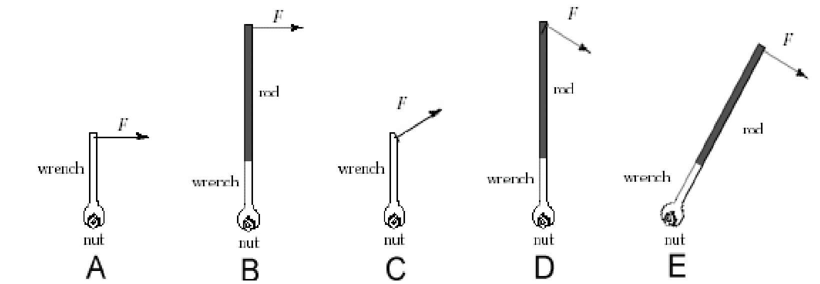
1. (30 points) *For each of the following three problems, rank the quantity indicated from the greatest to the smallest. The notation "A>B" means "A is greater than B". So, to say "A is greater than B and B is equal to C" write "A>B=C". (10 pts each)*

1.1 Rank the magnitude of the torques produced by the force F.



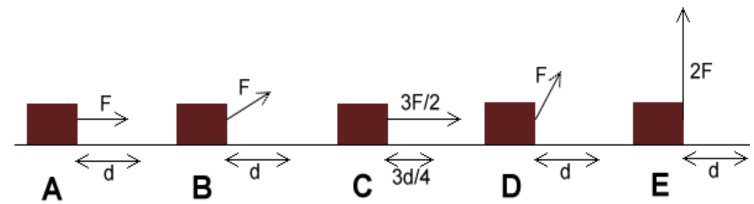
B = E> D > A > C

There are four relationships (> or =). 2 pts are given for each one where the relation between quantities on either side of the = or > are correct. 1 pt is given for the correct largest one (B or E) and 1 pt for the correct smallest one (C).

The key to this problem is understanding that the torque is = the product of the distance from the pivot to the point of application of the force multiplied by the component of the force perpendicular to that distance vector: $\tau = RF\sin\theta$. The component of the force along the distance doesn't do anything to produce a rotational effect. So B is greater than A because the distance is larger. B is greater than D because in D part of the force is not effective (the part that is parallel to the distance vector). This gives the following results.

A: (R)(F) = RF
B: (2R)(F) = 2RF
C: (R)(0.75 F) = 0.75 F (about)
D: (2R)(0.7 F) = 1.4 F (about)
E: (2R)(F) = 2RF

1.2 Rank the change in the kinetic energy produced by the force F if the box moves through the indicated distance.



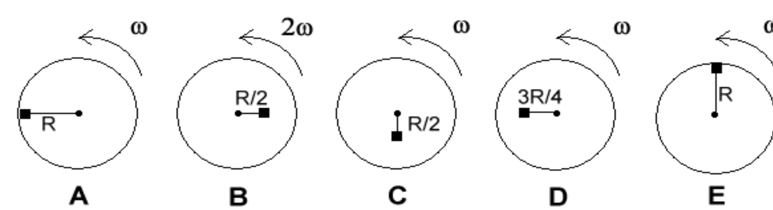
C > A > B > D > E

Grading pattern as in 1.1
The key to this problem is understanding that the work is the product of the distance the object moves times the part of the force in the direction of the object's motion, $\text{work} = \vec{F}\cdot\Delta\vec{r}$. In case B, only about 3/4 of the force is in the parallel direction, in case D, only about 1/2 is in the right direction, and in case E, none of it is. This makes the work as follows in the various cases.

A: (F)(d) = Fd
B: (0.75 F)(d) = 0.75 Fd
C: (1.5 F)(0.75 d) = (3/2)(3/4) Fd = (9/8) Fd = 1.125 Fd
D: (0.5 F)(d) = 0.5 Fd
E: (0 F)(d) = 0 Fd

These give the indicated order.

1.3 Rank the magnitude of velocity of the little boxes attached to a rotating disk. Each disk is rotating at the indicated angular velocity and the box is the indicated distance from the disk's center.



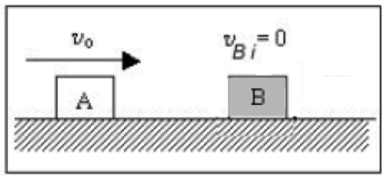
A = B > D > C = E

Grading pattern as in 1.1.
The key to getting this one is to recall that the velocity of each box is the product of the angular speed times the distance from the center: $v = \omega r$. This makes the linear speed (the magnitude of the velocity) as follows.

