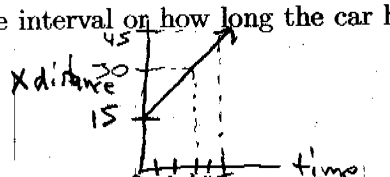
Chapter 2

5) Velocity is the relationship between an object's displacement and time. Note that the word dis-place-ment simply means shift in location. What we physicists mean when we use displacement is how much an object shifted in location and in what direction.

The displacement of our car is $x_f - x_i\hat{x} = 30 - 15\hat{x} = 15\hat{x}$. Where I'm assuming the book is being straight with me and gave me 30m and 15m along the x axis (for example) as opposed to 30m north and 15m east or some such (you can see how that would mess things up?

The time interval that our car spent traveling that distance is 3 seconds.

Now given this information I can construct a ratio that instantly tells me how far the car has been displaced given a time interval or how long the car has traveled given a displacement.



$velocity = \vec{v} = \frac{15m}{3s}\hat{x} = 5\frac{m}{s}\hat{x}$

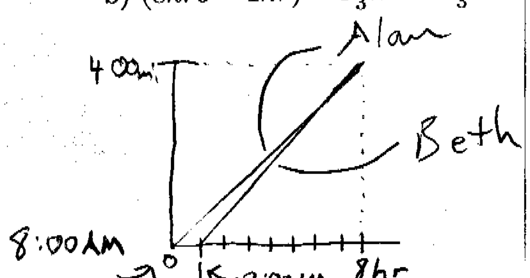
a) $5\frac{m}{s} \times 1.5s\hat{x} = 7.5m\hat{x}$. Are we done? Remember that the car started at 15m at t=0. So the final answer is $7.5m + 15m\hat{x} = 22.5m\hat{x} \approx 23m\hat{x}$

b) $5\frac{m}{s} \times 5.0s + 15m\hat{x} = 40m\hat{x}$

8) We can solve this problem in many ways but since both parts a and b require knowing the time elapsed while traveling why don't we calculate this immediately.

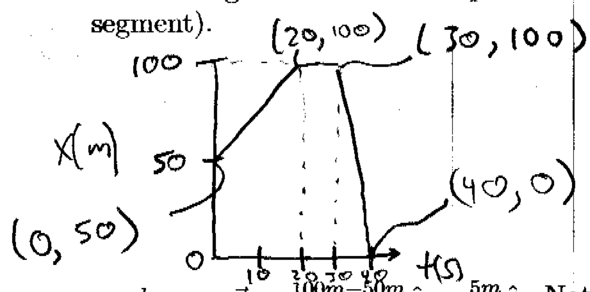
a) Alan: $t = 400mi \times \frac{1}{50\frac{mi}{hr}} = 8hrs$ Beth: $t = 400mi \times \frac{1}{60\frac{mi}{hr}} = 6\frac{2}{3}hrs$ Even though Alan has a one hour handicap it looks like Beth wins (barely)

b) $(8hrs - 1hr) - 6\frac{2}{3}hrs = \frac{1}{3}hrs.$



12) If you would recall question 5 the velocity is just a ratio (and a direction) where displacement is in the numerator and the time interval is in the denominator. Now as it happens the slope of a graph is the rise over run or change in the y axis over the change in the x axis. In this question the y axis is x (confusing I know) and the x axis is t. This just so happens to be the displacement over the time interval! To calculate the slope of

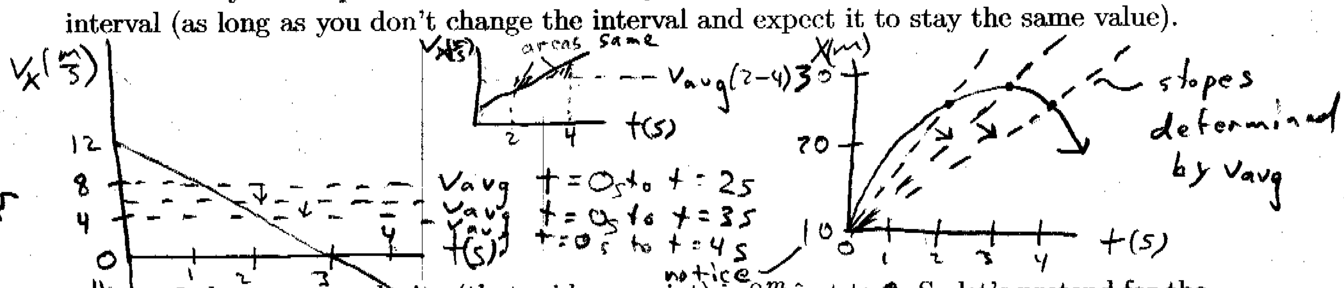
the first segment I subtract a point at t=20 from a point at t=0 (the endpoints of the line segment).



