Elizabeth and Lydia have just finished the pendulum lab, where they measured the period of a pendulum with different masses. They made a two-meter long pendulum, used six different masses and measured the period three times for each mass. Though they had a wonderful time measuring the period, they aren’t happy with just knowing whether or not the period depends on the mass. They want to know what function describes the dependence.

Lizzie: I think the graph looks like a square root function,

\[ T = C_1 \sqrt{m} \]

Lydia: How do you know? It could just as easily be a cube root,

\[ T = C_2 \sqrt[3]{m} \]

Lizzie: Well, if either function fits, then \( C_1 \) or \( C_2 \) must be constant for all of our data.

Lydia: Hmm?

Lizzie: If we use each of our data points to calculate the constant, then, if our data fits a function perfectly, we’ll get the same constant for all our data.

Let’s see, so \( C_1 = T / \sqrt{m} \) and \( C_2 = T / \sqrt[3]{m} \).
Lydia: Look – the cube root only varies from 1.408 to 2.503, but the square root varies from 0.995 to 2.384. I think the cube root fits the data better.

Lizzie: But they’re supposed to be constant. That’s not constant.

Lydia: Nothing’s going to be perfect in real life. That’s as close to constant as we’re going to get – we had to time these by hand with stopwatches.

Lizzie: No – look, we measured each period three times, so we know how scattered the data is going to be. For mass 2, our period was from 2.79 to 2.81 seconds. This made $C_1$ go from 1.704 to 1.716 and $C_2$ go from 2.009 to 2.023. So our range of 0.02 sec from timing can only cause a range in the $C_1$ of about 0.012 and $C_2$ of about 0.014. The constants have a lot larger range than that, so they aren’t constant.

Lydia: What?

Lizzie: In an ideal world, we’d get the same thing every time we measured the period, and the function that fit would have exactly the same constant for all the data.

Lydia: Yeah, that’s what I was saying, it’s not an ideal world. The constants are not going to be perfectly the same, so the function that has the smallest range of constants is the one that fits the best.

Lizzie: Right, but we know how ‘un-ideal’ we are because we measured the period three times for each mass. This range in periods for a single mass tells us how much the constant can vary and still be judged ‘constant’ because of the variation in periods. We need to find a function whose constants have a total range that is just as big as the range caused by the three different periods for one mass.

Since this is completely new material, you are not expected to give a perfect answer. The point is to get you thinking about issues that will arise in lab. Please don’t spend more than 20 minutes on this.

A. Do you agree with Lizzie or with Lydia? Explain why the particular reasoning makes sense to you.