SOLUTIONS 4: FORMULAE (L, T, M)

We WORK WITH INERTIAL SYSTEMS/OBSERVERS

SYSTEM/OBSERVER CAN MOVE WITH ANY VELOCITY BUT CANNOT HAVE AN ACCELERATION.

NEWTON'S LAWS: (POINT PARTICLES)

FIRST: DEFINES INERTIA: VELOCITY CANNOT CHANGE SPONTANEOUSLY

SECOND: INTRODUCES MASS (M)

IF MASS HAS ACCELERATION WE MUST PROVIDE A FORCE AT THAT POINT AT THAT TIME GIVEN BY:

$$M\vec{a} = \vec{F}$$

CORR I If $\vec{a} = 0$, MASS in EQUILIBRIUM ($\equiv$m)

VECTOR SUM OF ALL FORCES ACTING ON IT MUST BE ZERO: $\Sigma \vec{F}_i \equiv 0$

For Two Dimensions $\Sigma F_{ix} \equiv 0$; $\Sigma F_{iy} = 0$.

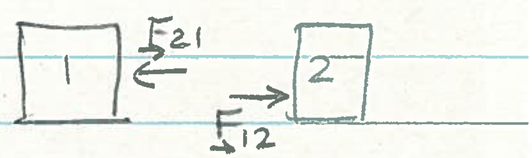
CORR II IF $\vec{a} \neq 0$,

$$M\vec{a} = \Sigma \vec{F}_i = \vec{F}_{net}$$

At that point at that time (FREE BODY DIAGRAM)

THIRD: TWO OBJECTS INTERACT, FORCES FORM ACTION-REACTION PAIR

$$\vec{F}_{12} + \vec{F}_{21} \equiv 0$$



FORCES i) Weight $\vec{W} = -Mg\hat{i}$ or $-Mg\hat{y}$

ii) Normal force or Contact force: ALWAYS PERPENDICULAR TO SOLID SURFACE — $\vec{n}$

(iii) TENSION IN STRING: SAME MAGNITUDE EVERYWHERE but at ends directed toward the middle,

$T = F$!

at any point within the string directed toward the ends

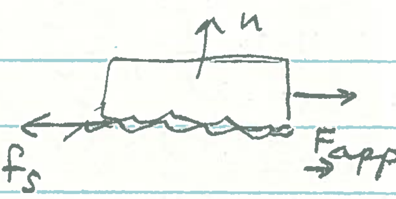
(iv) spring force: spring sets up a force which opposes change in its length. For small changes

$$\vec{F}_{sp} = -k\,\Delta x\,\hat{x}$$

(v) FRICTION COMES INTO PLAY IF ONE SOLID SURFACE SLIDES PAST ANOTHER. TWO regimes

static applied force less than the maximum static friction force

$$f_s \le \mu_s n$$



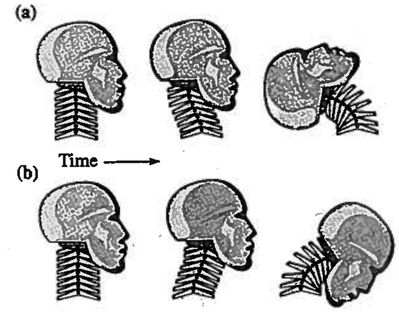
kinetic when $F_{app} > \mu_s n$

$$f_k = \mu_k n$$

Note: different symbol for friction. Frictional force CANNOT initiate motion. It always opposes motion

PROBLEMS

Week 4: Problems 4-1, 13, 18, 24, 25, 56, 66, 67
5 - 4, 6, 13, 16



P4.1

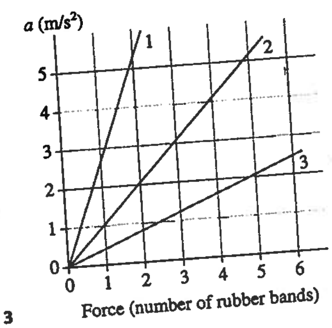
4-1

a) For this case, we note that the second picture happens after the first. Hence, if the body continues forward while the head appears to move backwards, since the body is attached to the car, by the principle of inertia the body is accelerated forward while the head is not; so this is a rear end collision.

b) In the second case the head continues moving forward whereas the body stops, so this is a head on collision

4-13

We assume that the forces and accelerations are all along $+\hat{x}$



E P4.13

We need to find the force per rubber band.

For object 2 $\quad F_{5bands}\hat{x} = m_2 a_2 \hat{x} = 0.2kg \cdot 5m/s^2 \hat{x} = 1.0N\hat{x}$

For Five Bands $\Rightarrow F\hat{x} = 1.0N\hat{x}$ so the force of one band is $\vec{F}_{1band} = 0.2N\hat{x}$

We see that object 1 accelerates about $5m/s^2\hat{x}$ with 2 rubber bands.