$slope = \vec{v} = \frac{100m-50m}{20s-0s}\hat{x} = \frac{5m}{2s}\hat{x}$. **Note that t=10s is in that time interval so this calculated velocity answers the first question.**

$slope = \vec{v} = \frac{0m-100m}{40s-30s}\hat{x} = -10\frac{m}{s}\hat{x}$. t=35s is in this time interval and velocity is constant so we have the velocity for the second part of the question.

14) There is a neat way to think about these problems. Find a point in a time interval in which the velocity is higher about half the time and lower the other half (try the midpoint : P). If the velocity is constantly changing then the distance you gain spending time above this point is exactly balanced out by the distance you lose spending time below this point. That means you can pretend that the velocity was at this middling value over the whole interval (as long as you don't change the interval and expect it to stay the same value).



At t=2 the average velocity (that midway point) is $8\frac{m}{s}\hat{x}$ ~~at t=2~~. So let's pretend for the purposes of calculating distance traveled at t=2 that v is constant at $8\frac{m}{s}$. $8\frac{m}{s} \times 2s = 16m$.

$16m + 10m\hat{x} = 26m\hat{x}$

By the way you can also use the formula $\vec{v_{average}} = \frac{v_f+v_i}{2}\hat{x} = \frac{4\frac{m}{s}+12\frac{m}{s}}{2}\hat{x} = 8\frac{m}{s}\hat{x}$. Multiplying this by 2s is equivalent to finding the area under the curve so don't worry that we're straying from the text or the class.

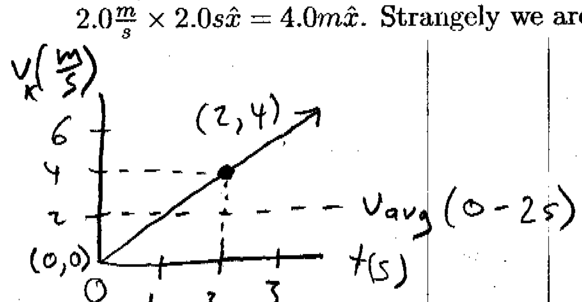
Car reverses direction at $t=3$

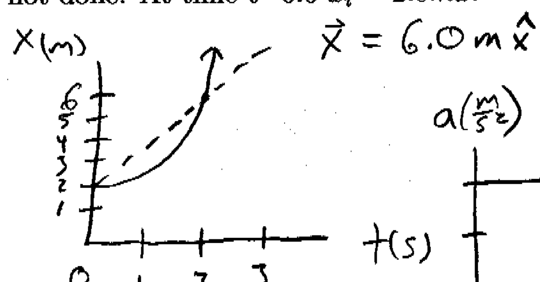
At 3s the average velocity is $6\frac{m}{s}\hat{x}$. The total distance is $6\frac{m}{s} \times 3s\hat{x} = 18m\hat{x}$. $18m + 10m\hat{x} = 28m\hat{x}$

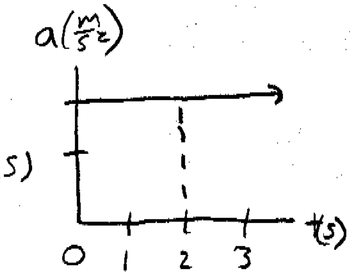
At 4s $\vec{v_{average}} = \frac{v_f+v_i}{2}\hat{x} = \frac{-4\frac{m}{s}+12\frac{m}{s}}{2}\hat{x} = 4\frac{m}{s}\hat{x}$. Distance is $4\frac{m}{s} \times 4s\hat{x} = 16m\hat{x}$

$16m + 10m\hat{x} = 26m\hat{x}$

17) At 2s the average velocity is $2.0\frac{m}{s}\hat{x}$. The total distance traveled in this time is $2.0\frac{m}{s} \times 2.0s\hat{x} = 4.0m\hat{x}$. Strangely we are not done. At time t=0.0 $\vec{x_i} = 2.0m\hat{x}$.



