{r}$$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

Kinematics with Vectors







What have we learned?

- Position $\hat{r} = x\hat{i}$ (where *x* is a signed length)
- Velocity $\langle \vec{v} \rangle = \dfrac{\Delta \vec{r}}{\Delta t}$ $\vec{v} = \dfrac{d\vec{r}}{dt}$
- Acceleration $\langle \vec{a} \rangle = \dfrac{\Delta \vec{v}}{\Delta t}$ $\vec{a} = \dfrac{d\vec{v}}{dt}$
- Seeing from the motion
- Seeing consistency (graphs & equations)

ILD 2

What if something
just doesn't make sense?

Acceleration at the peak