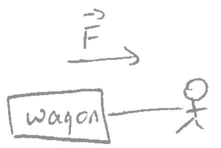
Hence $\quad F_{2bands}\hat{x} = m_1 \cdot a_1\hat{x} = \hat{x}\, m_1 \cdot 5m/s^2 \Rightarrow 0.4N\hat{x} = m_1 \cdot 5m/s^2\hat{x}$

$$\Rightarrow m_1 = \frac{0.4N}{5m/s^2} = 0.08kg$$

Similarly for object 3 acc is $2m/s^2$ with 5 bands
$F_{5bands}\hat{x} = m_3 a_3 \hat{x} \Rightarrow 1.0N\hat{x} = m_3 \cdot 2m/s^2\hat{x} \Rightarrow m_3 = \dfrac{1.0N}{2m/s^2} = 0.5kg$

Problem 18

$\hat{x}$



without the child

$$\vec{F} = m_{empty}\,\vec{a} \quad \Rightarrow \quad \vec{F} = m_{empty} \cdot (1.4 m/s^2)\,\hat{x} \qquad (1)$$

If the child jumps in the cart, + if her mass is 3 times the mass of the cart,

then $m_{full} = m_{empty} + m_{child} = m_{empty} + 3m_{empty} = 4m_{empty}$

The force of the man does not change. Hence

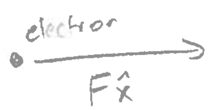
$$(2) \qquad F\hat{x} = 4\,m_{empty} \cdot a\,\hat{x}$$ where a is the new acceleration

Hence dividing equations 1 + 2 $\quad \dfrac{\cancel{F}}{\cancel{F}} = \dfrac{\cancel{m_{empty}} \cdot 1.4 m/s^2}{4\,\cancel{m_{empty}} \cdot a} \Rightarrow 4a = 1.4 m/s^2$

$$\Rightarrow a = \frac{1.4 m/s^2}{4} = 0.35 m/s^2$$

Problem 24)

$\hat{x}$



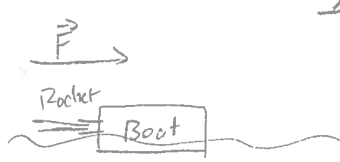
By Newton's second law, $\vec{F} = m\vec{a}$ or $\dfrac{\vec{F}}{m} = \vec{a}$

$\vec{F} = 2.5 \times 10^{-2} N\,\hat{x} \qquad m = 9.1 \times 10^{-31} kg$

so $$\vec{a} = \frac{\vec{F}}{m_e} = \frac{2.5 \times 10^{-2} N\,\hat{x}}{9.1 \times 10^{-31} kg} = 2.7 \times 10^{28} \frac{m}{s^2}\,\hat{x}$$

Problem 25

$\hat{x} \rightarrow$

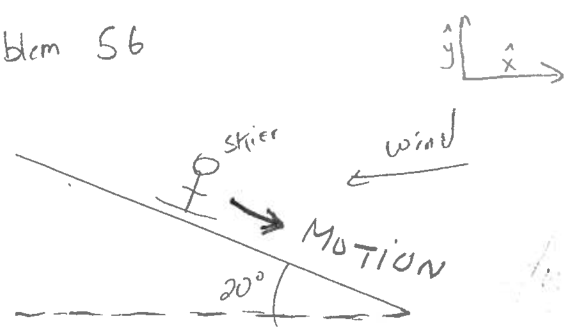


$m_{boat} = 3.0 \times 10^8\,kg$

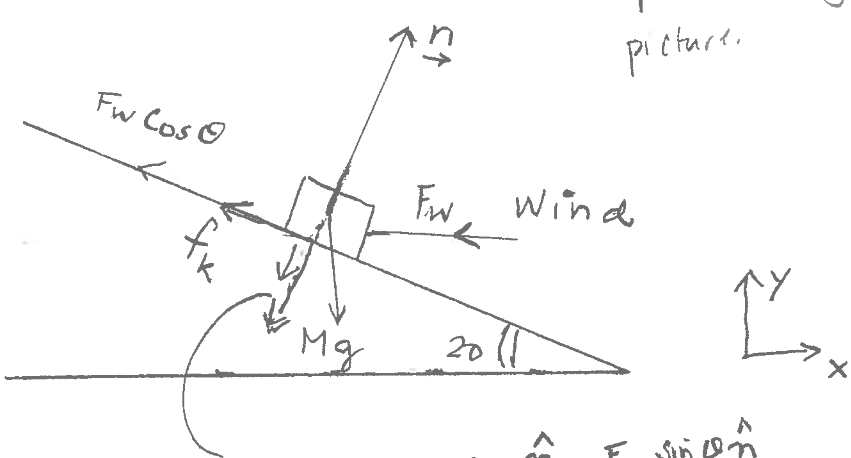
$\vec{F}_{rocket} = 5 \times 10^6\,N\,\hat{x}$