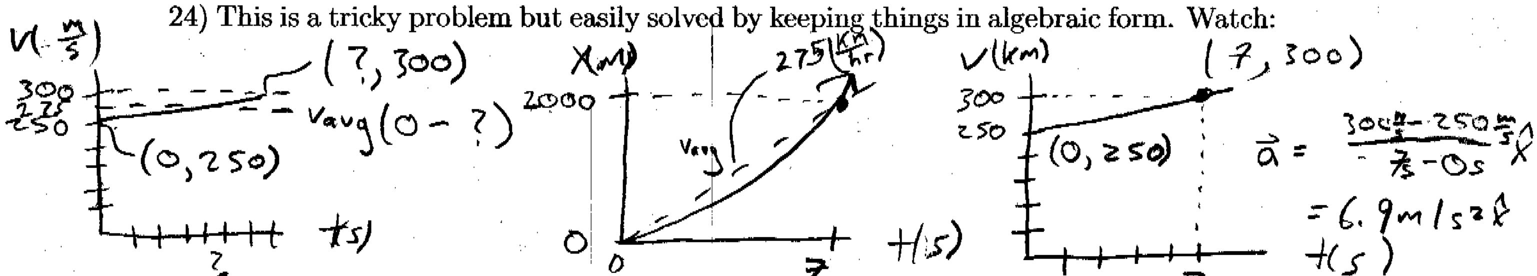




The velocity at t=2.0s is $4.0\frac{m}{s}\hat{x}$. Read it off the graph.

Loosely speaking acceleration is the velocity of velocity. Strictly speaking acceleration is the change of velocity over the time interval. In this graph the acceleration is the slope of the graph. $acceleration = \vec{a} = \frac{2.0\frac{m}{s} - 0.0\frac{m}{s}}{1.0s - 0.0s}\hat{x} = 2.0\frac{m}{s^2}\hat{x}$.

24) This is a tricky problem but easily solved by keeping things in algebraic form. Watch:



a) For free fall $v_f - v_i\hat{x} = acceleration \times \Delta t\hat{x} = -9.8\frac{m}{s^2}t\hat{x}$

But we don't have t! It could be anything! Se we need to use another equation that has t in it, solve for t, and forget about the variable.

$\vec{x} = v_{avg} \times t\hat{x} \rightarrow t = \frac{x}{v_{avg}}$.

Let's go back to the first equation:

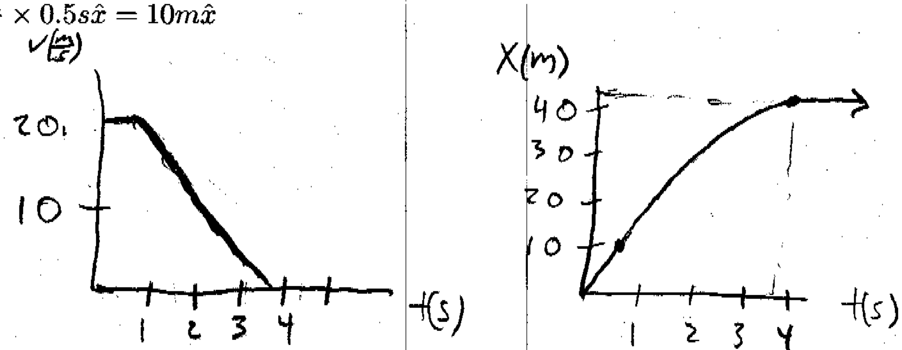
$v_f - v_i\hat{x} = a\frac{x}{v_{avg}}\hat{x} \rightarrow \vec{a} = (v_f - v_i)\frac{v_{avg}}{x}\hat{x} = (300m/s - 250m/s)\frac{300m/s+250m/s}{2\times 2000m}\hat{x} = 6.9\frac{m}{s^2}\hat{x}$

b) Is my answer reasonable? This is always a good question to ask yourself. Commercial aircraft definitely do not accelerate to the magnitude of gravity. So in that respect I believe that the answer is reasonable (at least to a factor of two).

26) Let's see what the stopping distance is. Using the equation derived in problem 24.

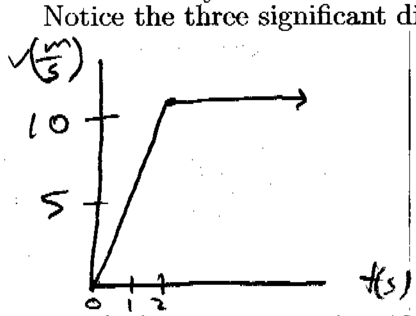
$\vec{a} = (v_f - v_i)\frac{v_{avg}}{x}\hat{X} \rightarrow \vec{x} = (v_f - v_i)\frac{v_{avg}}{a}\hat{X} = (0m/s - 20m/s)\frac{0m/s+20m/s}{2\times -6.0m/s^2}\hat{x} = 33m\hat{x}$

