SOLUTIONS 4: FORMULAE (L, T, M)

NEWTON WITH INERTIAL SYSTEMS/OBSERVERS
SYSTEM/OBSERVER CAN MOVE WITH ANY VELOCITY BUT CANNOT HAVE AN ACCELERATION.

NEWTON'S LAWS: (POINT PARTICLES)
FIRST: DEFINES INERTIAL VELOCITY CANNOT CHANGE SPONTANEOUSLY
SECOND: INTRODUCES MASS (M)
IF MASS HAS ACCELERATION WE MUST PROVIDE A FORCE AT THAT POINT AT THAT TIME GIVEN BY:

\[ \sum F = M \alpha \]

Case I: \( \alpha = 0 \), MASS IN EQUILIBRIUM (\( \equiv m \))
VECTOR SUM OF ALL FORCES ACTING ON IT MUST BE ZERO: \( \sum F = 0 \)
FOR TWO DIMENSIONS \( \sum F_x = 0, \sum F_y = 0 \).

Case II: \( \alpha \neq 0 \),
\[ Mg \rightarrow \sum F = F_{net} \]
AT THAT POINT AT THAT TIME (FREE BODY DIAGRAM)

THIRD: TWO OBJECTS INTERACT, FORCES FORM ACTION-REACTION PAIR

\[ F_{12} + F_{21} = 0 \]

\[ F_{12} \]
\[ F_{21} \]

FORCES:

i) WEIGHT: \( \mathbf{W} = -Mg \hat{y} \text{ or } -Mg \hat{j} \)

ii) NORMAL FORCE OR CONTACT FORCE: ALWAYS PERPENDICULAR TO SOLID SURFACE — \( \theta \)
(iii) TENSION IN STRING: SAME MAGNITUDE EVERYWHERE but at ends directed toward one middle, \( T = F \). At any point within the string, directed toward the ends.

(iv) Spring force: Spring sets up a force which opposes change in its length. For small changes:

\[
F_{sp} = -k \Delta x
\]

(v) Friction comes into play if one solid surface slides past another. Two regimes:

Static: applied force less than \( \mu_s \) maximum static friction force:

\[
f_s < \mu_s n
\]

Kinetic when \( F_{app} > \mu_s n \):

\[
f_k = \mu_k n
\]

Note. Different symbol for friction. Frictional force CANNOT initiate motion. It always opposes motion.
Problems

Week 4: Problems 4-1, 13, 18, 24, 25, 56, 66, 67
5 - 4, 6, 13, 16

4-1

a) For this case, we note that the second pickup happens after the first. Hence, if the body continues forward while the head appears to move backwards, since the body is attached to the car, by the principle of inertia, the body is accelerated forward while the head is not; so this is a rear-end collision.

b) In the second case, the head continues moving forward whereas the body stops, so this is a head-on collision.

4-13

2) We assume that the forces and accelerations are all along $+\hat{x}$.

We need to find the force per rubber band.

For object 2:
$$F_{\text{rubber}} = m_2 a \hat{x} = 0.2 \text{kg} \cdot 5 \text{m/s}^2 \hat{x} = 1.0 \text{N} \hat{x}$$

For five rubber bands:
$$F = 1.0 \text{N} \hat{x}$$

so the force of one band is
$$F_{\text{rubber}} = 0.2 \text{N} \hat{x}$$

We see that object 1 accelerates $5 \text{m/s}^2 \hat{x}$ with 2 rubber bands.