A: $(\omega)(R) = \omega R$
B: $(2\omega)(R/2) = \omega R$
C: $(\omega)(R/2) = 0.5\ \omega R$
D: $(\omega)(3R/4) = 0.75\ \omega R$
E: $(\omega/2)(R) = 0.5\ \omega R$

2. (20 points) *Two carts of equal mass are floating on a level air track so that they move horizontally with essentially no friction.*

*(a) Cart B is stationary and cart A approaches it from the left with a velocity v. It collides elastically (there are metal springs on the ends of the carts) so that no mechanical energy is lost in the collision. Will the momentum **of each cart** be conserved or not? Explain. (5 pts)*



No. (2 pts) When the moving block hits the second block, both their velocities will change. Since momentum = mv, neither block will maintain the same momentum after the collision. (3 pts)

*(b) Will the total momentum **of the system** (the two carts taken together) be conserved or not? Explain the reasons for your answer. (5 pts)*
Yes. (2 pts) Momentum of a system is conserved when all external forces cancel as is the case here. For each cart, the up and down forces cancel, there is no friction, leaving the only forces acting on them as the force from the other object in the system. When the momentum change of each is added together, the impulses cancel by N3, so however much momentum cart A loses is gained by cart B implying that the total is conserved. (3 pts)

(c) The metal springs on the carts are replaced by Velcro so the carts will stick together when they hit. Will the mechanical energy of the system (the two carts) be conserved or not? Explain the reasons for your answer. (5 pts)

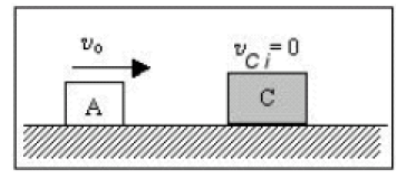
No. (2 pts) This is now an inelastic collision. Energy can go into other forms than kinetic or potential, such as into a deformation of the objects. (Imagine a spring between them that is latched together when they reach their closest distance. Some energy would be stored in the latched spring.) To be sure, we can calulcate precisely the energy lost (not needed for points) Momentum is conserved as described above, so if the final velocity of both together is V, momentum conservation says

$$mv_0 + 0 = (2m)V$$
$$V = v_0/2$$

$$KE_i = 1/2\ mv_0^2$$
$$KEf = 1/2\ (2m)(V)^2 = 1/2\ (2m)(v_0/2)^2 = 1/4\ mv_0^2$$

So 1/2 the KE is lost in the collision. (3 pts)

(d) Cart B is replaced by cart C, which has twice the mass of cart A. If cart A has a mass of 1 kg and approaches cart C with a speed of 30 cm/s, with what speed will they travel after they have hit and stuck together? (5 pts)



When both cars stick, the mass of the combined system is 3 kg. Momentum conservation gives

$$mv_0 = 3mV$$
$$V = v_0/3$$

The result is therefore 10 cm/s. (3 pts for setup from momentum conservation, 2 pts for calc. If mass of A is ignored in final combo giving 15 cm/se, +2. No credit for $m_A v_A = m_C v_C$ since this is not momentum conservation.)

***3. (15 points)**. One of the activities for the annual high school Physics Olympics competition is the "egg drop." Students create packaging that allows them to drop a raw egg off the top of the physics building to land on the ground undamaged. Some packages involve parachutes to slow the fall of the egg, but some don't. In one version, the packaging can have a mass equal to no more than the mass of the egg. For a packing that does NOT involve slowing the fall using air resistance, estimate how much energy the packing has to absorb when it hits the ground so that the egg does not break. Be sure to clearly state your assumptions and how you came to the numbers you* *estimated, since grading on this problem will be mostly based on your reasoning, not on your answer.*

The simplest way to do this is by energy conservation. At the top of the building (height h), the egg (mass m) and packaging (mass m) have a PE = 2mgh. (3 pts. –1 if miss factor of 2) Just before it hits the ground, this will be turned into all kinetic. (3 pts) The packaging must absorb almost all of this energy.

To see how much it is, we need to estimate the mass of the egg and the height of the building. It takes about 4 large eggs to make a pint and eggs aren't that different mass-wise from water. One pint of water weighs 16 oz, so an egg weighs about 2 oz. To convert this to a mass, recall 2.2 pound = 34 oz corresponds to 1 kg. 1 lb = 454 g (I know this from reading cereal boxes). This gives the mass of an egg to be about 454/8 g ~ 60 g. (3 pts. Everything from 40-150 g was accepted. –1 for 10-40 or 150-200, -2 for outside that. –1 for no reason.)

