- **<u>Theme Music:</u> Paul Simon**
  *When numbers get serious*
- **<u>Cartoon:</u> Bill Waterson**
  *Calvin & Hobbes*

# Math is different in science from in math class!

- Math in physics is fundamentally about <span style="color:red">relationships among measurements</span>, not primarily about calculating – or even solving (though we do some of that).

- <span style="color:red">Functional dependence</span> is critical. "More of this means more of that" is <u>not</u> good enough. *How much* more matters.

# An example from a math exam

- Writing the equation in this problem on a physics exam would receive 0 credit and the comment: "This is a meaningless equation!"

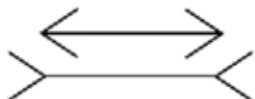The population density of trout in a stream is

$$r(x) = 20\frac{1+x}{x^2+1}$$

where $r$ is measured in trout per mile and x is measured in miles. $x$ runs from 0 to 10.

(a) Write an expression for the total number of trout in the stream. Do not compute it.

*How would you fix this?*

# Third icon: Measurement

- As we learned with our example, often our perceptions play us false.

- In order to be sure we understand what is going on we need to quantify. In order to do that we need to invent a process to assign a number to a physical quantity – measure.

# Measurement is basically about counting.
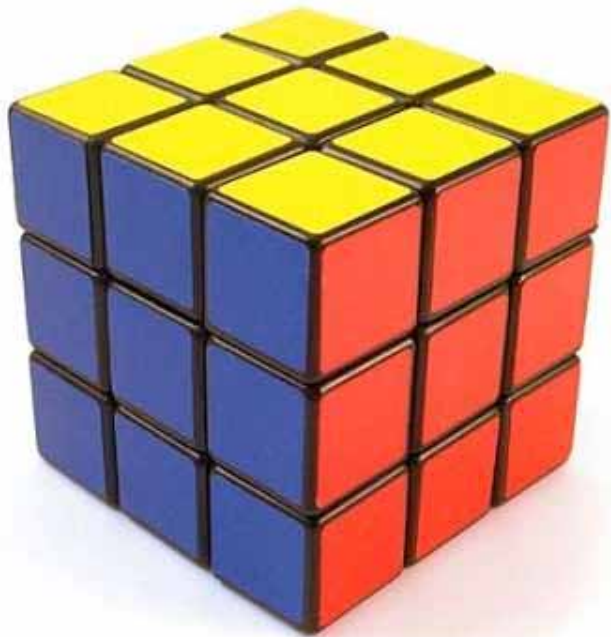


| N | Perim | Area | Vol |
|---|-------|------|-----|
| 1 |       |      |     |
| 2 |       |      |     |
| 3 |       |      |     |

# Operational definitions

- In science we often define something by telling how it is measured,
- To assign a number to anything you need
  - an idea about the character of the object
  - a process for assigning the number
  - a scale (usually arbitrary)
- In the case of our cubes what scale did we choose?

# Dimensions

- For every new arbitrary scale we choose, we assign a _dimension_.
  - A dimension specifies the kind of measurement (or combination of measurements) we are measuring to get the number.
  - The dimension we have just defined is LENGTH

- Dimensions are useful
  - for inventing new equations
  - for catching errors in math
  - for understanding how a quantity will change when its scale changes.

_"Unit" specifies the particular scale we have chosen to measure with._

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# Examples

- In the next week or so, we will discuss measurements of
  - length (L)
  - time (T)
  - mass (M)

- We write the dimensions of a combined quantity like this. If $v$ is a symbol representing velocty that is constructed by dividing a length measure by a time measure, we write

$$v = 87 \text{ km/hr}$$

$$[v] = L/T$$

# Measuring Length

- The first basic concept we are going to quantify is length.
- Propose a way of defining it a process for measuring it.
  - Scale?
  - Arbitrariness?
  - How "real" is this?

# Measuring Position

- What's the difference between length and position?

- What do you need to measure position?

# Dimensional Analysis and Scaling

- Since the measurement scale for a dimension is arbitrary, we could change it and the number assigned to a physical length would change.

- A <u>dimensional analysis</u> tells us how a quantity changes when the measurement scale is changed.

- Any equation which is supposed to represent a physical relation must retain its equality when we make a different choice of scale.

