University of Maryland
Department of Physics

Physics 121 | **Dr. E. F. Redish**
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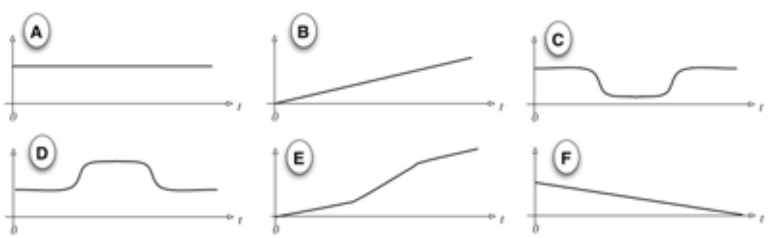
Exam 2 (Make up) Solution

1. (30 points) *In lecture we observed two billiard balls that were driven down parallel tracks by a spring gun. One ball traveled along a flat track, the other along a track with a dip. The apparatus is shown in the figure below.*

Below are shown 6 graphs and 10 physical quantities. Identify which of the graphs could match each physical quantity if the vertical axis were given the correct scale. The time t = 0 occurs just after the balls have been pushed by the spring. Choose the graphs that match each quantity a.-j. If none works, write N. Each graph may be chosen as many times as you like. You may assume friction can be neglected. No explanations are required on this problem. (3 pts each)



a. __**A**___ *The kinetic energy of ball A*
b. __**A**___ *The potential energy of ball A*
c. __**D**___ *The kinetic energy of ball B*
d. __**C**___ *The potential energy of ball B*
e. __**A**___ *The total energy of ball A*
f. __**A**____*The total energy of ball B*
g. __**A**___ *The velocity of ball A*
h. __**D**___ *The velocity of ball B*
i. __**B**___ *The position of ball A*
j. __**E**___ *The position of ball B.*

Ball A, after being driven forward by the spring, moves at a constant velocity (no forces act on it since we are told to assume friction is negligible). As a result, it's velocity, KE (1/2 mv^2), potential energy (*mgh*), and total energy are all constant. As a result, graph A works for all of them. The position increases linearly, so graph B is correct for that.

Note that the constant potential energy can have any value since it depends on where we take the 0 of height to be. If we use for the 0 of height a position on the top of the table on which the apparatus is resting, the height of track A is a positive value and graph A is correct.

For ball B, the total energy of the ball is constant, but the PE drops in the dip so the KE grows to compensate. This means the total energy is A, the PE is C (looking like the graph), and the KE is D. The velocity also looks somewhat like D, starting constant, going to a larger value, and then coming back to the original constant. But the ratio between the initial velocity and the velocity in the dip is different from the ratio between the initial KE and the KE in the dip, so while D is sort of correct, N is really better. Both are accepted.

The position of ball B grows at a uniform rate, then at a faster rate, then at the original rate, so graph E is correct.

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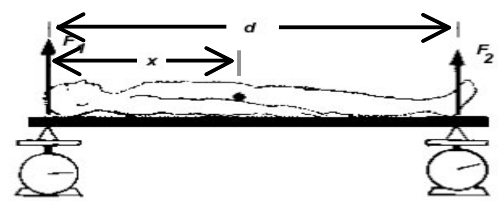
2. (20 points) *One way to measure the position of the center of gravity of a person (marked by a black circle) is to support them with two scales as shown in the figure at the right. The height of the person is d, the distance from the top of their head to their center of gravity is x, and the person's weight is W.*



(a) If the scale at the left reads F_1 and the scale at the right reads F_2 find the distance x in terms of the other given parameters. Be sure to explain carefully (but briefly!) what physics you are using. (8 pts)

This is a static situation so the forces and the torques are both balanced. The principles

$$F_{up} = F_{down}$$
$$\tau_{clockwise} = \tau_{counter-clockwise}$$

relate the forces and the distances. The force balance relation is

$$F_1 + F_2 = W$$

The torque balance relation depends on which pivot we use. Here are three possibilities for the torque balance condition:

$$Wx = F_2 d \qquad \text{(pivot at head)}$$
$$F_1 x = F_2(d-x) \qquad \text{(pivot at center of gravity)}$$
$$F_1 d = W(d-x) \qquad \text{(pivot at foot)}$$

