SOLUTIONS – 7 – FORMULAE

UNIVERSAL LAW OF GRAVITATION – Pt. masses

$$\underset{\rightarrow}{F_G} = -\frac{GM_1M_2}{r^2}\hat{r}$$

MINUS SIGN ENSURES THAT FORCE IS ATTRACTIVE.

Pt. mass (m) and Solid sphere of mass M and radius R.

$r < R$ $\quad \underset{\rightarrow}{F_G} = -\frac{4\pi}{3} d\, m\, G\, r\, \hat{r} \quad \left[d = \frac{M}{\frac{4\pi}{3}R^3} \right]$

$r > R$ $\quad \underset{\rightarrow}{F_G} = -\frac{GMm}{r^2}\hat{r}$

Two spheres

$$\underset{\rightarrow}{F_G} = -\frac{GM_1M_2}{r^2}\hat{r}$$

r is Center-to-Center Distance.

Satellites of Sun (Planets) – Circular orbits

$$T_P^2 = \frac{4\pi^2}{GM_S}R_P^3 \quad [\underset{\rightarrow}{F_G} \text{ provides } \underset{\rightarrow}{F_c}]$$

Satellites of Earth – Circular orbits

$$T_{sat}^2 = \frac{4\pi^2}{GM_E}r_{sat}^3 \quad [\underset{\rightarrow}{F_G} \text{ provides } \underset{\rightarrow}{F_c}]$$

Kinematic Eqns.	Previously
$\underset{\sim}{\alpha} = \alpha\hat{z}$	$\underset{\rightarrow}{a} = a\hat{x}$
$\underset{\sim}{\omega} = (\omega_i + \alpha t)\hat{z}$	$\underset{\rightarrow}{v} = (v_i + at)\hat{x}$
$\underset{\rightarrow}{\theta} = (\theta_i + \omega_i t + \frac{1}{2}\alpha t^2)\hat{z}$	$\underset{\rightarrow}{x} = (x_i + v_i t + \frac{1}{2}at^2)\hat{x}$
$\omega^2 = \omega_i^2 + 2\alpha(\theta - \theta_i)$	$v^2 = v_i^2 + 2a(x - x_i)$

Note: FORCE CAUSES $\underset{\rightarrow}{a}$ (linear acceleration)

TORQUE CAUSES $\underset{\sim}{\alpha}$ (angular acceleration)

Torque $\underline{\tau} = [\underline{r} \times \underline{F}]$

Magnitude $\tau = rF \sin(\underline{r}, \underline{F})$

$\underline{r}$ POINTS FROM PIVOT Pt. TO Pt. of application of $\underline{F}$

Direction of $\underline{\tau}$, Right hand rule. Stretch

Right hand: First vector ($\underline{r}$) along thumb

2nd Vector ($\underline{F}$) " Fingers

$\underline{\tau}$ perpendicular to palm.

Rigid Body ($\underline{r}_i - \underline{r}_j$) = Const. [No change of shape or size]

Center of Gravity

$$\underline{r}_{cg} = \frac{\sum m_i \underline{r}_i}{\sum m_i} \Rightarrow x_{cg} = \frac{\sum m_i x_i}{\sum m_i}$$

$$y_{cg} = \frac{\sum m_i y_i}{\sum m_i}$$

Dynamics.

Translation

$M\underline{a} = \sum \underline{F}_i$

at that pt.

at that time

Rotation (about fixed axis)

I replaces M

$I = \sum m_i r_i^2$.