Hence:
$$F_{\text{rubber}} = m_1 a \hat{x} = m_1 \cdot 5 \text{m/s}^2 \Rightarrow 0.4 \text{N} \hat{x} = m_1 \cdot 5 \text{m/s}^2 \hat{x}$$

$$\Rightarrow m_1 = \frac{0.4 \text{N}}{5 \text{m/s}^2} = 0.08 \text{kg}$$

Similarly for object 3, it is $2 \text{m/s}^2 \hat{x}$ with 5 rubber bands:
$$F_{\text{rubber}} = m_3 a \hat{x} = 1.0 \text{N} \hat{x} = m_3 \cdot 2 \text{m/s}^2 \hat{x}$$

$$\Rightarrow m_3 = \frac{1.0 \text{N}}{8 \text{m/s}^2} = 0.125 \text{kg}$$
Problem 18

\[ F = m \cdot a \]

without the child

\[ F = M_{\text{empty}} \cdot a \quad \Rightarrow \quad F = M_{\text{empty}} \cdot (1.4 \text{m/s}^2) \cdot x \quad (1) \]

If the child jumps in the cart, let her mass be 3 times the mass of the cart,

then \[ M_{\text{full}} = M_{\text{empty}} + M_{\text{child}} = M_{\text{empty}} + 3M_{\text{empty}} = 4M_{\text{empty}} \]

The force of the man does not change. Hence

\[ F = 4M_{\text{empty}} \cdot a \cdot x \quad \text{where } a \text{ is the new acceleration} \quad (2) \]

Hence dividing equations 1 and 2

\[ \frac{F}{4M_{\text{empty}} \cdot a} = 1.4 \text{m/s}^2 \]

\[ \Rightarrow \quad a = \frac{1.4 \text{m/s}^2}{4} = 0.35 \text{m/s}^2 \]

Problem 24

\[ F = m \cdot a \quad \text{or} \quad \frac{F}{m} = a \]

By Newton's second law,

\[ F = 2.5 \times 10^{-2} \text{ N} \cdot x \quad m = 9.1 \times 10^{-3} \text{ kg} \]

so

\[ a = \frac{F}{m} = \frac{2.5 \times 10^{-2} \text{ N} \cdot x}{9.1 \times 10^{-3} \text{ kg}} = 2.7 \times 10^{-2} \text{ m/s}^2 \]
Problem 25

\[ \vec{F} \rightarrow \text{Puck} \]
\[ \vec{F}_{\text{net}} \rightarrow \text{Boot} \]

\[ m_{\text{boat}} = 3.0 \times 10^{-4} \text{ kg} \]
\[ F_{\text{net}} = 5 \times 10^{-2} \text{ N} \]

Hence \[ \vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{5 \times 10^{-2} \text{ N}}{3 \times 10^{-4} \text{ kg}} = 0.0167 \text{ m/s}^2 \approx 0.02 \text{ m/s}^2 \]

Problem 56

There are four forces acting on the skier. He is going down the hill. Gravity pulls him down. The hill pushes upwards on the skier at an angle to the \( \hat{x} \) axis. The wind pushes in the \( -\hat{x} \) direction, friction acts backwards up at an angle of 20° as shown in the picture.

\( \hat{n} \) is perpendicular to incline
Problem 66

During the pitch, we assume the force to be constant.

a) The free body diagram for the ball is: $\vec{n}$ and $-Mg \hat{y}$ play no role.

b) To find acceleration, we use $(V_f)^2 = V_i^2 + 2ax$ for one dimensional motion.

$V_i = 0 \Rightarrow V_f = 46 m/s \hat{x}$, $\Delta x = 1m$

so $V_f^2 = 2ax \Rightarrow a = \frac{V_f^2}{2x} = \frac{(46m/s)^2}{2 \times 1m} = 1058 m/s^2$ \(x\)

But $F = ma \Rightarrow F = 0.145 kg \times 1058 m/s^2 \times 1 = 150 N \hat{x}$

c) If a pitcher has a mass of 75 kg, then $W = Mg = 75 kg \times 9.8 m/s^2 = 750 N$.