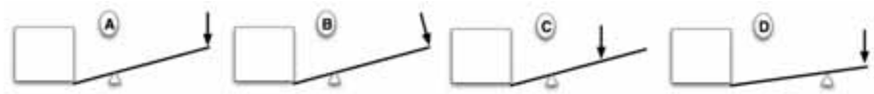
We can use as many of these as we want and combine them in any way we want, but we must solve for x. Solving the torque balance equation with the pivot at the center of gravity easily expresses the result in terms of the two measurements.

$$F_1 x = F_2(d-x)$$
$$F_1 x = F_2 d - F_2 x$$
$$F_1 x + F_2 x = F_2 d$$
$$\left(F_1 + F_2\right)x = F_2 d$$
$$x = \frac{F_2}{F_1 + F_2} d$$

Other forms are possible.

[Grading: 3 pts for a correct set of principles that would allow you to solve. 2 pts for creating the correct equation and 3 pts for solving it correctly in terms of symbols.]

(b) You want to use a long metal bar to lift up the corner of a heavy stone block. For the four arrangements shown below, rank them in order of their effectiveness in raising the stone. The arrow in each figure indicates where you are pushing and in what direction. If two are equal, indicate that. (Your answer should be a string of letters that looks something like E = F > G > H meaning E and F are the biggest and H is the smallest.) No explanation is required on this problem. (12 pts)



We want to move the block, so we should be looking for the force to exert that gives the greatest force on the block. If *a* is the distance from the block to the pivot and *b* is the distance from the pivot to my force, then the force on the block will be $F_{\|} b/a$. We want my force to be as far away as possible from the pivot, the block to be as close to the pivot as possible, and my force to be perpendicular to the board so as to have the greatest effect. This gives the ordering

B > A > C > D.

[Grading: 4 pts for each inequality. These are evaluated by whatever is on either side. Thus, if you wrote A>B>C>D, since A>B is wrong but B>C and C>D are right you get 8 pts. No pts are given for second level inferences – so the fact that A>C is correct doesn't buy you anything.

Note: One could plausibly interpret the question as asking in which case am I exerting the most torque. In that case the answer would be B>A>C=D. Credit is also given for this answer.]

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3. ***(15 points)*** *In a proposed episode that Disney is considering for teen superhero Kim Possible®, Kim is planning to lasso the (large) bad guy Duff Killigan and swing him around her head.*

If Kim is holding the rope so that there is 3 m of rope from her hand to the lasso's loop and if Duff has a mass of 150 kg, estimate how much force Kim would have to exert on the rope in order to keep him swinging around in a circle. Is this a plausible amount of force for a teenage superhero to exert? Be sure to clearly state your assumptions and how you came to the numbers you estimated, since grading on this problem will be mostly based on your reasoning, not on your answer.

We know to keep an object going in a circle you need a force pulling inward (to keep it from going out on a straight line path). For Duff, this is provided by the rope. If there is a tension *F* of the rope pulling on him, then Kim must also be pulling on the other end of the rope with a force *F*.

She has to both exert a force to lift him and to keep him going in a circle. But we are only asked about the latter, so let's calculate that. We know the force needed to keep him moving in a circle is

$$F = \frac{mv^2}{R}$$

from our analysis of uniform circular motion. We know his mass (given as 150 kg) and the radius of the circle he is moving in (given as 3 m). We have to estimate his speed, v.