$\underline{\alpha}$ replaces $\underline{a}$

$\underline{\tau}$ " $\underline{F}$

$I\underline{\alpha} = \sum \underline{\tau}_i$

about the axis for which I is calculated

~~Solution week~~

CHAPTER 6

6.56 On the surface of a sphere $\vec{F_G} = -\frac{GMm}{r^2}\hat{r}$ hence Equation ~~about~~ for free-fall acceleration on the surface of a planet becomes

$$\vec{g}_{planet} = -\frac{G M_{planet}}{R^2_{planet}}\hat{r} \qquad (6.24)$$

X : the radius of the earth after it's shrunk. ; we have

$$\vec{g}_{surface} = -\frac{G M_e}{R_e^2}\hat{r} \quad \text{and} \quad 3 \cdot \vec{g}_{surface} = -\frac{G M_e}{X^2}\hat{r} \text{ (mass doesn't change)}$$

Compare magnitudes

$$\Rightarrow \frac{3 G M_e}{R_e^2} = \frac{G M_e}{X^2} \Rightarrow X^2 = \frac{R_e^2}{3} \quad \text{or} \quad X = \frac{R_e}{\sqrt{3}} \approx 0.58\, R_e$$

$\Downarrow$ The fraction should be 0.58

6.63 ~~Assume~~ Assume that, Mars and Phobos are spherical masses and the orbit is a circle.

We will use equation 6.28, which is

$$T^2 = \left(\frac{4\pi^2}{G M_{mars}}\right) \cdot r^3$$

T = 7hour 39 min = 27540 second.

$r = 9.4 \times 10^6$ m

$T = 6.67 \times 10^{-11}\ N.m^2/kg^2$

$$M_{mars} = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 . (9.4\times10^6)^3}{(6.67\times10^{-11})(27540)^2} = 6.5\times10^{23}\ (kg)$$

$\Rightarrow$ Mass of Mars is 6.5×10^{23} (kg)

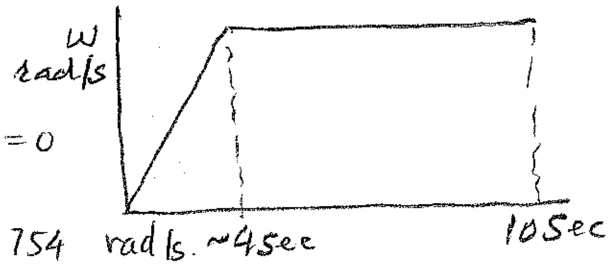
CHAPTER 7

7.2 : There are two parts happening in this motion.

* The 1st part: The disk accelarates until it reaches a constant angular velocity.
* The 2nd part: The disk rotates at this constant [redacted] velocity for the remainder of the time $\vec{\omega}_f = (\omega_i + \alpha t)\hat{z}$

Use equation $\Delta\omega = \alpha . \Delta t$ (Table 7.2)

where $\Delta\omega = \omega_f - \omega_i$



The disk starts from rest $\Rightarrow \omega_i = 0$

$\omega_f = 7200$ rpm $= 7200 . \dfrac{2\pi}{60} = 754$ rad/s.

$\Rightarrow$ Accelerating time Δt is $\Delta t = \dfrac{\Delta\omega}{\alpha} = \dfrac{754 - 0}{190} = 3.97$ (s)

+ Total angular displacement of disk during part I :

Use equation $\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha . \Delta t^2$ (Table 7.2)

$\Rightarrow \Delta\theta_1 = \frac{1}{2}\alpha . \Delta t^2 = \frac{1}{2} \times 190 \times (3.97)^2 = 1500$ (rad)

Time remaining will be

$t_2 = t - \Delta t = 10 - 3.97 = 6.03$ (s)

Total angular displacement of disk during part II

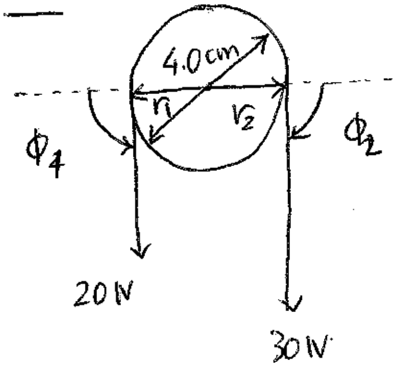
$\Delta\theta_2 = \omega_f . t_2 = 754 \times 6.03 = 4550$ (rad)

Total angular displacement will be :

$\Delta\theta = \Delta\theta_1 + \Delta\theta_2 = 1500 + 4550 = 6050$ (rad)

Convert to revolutions, we have $\dfrac{6050}{2\pi} = 960$ (revolutions)

7.7



diam = 4 cm

We will use the formula. $\vec{\tau} = [\vec{r} \times \vec{F}]$

Magnitude $\tau = F.\ r.\ \sin\phi$ (7.4).