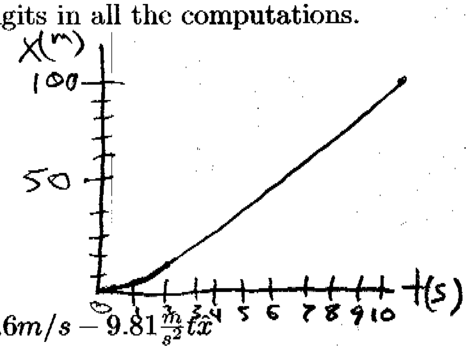
Now let's add the distance traveled due to the reaction time delay. $\vec{x} = \vec{v} \times t = 20\frac{m}{s} \times 0.5s\hat{x} = 10m\hat{x}$



Adding the two components together we get $33m + 10m\hat{x} = 43m\hat{x}$. Since this is less than her 50m leeway we can conclude that she stops in time.

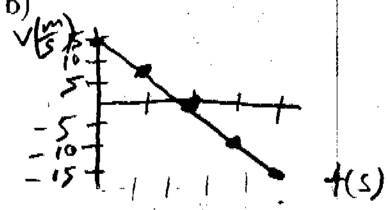
29) The average velocity for the first 2.14s is half $11.2m/s\hat{x}$ or $5.60m/s\hat{x}$. $5.60m/s \times 2.14s\hat{x} = 12.0m\hat{x}$. The runner has $100m - 12.0m\hat{x} = 88.0m\hat{x}$ to run. This requires $t = \frac{x}{v} = \frac{88m}{11.2\frac{m}{s}} = 7.86s$ for the remaining stretch. The total time is $2.14s + 7.86s = 10.0s$. Notice the three significant digits in all the computations.

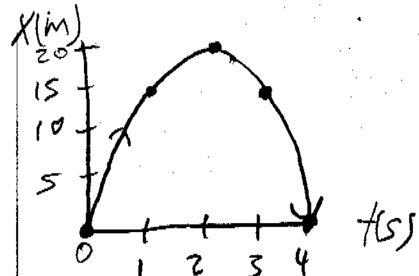




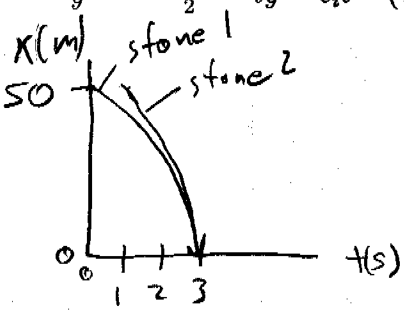
32) a) $v_f\hat{x} = v_i + at\hat{x} = 19.6m/s - 9.81\frac{m}{s^2}t\hat{x}$
$\vec{x} = v_{avg}t\hat{x}$
t=1.00s $\vec{v} = 9.79m/s\hat{y}$ $\vec{v_{avg}} = 14.7m/s\hat{y}$ $\vec{y} = 14.7m\hat{y}$
t=2.00s $\vec{v} = -0.02m/s\hat{y}$ $\vec{v_{avg}} = 9.79m/s\hat{y}$ $\vec{y} = 19.6m\hat{y}$
t=3.00s $\vec{v} = -9.83m/s\hat{y}$ $\vec{v_{avg}} = 4.91m/s\hat{y}$ $\vec{y} = 14.7m\hat{y}$
t=4.00s $\vec{v} = -19.6m/s\hat{y}$ $\vec{v_{avg}} = 0.00m/s\hat{y}$ $\vec{y} = 0.00m\hat{y}$

b)





35) a) $\vec{y} = v_{avg}t\hat{y} = \frac{v_f + v_i}{2}t\hat{y}$
We don't know $v_f\hat{y}$!
$v_f\hat{y} = v_i - 9.81\frac{m}{s^2}t\hat{y}$
$\vec{y} = \frac{2v_i - 9.81\frac{m}{s^2}t}{2}t\hat{y} = v_i t - (4.91\frac{m}{s^2})t^2\hat{y}$. This may be a familiar form to some of you.



$$t = \frac{-v_i \pm \sqrt{v_i^2 - 4(-4.91\frac{m}{s^2})(-x)}}{-9.81\frac{m}{s^2}} = \frac{2\frac{m}{s} - \sqrt{4\frac{m^2}{s^2} - 4(-4.91\frac{m}{s^2})50m}}{-9.81\frac{m}{s^2}} = 3s.$$

Note: all the variables in the above equation had negative initial values. I also selected the root in the quadratic equation that gave a positive time.

b) The second stone has only 2s to reach the water.