To estimate v, I will think about the time it takes to make a full circle. A good starting speed would be about 1 second to go around. (If I know my cartoons, he would quickly be sped up until he became a blur!) At one second to go the distance around the circle (=2πR ~ 18 m – call it 20 for estimation purposes) his speed would be his distance over time, or

$$v = \frac{2\pi R}{T} = \frac{20 \text{ m}}{1 \text{ s}} = 20 \text{ m/s} \cdot$$

Putting this into the formula for the force gives

$$F = \frac{mv^2}{R} = \frac{(150 \text{ kg})(20 \text{ m/s})^2}{3 \text{ m}}$$
$$= \frac{(15\times 10^1)\times(4\times 10^2)}{3}\frac{\text{kg-m}^2}{\text{m-s}^2}$$
$$= 5\times 4\times 10^3 \frac{\text{kg-m}}{\text{s}^2} = 2\times 10^4 \text{ N}$$

20,000 N! That's a lot, since 1 pound ~ 5 N, so its about 4000 lbs.

As a superhero, she might be able to exert that force, but the rope would pull back on her with an equal force and, unless she had a tap root, or was way heavier than she looks, there is no way the friction from the ground would be able to keep her in place.

[Grading: Getting the right formula for the force = 5, figuring out what to estimate and a plausible way to estimate it = 3, a reasonable estimation number (for velocity or time) = 2, calculation = 3, some statement about whether she could do it = 2. As usual, -2 for too many (3 or more) sig figs.]

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4. *(10 points)* *In the middle part of the class we have studied momentum conservation and energy conservation. Pick one of these topics and explain briefly what it means, when it holds, and how it relates to the material studied in the first part of the class (Newton's laws). Note: This is an essay question. Your answer will be judged not solely on its correctness, but for its depth, coherence, and clarity.*

Here are sample essays covering each topic. You only had to do one.

Momentum conservation –

Momentum conservation states that if you add together the momenta ($\vec{p} = m\vec{v}$) of a set of interacting objects, and if the forces acting on those objects from objects outside the set add to 0, then the vector sum of the momenta of the objects in the set remains the same no matter how the objects interact among themselves.

This relates very intimately to the Newton's laws we studied in the first part of the course. Newton's 0^{th} law leads to free-body diagrams and our ability to identify forces acting on individual objects. The way momentum changes, as specified in the impulse momentum theorem, $\vec{F}^{net}\Delta t = \Delta\vec{p}$, is an alternative statement of Newton's 2^{nd} law and is what controls how the individual momenta change. Newton's 3^{rd} law, which says that the forces objects exert on each other when they interact are equal and opposite, is what is responsible for the cancellation of the internal forces within the set that leads to momentum conservation. It relates to Newton's 1^{st} law in that it tells us we can treat a system of objects as an object: if all external forces on the system balance it's momentum doesn't change – the analog of staying at the same velocity for a single object.

Energy conservation –

Energy conservation as used so far in this course refers to conservation of mechanical energy, kinetic ($\frac{1}{2}mv^2$) and potential (so far, gravitational and spring). It states that if you have an object moving under the influence of forces and if those forces are all conservative (do not include friction or applied forces), then the total mechanical energy of the system does not change.

This relates directly to the Newton's laws we studied in the first part of the course. Newton's 0^{th} law leads us to free-body diagrams and our ability to identify which forces are acting on the object and when so as to see if they are conservative. The way energy redistributes is specified by the work-energy theorem, $F_{\parallel}^{net}\Delta x = \Delta\left(\frac{1}{2}mv^2\right)$, which is a direct consequence of Newton's 2^{nd} law.

[Grading: A correct statement of the principle is worth 3, while the explication of the correct conditions is worth 3. Your discussion of the relation to Newton's laws is worth 4.]

5. ***(25 points)*** *Making a rest stop at a service station on the highway, the driver of a moving van turned off his truck, left it in neutral, and ran into the building, forgetting to set his emergency brake. Unfortunately, he was parked on a slight incline and the truck started to roll down.*

When the van got to the bottom of the incline, the road was flat and smooth and the van rolled on without slowing significantly. After traveling a short distance, it ran into a Porsche. The driver of the Porsche also had left her car in neutral without setting the emergency brake. (Bad idea!) The truck was going pretty slowly and had a rubber bumper so when it hit the car, the car bounced off. For the period of the collision, friction with the ground was negligible.

Here are the parameters of what happened.