We calculate $\vec{\tau}_1$ for 20 N force and $\vec{\tau}_2$ for 30 N force then add them together.

For 20 N force, force vector makes an angle + 90° relative to radius vector $\vec{r}_1$

$$\vec{\tau}_1 = F_1.\ r_1.\ \sin\phi_1 \hat{z} = 20.\ r_1.\ \sin(+90)\ N\text{-}m\ \hat{z}$$

For 30N force, force vector makes angle ~~+~~90° relative to $\vec{r}_2$

$$\vec{\tau}_2 = -F_2.\ r_2\ \sin\phi_2 \hat{z} = -30.\ r_2.\ \sin(+90)\ N\text{-}m\ \hat{z}$$

$$r_1 = r_2 = \frac{4.0}{2} = 2.0\ \text{cm}) \Rightarrow$$

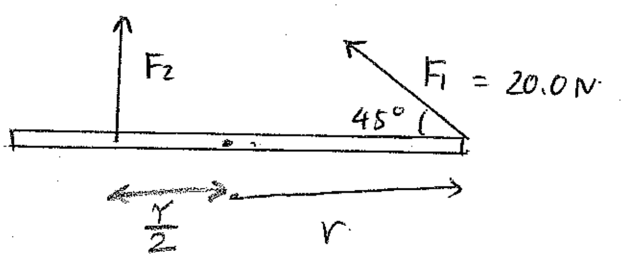
$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 = 20 \cdot \frac{2}{100} \sin(+90°)\hat{z} - 30.\ \frac{2}{100}.\ \sin(+90°)\ \hat{z}$$

$$= -\frac{0.2}{100}\ Nm\ \hat{z}$$

$$\Downarrow\ \vec{\tau} = -\frac{0.2}{100}\ Nm\ \hat{z} \quad \text{(causing clock wise motion)}$$

$$= -2 \times 10^{-3}\ N\text{-}m\ \hat{z}$$

7.10



We will use equation $\vec{\tau} = [\vec{r} \times \vec{F}]$

Magnitude $\tau = F.\ r.\ \sin\theta$.

The torque due to F_1 is

$$\vec{\tau}_1 = +F_1.\ r.\ \sin 45°\ \hat{z}$$
$$= 20.\ r.\ (\frac{\sqrt{2}}{2})\ \hat{z}$$

The torque due to F_2 is

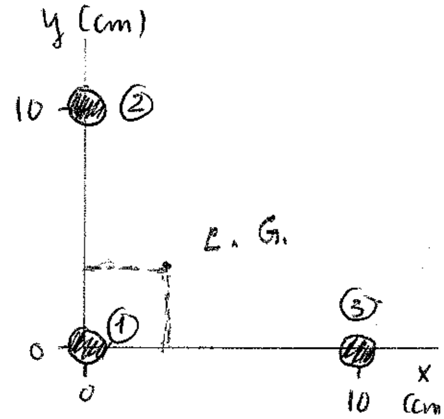
$$\vec{\tau}_2 = F_2.\frac{r}{2}\ \sin(90°)\hat{z} - F_2.\ \frac{r}{2}\ \hat{z}$$

The net torque on the rod is zero, meaning $\vec{\tau}_1 + \vec{\tau}_2 = 0$

$$\Rightarrow\ 20.r.\ (\frac{\sqrt{2}}{2})\ \hat{z} - F_2.\frac{r}{2}\hat{z} = 0 \Rightarrow F_2 = 20.\ \sqrt{2} \quad (N)$$

or $F_2 \simeq 28.3$ (N)

7.16 Using x - Y plane as a picture.



The coordinates for three masses are

$x_1 = 0$ cm ; $y_1 = 0$ cm

$x_2 = 0$ cm ; $y_2 = 10$ cm

$x_3 = 10$ cm ; $y_3 = 0$ cm.

We will use two formulas in " Tactic box 7.1 ".