So \(\frac{150 N}{750 N} = \frac{1}{5}\) so the force on the ball is $\frac{1}{5}$ the pitcher's weight.
Problem 6)

\[ \theta - \frac{2}{3} \]

\[ \begin{align*}
\text{a) The legs push up while gravity pulls down, so the force diagram is:} \\
& \begin{cases}
\text{F}\text{net} \\
\text{a} \\
\text{F}
\end{cases}
\end{align*} \]

\[ b) \text{ Since the bug is accelerating up, the force of the ground on the bug, which is the only upward force, must be greater than the weight in order for there to be a net upward acceleration.} \]

\[ \text{CHAPTER 5} \]

Problem 4)

The weight of the beam is
\[ W = mg = 1000 \text{kg} \cdot 9.8 \text{m/s}^2 \cdot g = -9800 \text{ N} \]

In order to hold the weight of the slab, each rope must hold the weight. So the tensions in the ropes must be less than the maximum tension in order to hold the slab.

For \( \Theta \):
\[ 2T \cdot \cos(30) \cdot g = 9800 \text{ N} \cdot g = 0 \]

\[ T = \frac{9800 \text{ N}}{2 \cdot \cos(30)} \Rightarrow T = \frac{4900 \text{ N}}{\cos(30)} \]

\[ = \frac{4900 \text{ N}}{\cos(\pi/6)} = 5670 \text{ N} \]

This tension is greater than the \( \pi/6 \) tension each string can support, so they will break.
The point where the three forces meet is in \( \Xi M \).

The sum of forces in \( x \) and \( y \) directions must be zero.

Let \( \mathbf{F}_{\text{sum}} = F_{\text{mx}} \hat{x} + F_{\text{my}} \hat{y} \)

Hence

\[
F_{\text{mx}} \hat{x} - F_{\text{my}} \hat{y} - F_p \sin(42^\circ) \hat{y} = 0
\]

\[
F_p \cos(42^\circ) \hat{x} - F_{\text{mx}} \hat{x} = 0
\]

For the first equation

\[
F_{\text{my}} = F_q - F_p \sin(42^\circ) = 60N - 60N \sin(42^\circ) = 20N
\]

So \( F_{\text{my}} = -20N \hat{y} \)

For the second equation \( F_{\text{mx}} = 60N \cos(42^\circ) = 45N \)

So \( F_{\text{mx}} = -45N \hat{x} \)

\[
\mathbf{F}_f = -45N \hat{x} - 20N \hat{y}
\]

Magnitude of \( \mathbf{F}_f = \sqrt{45^2 + 20^2} = 49N \).

\[
\tan \Theta = -\frac{20}{-45} = 0.44
\]

\( \Theta = 24^\circ \) in Third Quadrant, i.e. 204° from x-axis
13) 

[Image of a force diagram with a box and forces labeled]

a) If the box is at rest, $\ddot{a} = 0$ so $F = F_e = ma = m \cdot 0$

so $\ddot{F} = 0$

b) if $\vec{V} = \vec{m}/s \hat{x}$ then $a = 0 \text{ m/s}^2 \hat{x}$, so $\ddot{F} = ma = 0$

c) If $\ddot{a} = 5 \text{ m/s}^2 \hat{x}$ then $\ddot{F} = ma$ \implies $\ddot{F} = 50 \text{ kg} \cdot 5 \text{ m/s}^2 \hat{x} = 250 \text{ N} \hat{x}$ = $\ddot{T}$

16) Weight is given by

$\vec{\omega} = -mg \hat{y}$, where $g$ is acceleration due to gravity.

If the man weighs 800N on Earth, then,

$\vec{\omega} = -800 \text{ N} \hat{y}$

$\text{His mass is } m = \frac{800 \text{ N}}{-9.8 \text{ m/s}^2} = 81.6 \text{ kg}$

Mass does not change!

On Mars, $\ddot{g} = -3.76 \text{ m/s}^2 \hat{y}$

\[ \text{so } \vec{\omega}_{\text{Mars}} = -(81.6 \text{ kg}) \cdot (-3.76 \text{ m/s}^2 \hat{y}) \]

\[ = -310 \text{ N} \hat{y} \]