Hence $\vec{a} = \frac{\vec{F}}{m} = \frac{5 \times 10^6\,N\,\hat{x}}{3 \times 10^8\,kg} = 0.0167\,m/s^2\,\hat{x} \approx 0.02\,m/s^2\,\hat{x}$

Problem 56



There are four forces acting on the skier. He is going down the hill. Gravity pulls him down. The hill pushes upwards on the skier at an angle to the $\hat{x}$ axis. The wind pushes in the $-\hat{x}$ direction, + friction acts backwards + up at an angle of 20° as shown in the picture.

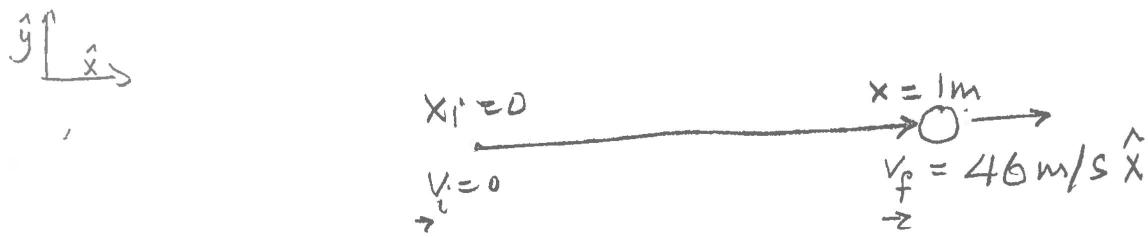


$\hat{n}$ is perpendicular to incline

$\vec{n} = -Mg \cos 20\,\hat{n} - F_W \sin\Theta\,\hat{n}$

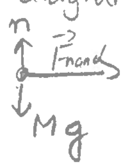
$\vec{f_k} = -\mu_k n$ up the incline

Problem 66



During the pitch, we assume the force to be constant

a) The free body diagram for the ball is:



$\vec{n}$ and $-Mg\,\hat{y}$ play no role

b) To find acceleration, we use $(v_f)^2 = v_i^2 + 2a(x - x_i)$ for one dimensional motion.

$\vec{v}_i = 0$ + $\vec{v}_f = 46 m/s\ \hat{x}$ $\qquad \Delta x = 1m$

so $\quad v_f^2 = 2a\Delta x \implies a = \dfrac{v_f^2}{2\Delta x} \quad \vec{a} = \dfrac{(46 m/s)^2}{2 m}\ \hat{x}$

$= 1058\ m/s^2\ \hat{x}$

But $\vec{F} = m\vec{a} \implies \vec{F} = 0.145kg \cdot 1058\ m/s^2\ \hat{x} = 150N\ \hat{x}$

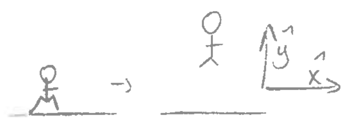
c) If a pitcher has a mass of 75 kg then $w = |m\vec{g}| = 75kg \cdot 9.8\ m/s^2$

$\approx 750\ N$

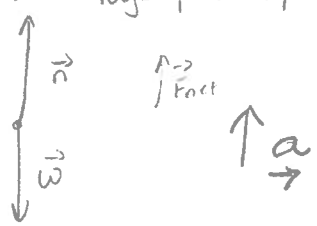
So $\dfrac{150\ N}{750 N} = \dfrac{1}{5}$, so the force on the ball is $\frac{1}{5}$ the pitcher's weight

Problem

67)



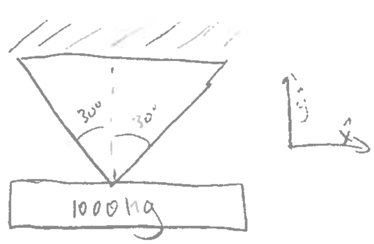
a) The legs push up while gravity pulls down. So the force diagram is:



b) Since the bug is accelerating up, the force of the ground on the bug, which is the only upward force, must be greater than the weight in order for there to be a net upward force. because it has an upward acceleration

CHAPTER 5

Problem 4)