X- coordinate of center of gravity is

$$x_{cg} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3}$$

Y - coordinate :

$$y_{cg} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3}{m_1 + m_2 + m_3}.$$

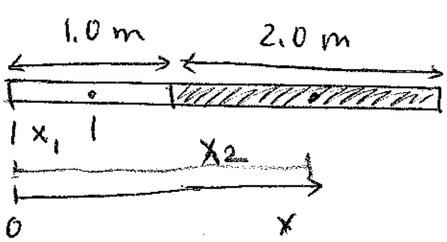
NOTE: C.G. is at a point other than any of the 3 masses.

Three masses are identical, $m_1 = m_2 = m_3 = m \Rightarrow$

$$x_{cg} = \frac{0 \cdot m + 0 \cdot m + 10 \cdot m}{m + m + m} = \frac{10}{3} \simeq 3.33 \text{ (cm)}$$

$$y_{cg} = \frac{0 \cdot m + 10 \cdot m + 0 \cdot m}{m + m + m} = \frac{10}{3} \simeq 3.33 \text{ (cm)}$$

7.23



Assume each beam is of uniform density, its own center of gravity will be at its geometrical center.

Use x - coordinate only.

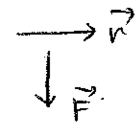
$\Rightarrow x_1 = \frac{1.0}{2}$ m; $x_2 = \left(\frac{2.0}{2} + 1.0\right)$ m

Use the same equation with problem 7.16

$$x_{cg} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} = \frac{\frac{1.0}{2} \times 10 + \left(\frac{2.0}{2} + 1.0\right) \times 40}{10 + 40} = 1.70 \text{ (m)}$$

b) Use equation $\vec{\tau} = \vec{r} \times \vec{F}$

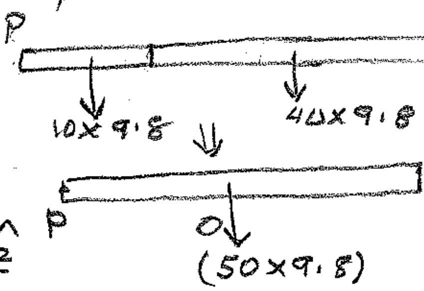
~~Hequ:~~ $\tau = F_1 r_1 \sin\theta$



where F is total weight acting at the center of gravity.

r is x- coordinate of center of gravity

$\theta = +90^\circ$

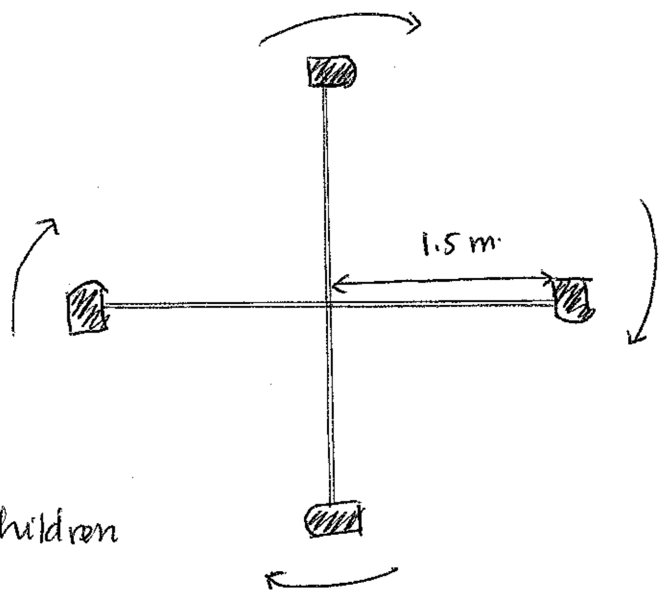


$\Rightarrow \vec{\tau} = -(m_1+m_2) \times g \times x_{cg} \times \sin 90^\circ \hat{z}$

$= -(10+40) \times 9.8 \times 1.7 \hat{z} = -833 (N.m) \hat{z}$

Note that this torque has clockwise direction, so $\vec{\tau} = -833 \ (N.m)\hat{z}$

7.21



We will use the formula

$I = MR^2$.