For the height of the building, the physics building is 4 stories high and 1 story is about 1.5 my height, so about 10 ft giving a total height of about 40 ft or 10-15 m. (3 pts. Everything from 10-15 was accepted. Outside a bit was –1. No reason was –1.)

So the energy needed to be absorbed is

$$2mgh = 2\ (0.06\ \text{kg})\ (10\ \text{N/kg})\ (10\ \text{m}) = 12\ \text{N-m} = 12\ \text{J}$$

About 10 J. (2 for calculation. –1 for wrong units. –1 for using mixed units and not noticing.)

4. (10 points) *In this class, we defined two ways of multiplying two vectors: a "dot product" and a "cross product." Define how one constructs each product given a general pair of vectors. For each of these two products identify a physical quantity that one might construct using them and explain why the particular product is the right one to use for that quantity.*

The cross product of two vectors is a vector designed to show that plane that the 2 vectors determine. It's magnitude is the area spanned by the two vectors (= AB sinθ □□□□□θ is the angle between the two vectors). The direction is the perpendicular to the two vector determined by a right-hand rule.

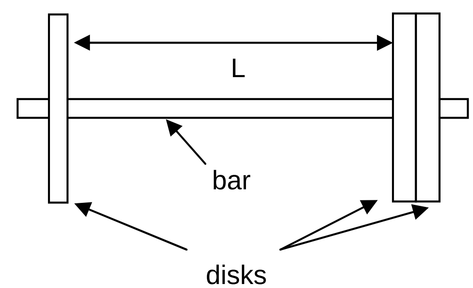
The cross product picks out the component of one vector perpendicular to the other. We need this to construct the torque, which represents the rotational effect of a force, since only the perpendicular component of the force produces a rotational effect.

The dot product of two vectors is a number – a scalar, not a vector. It is designed to determine the amount of one vector that points in the same direction as the second vector. Its magnitude is AB cos θ where θ is the angle between the two vectors.

The dot product picks out the part of one vector in the samne direction as another, just what we need to specify the work, since only the part of the force in the same direction as the object's displacement has any effect on changing the object's KE. (5 pts for each product, 2 for a construction rule, 1 for specifying a physical quantity, and 2 for an explanation of why it is used there.)

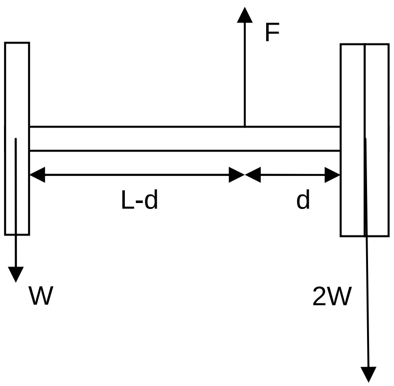
5. (25 points)

(a) A weightlifter wants to do exercise lifting 75 pounds with one hand, but he only has a light (but strong) aluminum bar and three steel disks weighing 25 pounds each. He can set them up as in the figure, but then finds that they are unbalanced if he tries to lift the bar holding it in the middle. Where should he hold the bar so that the weights are balanced? Assume you can ignore the weight of the bar and that the disks are thin compared to the distance L between them. (15 pts)



If he's holding it by one hand, that hand serves as a convenient pivot. We have to balance the torques of the disks on either side around that pivot. (5 pts)

How to do it can be easily seen from an extended free-body diagram.



Around the pivot of his hand, the one disk exerts half the force of the two disks, so it must be at twice the distance to balance. This means his hand should be 1/3 of the way from the right end. (5 pts for answer, 5 pts for reasoning)

To see this in equations, write the torque balance:

$$(W)(L-d) = (2W)(d)$$

$$WL - Wd = 2Wd$$

$$WL = 3Wd$$

$$d = L/3$$

(b) While waiting to have the disks attached, he balances a baseball bat on his finger as shown. His friend comments, "That must be where the center of mass is." Is the friend right? How do you know? (5 pts)

The friend is right. (2 pts) The center of mass is where the total force of gravity on the bat appears to act. If it were anywhere except right about the pivot, it would exert a torque about the pivot and tip it, since there is no other force acting on the bat away from the pivot. (3 pts)

(c) When the bat is balanced is there more of the bat's mass to the left of the weightlifter's finger, more to the right, or is it the same on both sides? How do you know? (5 pts)

More to the right. (2 pts) When you consider the bat as two pieces, it's like a seesaw with two different kids on it. Each piece has its weight acting at its own CM. The right piece has its CM closer to the pivot than the left piece. This means that the right piece is like the heavier kid sitting closer to the pivot and the left piece is like the lighter kid sitting farther away to balance. (3 pts)