- Dimensional analysis tells us *how* something changes when we change our arbitrary scale.

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# Dimensioned quantities are not just numbers

$$1 \text{ inch} = 2.54 \text{ cm}$$

$$1 = \frac{2.54 \text{ cm}}{1 \text{ inch}}$$

You can multiply anything by 1 without changing it!

$$12 \text{ inches} = (12 \text{ inches}) \times 1 = (12 \text{ inches}) \times \frac{2.54 \text{ cm}}{1 \text{ inch}}$$

$$= 12 \times 2.54 \text{ cm} = 30.48 \text{ cm}$$

# Changing units

- To change units, we use the fact that the unit clings to a number like a multiplied symbol in algebra, 6*a* or 3*b*.

- We then manipulate our units like symbols.

$$1 \text{ hour} = 60 \text{ minutes}$$

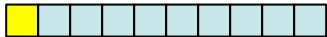$$1 = \frac{60 \text{ minutes}}{1 \text{ hour}}$$

$$\frac{1}{6} \frac{\text{mi}}{\text{min}} = \frac{1}{6} \frac{\text{mi}}{\text{min}} \times 1 = \frac{1}{6} \frac{\text{mi}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ hour}} = \frac{60}{6} \frac{(\text{mi})(\text{min})}{(\text{min})(\text{hour})} = 10 \frac{\text{mi}}{\text{hour}}$$
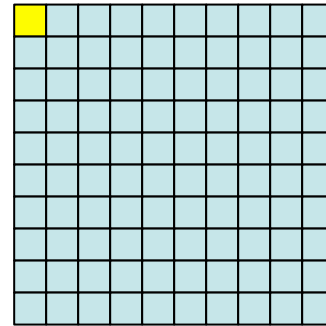
# The unit is part of the expression.

$$1 \text{ m} = 100 \text{ cm}$$

$$\left(1 \text{ m}\right)^2 = \left(100 \text{ cm}\right)^2$$

$$1 \text{ m}^2 = (10^2)^2 \text{ cm}^2 = 10^4 \text{ cm}^2$$

10 cm is what part of a meter?

$(10 \text{ cm}) \times (10 \text{ cm}) = 100 \text{ cm}^2$
is what part of a meter$^2$?

# Careful!

- Dimensions are not algebraic symbols – they are type labels.

$$6 \text{ ft} + 9 \text{ ft} = 15 \text{ ft}$$

$$[6 \text{ ft}] + [9 \text{ ft}] = [15 \text{ ft}]$$

$$L \quad + \quad L \quad = \quad L$$

- We sometimes use "L" (or "M" or "T") for algebraic symbols – to specify a particular length or mass or time.

  You have to know whether you are doing a dimensional analysis or a calculation!

# An example

- If I run a 6 minute mile, what is my speed?

- "6 minute mile" is a *pace*
$$p = (time)/(distance).$$

- Speed is the upside-down pace
$$v = (distance)/(time)$$

- Therefore, $v = 1/p$

$$p = \frac{(6 \text{ min})}{(1 \text{ mi})} \qquad v = \frac{1}{p} = \frac{1}{\left(6 \text{ min} \middle/ 1 \text{ mi}\right)} = \frac{1 \text{ mi}}{6 \text{ min}} = \frac{1}{6} \ \frac{\text{mi}}{\text{min}}$$

# What have we learned? ☑

- In physics we have different kinds of quantities depending on how they were measured.
- These quantities change in different ways when you change your measuring units.
- Only quantities of the same type may be equated (or added) otherwise an equality for one person would not hold for another.

$$1\,cm^3 + 4\,cm^3 = 5\,cm^3 \quad ✓ \qquad 1\,cm + 4\,cm^2 \neq 5\,(anythings) \quad ✗$$

- Measurements are not numbers. They represent physical quantities and therefore contain units as part of them.

# Letting dimensional analysis work for you

- In physics, if we try to add or equate quantities of different dimensions we get nonsense.

- If we didn't maintain dimensional correctness, an equality that worked in one measurement system wouldn't work in another.

- This is a very good way to check your work with equations. (But it's hard to do if you put numbers in too early!*)

*You also won't get much partial credit on exams if you put numbers in too early since we may not be able to tell what equations you are using and why!*

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