KINEMATICS: EQUATIONS FOR MOTION WHEN ACCELERATION IS CONSTANT. — FREE FALL

INTRODUCTION

We use the coordinate system shown on the right. Consequently,

i) any vector along x can be written as

$$\underset{\rightarrow}{A} = +A\,\hat{x}$$

or

$$\underset{\rightarrow}{A} = -A\,\hat{x}$$

A is magnitude, $+\hat{x}$ vector points right

$-\hat{x}$ vector points left.

ii) Any vector can be written as

$$\underset{\rightarrow}{A} = A_x\,\hat{x} + A_y\,\hat{y}$$

A_x, A_y are components

$$A_x = A\cos(\underset{\rightarrow}{A}, \hat{x}), \quad A_y = A\cos(\underset{\rightarrow}{A}, \hat{y}).$$

ONE DIMENSIONAL MOTION ALONG X.

$$\underset{\rightarrow}{a} = a\,\hat{x} \qquad (1)$$

$$\underset{\rightarrow}{V} = (V_0 + at)\,\hat{x} \qquad (2)$$

$$\underset{\rightarrow}{X} = (X_0 + V_0 t + \tfrac{1}{2} a t^2)\,\hat{x} \qquad (3).$$

Here $V_0\,\hat{x}$ is velocity at $t=0$,

$X_0\,\hat{x}$ is position vector at $t=0$.

From Eq (2) $\quad t = \dfrac{V - V_0}{a}$

Substitute in Eq. (3)

$$x = x_0 + \frac{v_0(v - v_0)}{a} + \frac{1}{2} a \left(\frac{v - v_0}{a}\right)^2$$

On simplifying this gives

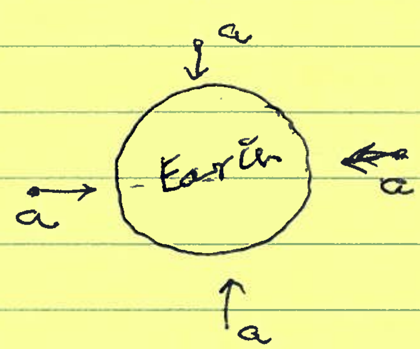
$$v^2 = v_0^2 + 2a(x - x_0) \qquad (4).$$

Please Note that in the Book, $x_0 = 0$.

<u>FREE FALL</u>

Near the surface of the Earth every unsupported object has a <u>CONSTANT</u> acceleration of $9.8\,m/s^2$ directed along the radius (of the Earth) and pointing toward the center.

Locally, we pretend that the Earth is flat, Take x along horizontal, y along vertical, +ive up so



$$\underset{\rightarrow}{a} = -9.8\,m/s^2\,\hat{y}.$$

CONSEQUENTLY, The kinematic EQUATIONS for free fall are

$$\underset{\rightarrow}{a} = -9.8\,m/s^2\,\hat{y} \qquad \rightarrow (5)$$

$$\underset{\rightarrow}{v} = (v_0 - 9.8t)\,\hat{y} \qquad \rightarrow (6)$$

$$\underset{\rightarrow}{y} = (y_0 + v_0 t - 4.9t^2)\,\hat{y} \qquad \rightarrow (7)$$

$$v^2 = v_0^2 - 19.6(y - y_0). \qquad \rightarrow (8).$$

Note 1 if you throw a ball along the vertical, once it leaves your hand the motion is controlled by the Earth via Eqs. (5) through (8). The acceleration is the <u>SAME</u> AT ALL TIMES DURING THE FLIGHT OF THE BALL

Note 2 The same Eqns. apply for free fall on the Moon or any planet. Only the magnitude of $\vec{a}$ changes. On the Moon

$$\vec{a} = -1.63\,\mathrm{m/s^2}\,\hat{y} \qquad (\mathrm{MOON}).$$

Note 3 If $\vec{v}_0 = +v_0\,\hat{y}$, $y_0 = 0$

That is ball is thrown straight up. Its acc., vel. and position vary with time as