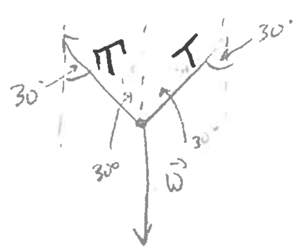


The weight of the beam is

$$\vec{w} = m\vec{g} = 1000kg \cdot -9.8\,m/s^2\,\hat{y}$$

$$= -9800\,N\hat{y}$$

In order to hold the weight of the slab, each rope must hold half this weight. So the tensions in the ~~ropes~~ must be less than the maximum tension in order to hold the slab



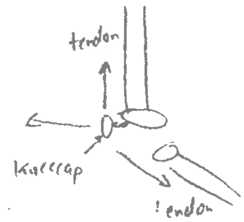
For Σm: $2T\cos(30)\hat{y} - 9800N\hat{y} = 0$

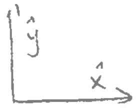
$$\therefore T = \frac{9800N}{2\cos(30)} \Rightarrow T = \frac{4900N}{\cos(30^\circ)}$$

$$= \frac{4900N}{\cos(\pi/6)} = 5670N$$

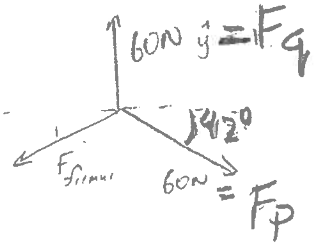
This tension is greater than the ... tension each string can support, so they will break

6)





The point where the three forces meet is in $\equiv$m.



The sum of forces in $\hat{x}$ + $\hat{y}$ directions must be zero.

Let $\vec{F}_{femur} = F_{fx}\hat{x} + F_{fy}\hat{y}$

Hence $F_q\hat{y} - F_{fy}\hat{y} - F_p\sin(42°)\hat{y} = 0$ $\Sigma F_{iy} = 0$

$F_p\cos(42°)\hat{x} - F_f\hat{x} = 0$ $\Sigma F_{ix} = 0$

For the first equation

$$F_{fy} = F_q - F_p\sin(42°) = 60N - 60N\sin(42°)$$
$$= 20N$$

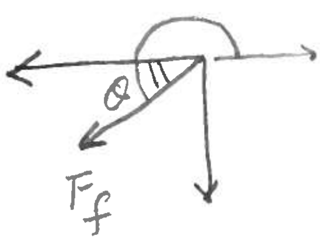
So $\vec{F}_{fy} = -20N\hat{y}$

For the second equation $F_f = 60N\cos(42°) = 45N$

so $\vec{F}_f = -45N\hat{x}$

$$\vec{F}_f = -45N\hat{x} - 20N\hat{y}$$

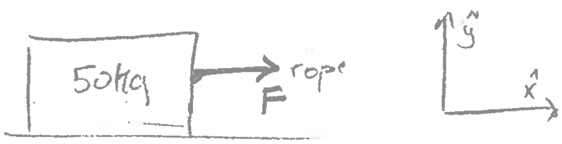
Magnitude of $\vec{F}_f = \sqrt{45^2+20^2}\ N = 49N.$



$$\tan\theta = \frac{-20}{-45} = 0.44$$

θ = 24° in Third Quadrant
or 204° from x-axis

13)



a) If the box is at rest $\vec{a}=0$ so $\vec{F}=\vec{T}=m\vec{a}=m\cdot\vec{0}$

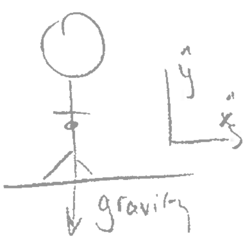
so $\vec{T}=0$

b) if $V = m/s\ \hat{x}$ then $a = 0\ m/s^2\ \hat{x}$, so $\vec{T}=m\vec{a}=0$

c) If $\vec{a}=5m/s^2\ \hat{x}$ then $\vec{F}=m\vec{a}$ $\Rightarrow$ $\vec{F}=50kg\cdot 5m/s^2\ \hat{x}=$

$250\ N\hat{x}=\vec{T}$

16) Weight is given by



$\vec{w}=-Mg\hat{y}$ where g is acceleration due to gravity

if the man weighs 800N on earth then

$\vec{W}= -800N\hat{y} = -m\ 9.8\ m/s^2\hat{y}$

His mass is $m = \dfrac{-800N}{-9.8m/s^2} = 81.6kg$

Mass does not change!

On Mars, $\vec{g} = -3.76\ m/s^2\ \hat{y}$

so $\vec{W}_{Mars} = (81.6\ kg)(-3.76\ m/s^2\ \hat{y})$

$= -310N\hat{y}$