- *Distance van traveled after coming off the incline before it hit the car = 37 ft.*
- *Mass of the van = 4000 kg.*
- *Mass of the car = 1000 kg.*
- *Time car and tuck are in contact = 1.5 s.*
- *Van's speed just before it hit the car = 5 km/hr.*
- *Van's speed just after the car and van separate = 2 km/hr.*

a) What was the speed of the car after the collision? Explain how you know. (10 pts)

This is determined by momentum conservation since there are no unbalanced external forces on the pair of objects. Since the initial velocity of the car is 0, momentum conservation becomes:

$$m_T v_T^i = m_T v_T^f + m_C v_C^f$$

(We don't explicitly write vector signs since we are in one dimension where we use the sign to specify direction.) We know everything except the velocity of the car. Solving for this gives

$$v_C^f = \frac{m_T}{m_C}\left(v_T^i - v_T^f\right) = \frac{4000\text{ kg}}{1000\text{ kg}}\left(5\text{ km/hr - 2 km/hr}\right) = 12\text{ km/hr}$$

[Grading: 4 pts for identifying momentum conservation as the relevant principle and writing it correctly. 3 pts for putting the right parameters in for the right variables. 3 pts for doing the calculation correctly.]

b) Is the information given enough to calculate the average force felt by the car? If so, calculate it. If not, explain why not. (5 pts)

Yes it is, by the impulse-momentum theorem. If we know the change in momentum and the time the force is applied, we can calculate it. (This is the average force over the time interval.) To get to N we would have to convert to m/s, but I won't bother. It's OK as long as I specify what units I am in.

$$F_{T\to C}\Delta t = \Delta p_C$$

$$F_{T\to C} = \frac{\Delta p_C}{\Delta t} = \frac{p_C^f - p_C^i}{\Delta t} = \frac{\left(1000\text{ kg-km/hr - 0 kg-km/hr}\right)}{1.5\text{ s}}$$

$$= 667\text{ kg-km/hr-s}$$

[Grading: 1 pts for getting the answer (yes), 2 pts for saying why. 2 pts for the calculation and answer.]

c) Is the information given enough to calculate the average force felt by the truck? If so, calculate it. If not, explain why not. (5 pts)

Yes it is. Either by the impulse-momentum theorem as in (b), or by Newton's 3rd law which says the forces the car and the truck exert on each other have to be equal and opposite. So the answer is again 667 N.

[Grading: 1 pts for getting the answer (yes), 2 pts for saying why. 2 pts for the value (could just be stated).]

d) How much mechanical energy was lost in the collision, if any? (This will determine how much damage the bumpers suffered.) (5 pts)

Since we know the initial and final velocities and masses we can calculate the loss of KE (PE doesn't change).

$$KE_f - KE_i = \left(\tfrac{1}{2}m_T v_T^{f\,2} + \tfrac{1}{2}m_C v_C^{f\,2}\right) - \left(\tfrac{1}{2}m_T v_T^{i\,2} + \tfrac{1}{2}m_C v_C^{i\,2}\right)$$

$$= \tfrac{1}{2}m_T\left(v_T^{f\,2} - v_T^{i\,2}\right) + \tfrac{1}{2}m_C v_C^{f\,2}$$

$$= \tfrac{1}{2}\left(4000\text{ kg}\right)\left(\left[2\text{ mi/hr}\right]^2 - \left[5\text{ mi/hr}\right]^2\right) + \tfrac{1}{2}\left(1000\text{ kg}\right)\left(12\text{ mi/hr}\right)^2$$

$$= (2000)(4-25)\text{kg-mi}^2/\text{hr}^2 \;\; + (500)(144)\text{kg-mi}^2/\text{hr}^2$$

$$= \text{-42,000kg-mi}^2/\text{hr}^2 \;\; + \text{72,000kg-mi}^2/\text{hr}^2$$

$$= 30,000\text{ kg-mi}^2/\text{hr}^2$$

Energy was gained! Either there was an error in the measurements (or a typo) or an explosion that added energy.

[Grading: 3 for the equation of energy change, 1 for putting in the right numbers, 1 for the value.]