+ We ignore rods since problem said they're very light

+ Since all the masses, including children are far from the axis of ~~no~~ rotation equally, we will use the same R for all masses & kids.

$R = 1.5$ m.

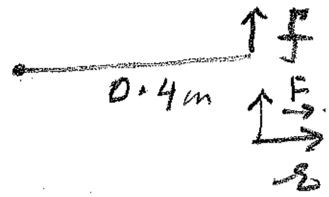
$M = 4 \times m_{max} + m_{kid\,1} + m_{kid\,2} \Rightarrow$

$I = (4 \times 5.0 + 15 + 20) \cdot (1.5)^2 = 120 \ (Kg \cdot m^2)$ [actually 124 kg-m^2]

$\Rightarrow I = 120 \ Kg \cdot m^2$.

7.33 We use equation $I\vec{\alpha} = \vec{\tau}$ both $\vec{\alpha}$ and $\vec{\tau}$ are along $+\hat{z}$

magnitude of $\alpha = \dfrac{\tau_{net}}{I}$



+ τ_{net} = torque given by the frictional force.

$\vec{\tau}_{net} = F.\, r.\, \sin\theta \cdot \hat{z}$ where $F = 7.0$ N

$r = 40$ cm $= 0.4$ m.

We're told that, frictional force applied in a direction that cause the greatest angular acceleration, meaning ~~ϕ~~ $\theta = +90°$

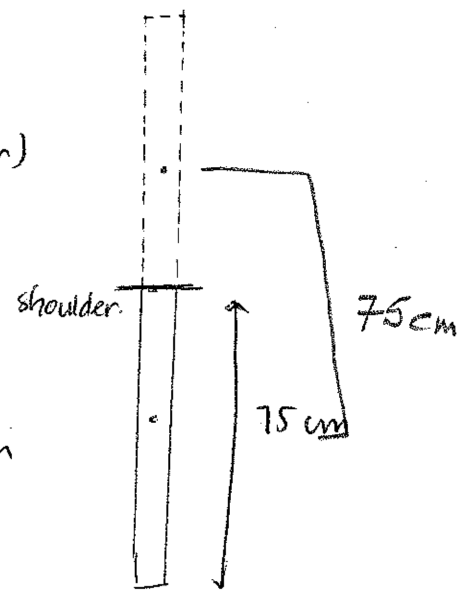
$\Rightarrow \quad \vec{\tau}_{net} = 7.0 \times 0.4 \times \sin 90° \hat{z}$

+ $\vec{\alpha} = +1.8$ rad/s$^2 \hat{z}$

$\Rightarrow \quad I = \dfrac{\tau_{net}}{\alpha} = \dfrac{(7)(0.4)(\sin 90°)}{1.8} = 1.6\,(\mathrm{Kg.m^2})$

7.49 Modelling arm as 75-cm long uniform cylinder, its center of gravity is its geometrical center; should be

$\dfrac{75}{2} = 37.5$ (cm) from the pivot point (shoulder)



When he raising both ~~hands~~ arms,
from hanging down to straight up,
the height of center of gravity of the arm
should ~~be~~ change by $2 \times (37.5) = 75$ (cm)

or $Y_{arm\,up} - Y_{arm\,down} = 75$ cm $= 0.75$ (m)

We will use the formula

$$Y_{cg} = \frac{Y_1 \cdot m_1 + 2 Y_{arm} \cdot m_{arm}}{m_{body}}, \quad m_{body} = 70\,\mathrm{kg}$$

$m_{body} = m_1 + 2m_{arm}$

with m_1 is body without two arms (!!)

y - coordinator of m_1 doesn't change since he just moves his arms.