$\vec{x} = v_i t + \frac{1}{2}at^2\hat{y}$

$v_i\hat{y} = \frac{x + 4.91\frac{m}{s^2}t^2}{t}\hat{y} = \frac{-50m + 4.91\frac{m}{s^2}4s^2}{2s}\hat{y} = -15.2\frac{m}{s}\hat{y}$

c) $v_f\hat{y} = v_i + at\hat{y}$

Stone 1: $v_f\hat{y} = -2\frac{m}{s} - 9.81\frac{m}{s^2}3s\hat{y} = -31\frac{m}{s}\hat{y}$

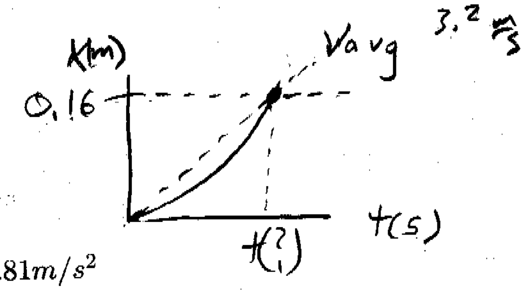
Stone 2: $v_f\hat{y} = -15.2\frac{m}{s} - 9.81\frac{m}{s^2}2s\hat{y} = -35\frac{m}{s}\hat{y}$

52) The bush baby only remains in free fall for $2.26m - 0.16m\hat{y} = 2.1m\hat{y}$.

$v_f - v_i\hat{y} = -9.81\frac{m}{s^2}\frac{y}{v_{avg}}\hat{y}$ from problem 24.

$\frac{1}{2}(v_f - vi)(v_f + v_i)\hat{y} = -9.81\frac{m}{s^2}y\hat{y} \to v_f^2 - v_i^2\hat{y} = 2(-9.81\frac{m}{s^2})y\hat{y} \to v_i\hat{y} = \sqrt{v_f^2 - 2(-9.81\frac{m}{s^2})y\hat{y}} = \sqrt{0 + 4.91\frac{m}{s^2}2.1m\hat{y}} = 6.42\frac{m}{s}\hat{y}$

Now we look at the initial jump stage.

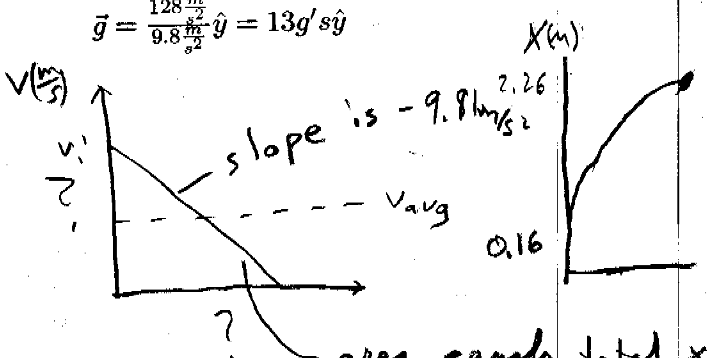


$\vec{x} = v_{avg}\cancel{t}\hat{y} \to 0.16m = \frac{0 + 6.42\frac{m}{s}}{2}t\hat{y} \to t = 0.0498s$

$v_i\hat{y} = at\hat{y} \to \vec{a} = \frac{v_i}{t}\hat{y} = \frac{6.42\frac{m}{s}}{0.0498s}\hat{y} = 129\frac{m}{s^2}\hat{y}$

The number of g's is then this number divided by $9.81m/s^2$

$\vec{g} = \frac{128\frac{m}{s^2}}{9.8\frac{m}{s^2}}\hat{y} = 13g's\hat{y}$



area equals total x to top

$2.1m\hat{x} = \frac{1}{2}v_i t\hat{x}$ $\quad v_i = 9.81\ m/s^2\ t$

$2.1m\ \hat{x} = \frac{1}{2}v_i^2\frac{1}{9.8m/s^2} \to v_i = \sqrt{(2.1m)(9.81m/s^2)(2)} = 6.4\ m/s$

56) $\vec{x} = v_i t + \frac{1}{2}(-9.81\frac{m}{s^2})t^2\hat{y}$

$$t = \frac{-v_i \pm \sqrt{v_i^2 + 4(4.91\frac{m}{s^2})(-x)}}{-9.81\frac{m}{s^2}} = \frac{-15\frac{m}{s} - \sqrt{(15\frac{m}{s})^2 + 4(4.91\frac{m}{s^2})2m}}{-9.81\frac{m}{s^2}} = 3.2s$$