So, the changing of his center of gravity is.

$$\Delta(y_{cg}) = Y_{cg\,(up)} - Y_{cg\,(down)}.$$

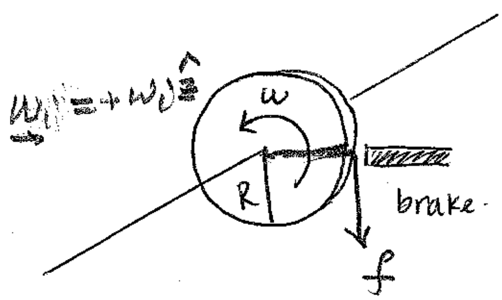
$$= \frac{Y_1 . m_1 + 2 . Y_{arm-up} . m_{arm}}{m_{body}} - \frac{Y_1 . m_1 + 2 Y_{arm-down} . m_{arm}}{m_{body}}$$

$$= \frac{2 m_{arm}}{m_{body}} (Y_{arm-up} - Y_{arm-down})$$

$$= \frac{2 \times (3.5)}{70} . (0.75) = 0.075 \text{ (m)} = 7.5 \text{ cm}$$

He raised his center of gravity by 7.5 cm.

7.64 $\vec{\omega}_f = (\omega_i + \alpha t)\hat{z}$ and $I \vec{\alpha} = \sum \vec{\tau}$ [compare $M\vec{a} = \sum \vec{F}_i$]



To stop it
f must be along $-\hat{y}$

Mass of Disk
M = 2 kg
Dia
2r = 30 cm

We will use the equation

$$\tau = I . \alpha \quad \text{or} \quad \alpha = \frac{\tau}{I}$$

where:

+ the torque τ is due to friction force.

$$\vec{\tau} = -F . R . \sin(+90^\circ)\hat{z}; \quad R = 30\text{cm} = 0.3 \text{ m}$$

+ I is ~~inertial~~ moment of inertia of disk, about axis through center
in this case, it is a rigid body.

$$I = \frac{1}{2} m . R^2$$

+ $\vec{\alpha}$ is angular acceleration and must be along $-\hat{z}$

To calculate α, we use formular in Table 7.1.

$$\vec{\Delta\omega} = \vec{\alpha} \cdot \Delta t \quad ; \quad \vec{\Delta\omega} = \vec{\omega_f} - \vec{\omega_i}$$
$$\Delta t = 3.0 \text{ s.}$$

After 3.0 second, the disk stops $\Rightarrow$ $\omega_f = 0$

$$\Rightarrow \vec{\Delta\omega} = -\vec{\omega_i} = -300\,rpm\,\hat{z} = -300 \cdot \left(\frac{2\pi}{60}\right)\hat{z} = -10\pi \text{ rad/s}\,\hat{z}$$

So, $$\vec{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{-10\pi}{3} \cdot (\text{rad/s}^2)\,\hat{z}$$

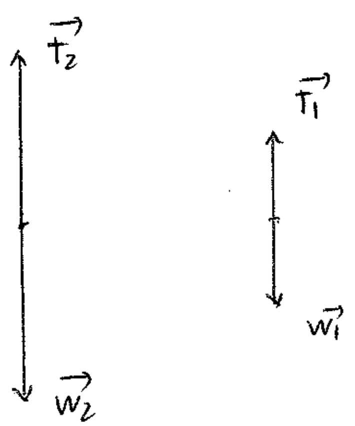
$\Rightarrow$ We have $\vec{\tau} = I \cdot \vec{\alpha}$

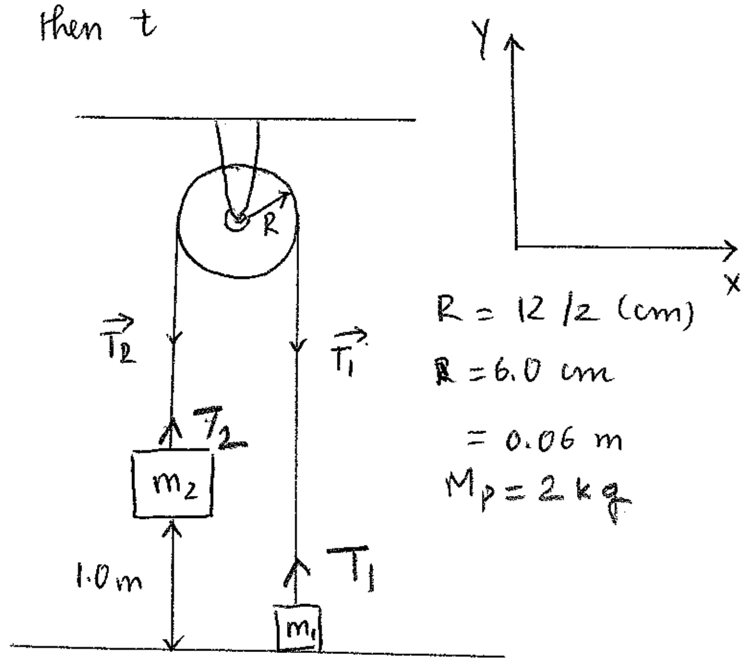
$$\Rightarrow f \cdot R \cdot \sin(-90^\circ) = \frac{1}{2} m R^2 \cdot \alpha.$$

$$\Rightarrow \vec{f} = -\frac{1}{2}(mR) \cdot \alpha\hat{y} = -\frac{1}{2}\left(2.0 \times \frac{0.3}{2}\right) \cdot \left(-\frac{10\pi}{3}\right)\hat{y}$$
$$\approx 1.6 \text{ N} \cdot \hat{y}$$

$\Rightarrow$ friction force has magnitude 1.6 N.

7.65 We find acceleration first and then t





R = 12 / 2 (cm)
R = 6.0 cm
= 0.06 m
$M_p = 2$ kg

Note: The pulley is a rigid body, (it has mass) ~~and~~ and friction at the axle. So, tensions ~~on the string~~ on the both sides not the same.

We will use the Newton's 2nd law for m_1, m_2 and the pulley.

For m_1 : $\quad T_1 - W_1 = m_1 a_1$

For m_2 : $\quad -W_2 + T_2 = +m_2 a_2$

For pulley : $\quad T_2 R - T_1 R - \tau_{friction} = I . \alpha$

where : $\quad W_1 = m_1 . g \; ; \; W_2 = m_2 . g$

$\tau_{friction} = 0.5 \; (N.m)$

$I = \frac{1}{2} m_p . R^2$,

Because string is massless and there is no slip. $\Rightarrow$ $\quad a_1 = -a_2 = a$

$\alpha = \frac{a}{R}$

$\Rightarrow \quad T_1 - m_1 g = m_1 a \quad (1)$

$-m_2 g + T_2 = -m_2 a$ or $m_2 g - T_2 = m_2 a \quad (2)$

$(T_2 - T_1) R - \tau_{friction} = \frac{1}{2} m_p . R^2 . \frac{a}{R}$

or $\quad T_2 - T_1 - \frac{\tau_{friction}}{R} = \frac{1}{2} m_p . a \quad (3)$

Adding three equation (1), (2) & (3) together, we have

$$T_1 - m_1 g + m_2 g - T_2 + T_2 - T_1 - \frac{\tau_{friction}}{R} = m_1 a + m_2 a + \frac{1}{2} m_p . a$$

$$\Rightarrow \quad (m_2 - m_1) g - \frac{\tau_{friction}}{R} = (m_1 + m_2 + \frac{1}{2} m_p) a .$$

$$a = \frac{(m_2 - m_1) g - \frac{\tau_{friction}}{R}}{m_1 + m_2 + \frac{1}{2} m_p} = \frac{(4.0 - 2.0) \times 9.8 - \frac{0.5}{0.06}}{4.0 + 2.0 + \frac{1}{2} . (2.0)}$$

$$= 1.61 \; m/s^2 .$$

√ Use equation from chapter II to calculate time

$y_f = y_i + v_i t + \frac{1}{2} a_2 t^2$ where $\quad y_f = 0$

$y_i = 1.0 \; m$

$v_i = 0$

$\vec{a}_2 = -\vec{a} = -1.61 \; m/s^2 \hat{y}$

$$\Rightarrow \quad 0 = 1.0 + 0 + \frac{1}{2}(-1.61)t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2 \times (1.0)}{1.61}} = 1.1 \text{ (s)}.$$

It takes 1.1 second for 4.0 kg block reaching the floor!