SOLUTIONS - Wk 8

8-1 If an object of Mass M has a linear momentum $\underline{p}$, its kinetic Energy K is given by

$$K = \frac{p^2}{2M}$$

So if we have two objects

$$K_1 = \frac{p_1^2}{2M_1} \qquad K_2 = \frac{p_2^2}{2M_2}$$

If $K_1 = K_2$ $\qquad \frac{p_1^2}{2M_1} = \frac{p_2^2}{2M_2}$

$$\left(\frac{p_1}{p_2}\right)^2 = \frac{M_1}{M_2}$$

$$\left(\frac{p_1}{p_2}\right) = \left(\frac{M_1}{M_2}\right)^{1/2}$$

Here $M_2 = M$, $M_1 = 3M$

so

$$\frac{p_1}{p_2} = \left(\frac{3}{1}\right)^{1/2} = \sqrt{3} = 1.732.$$

The larger mass has a larger lin. momentum by $\sqrt{3}$.

8-2 From 8-1

$$p_1^2 = 2M_1K_1 \qquad p_2^2 = 2M_2K_2.$$

If $p_1 = p_2$ $\qquad 2M_1K_1 = 2M_2K_2.$

$$\frac{K_1}{K_2} = \frac{M_2}{M_1} = \frac{1}{3}.$$

The smaller mass has the larger kin En by a factor of 3.

8-3 IF THE External force is non-zero the total vector momentum

$$\vec{P} = \sum \vec{p_i}$$

of the system Cannot be a Constant. Newton's second law would say that

$$\frac{\Delta \vec{P}}{\Delta t} = \sum \vec{F_i}$$

8-4 In a totally Elastic Two body Collision the total kinetic Energy is also conserved

$$\underbrace{(K_1' + K_2')}_{\text{After}} = \underbrace{(K_1 + K_2)}_{\text{Before}}$$

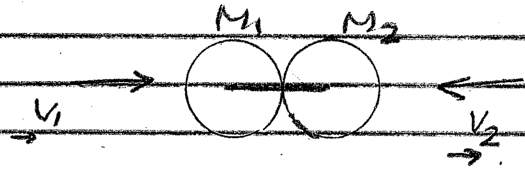
In a totally Inelastic Collision the two objects <u>Stick together</u> after the Collision

$$\vec{V_1}' = \vec{V_2}'$$

In both cases if $\vec{F}_{ext} = 0$ the total Vector momentum is Conserved

$$\underbrace{(\vec{p_1}' + \vec{p_2}')}_{\text{After}} = \underbrace{(\vec{p_1} + \vec{p_2})}_{\text{Before}}$$

8-5 In a totally Elastic head on Collision the line of Centers at the instant of Collision is parallel to the relative velocity.



If we take $(\vec{V_1}-\vec{V_2})$ along x-axis the collision is essentially one dimensional and the conservation Equations are

Linear Momentum $\quad M_1 V_1' + M_2 V_2' = M_1 V_1 + M_2 V_2 \quad \rightarrow (1)$

Kinetic Energy $\quad \frac{1}{2} M_1 V_1'^2 + \frac{1}{2} M_2 V_2'^2 = \frac{1}{2} M_1 V_1^2 + \frac{1}{2} M_2 V_2^2 \quad \rightarrow (2)$

Rewrite Eq (1)

$$M_1 (V_1' - V_1) = M_2 (V_2 - V_2') \quad \rightarrow (3)$$

Rewrite Eq (2) $\quad M_1 (V_1'^2 - V_1^2) = M_2 (V_2^2 - V_2'^2) \quad \rightarrow (4)$

Divide Eq (4) by Eq (3)

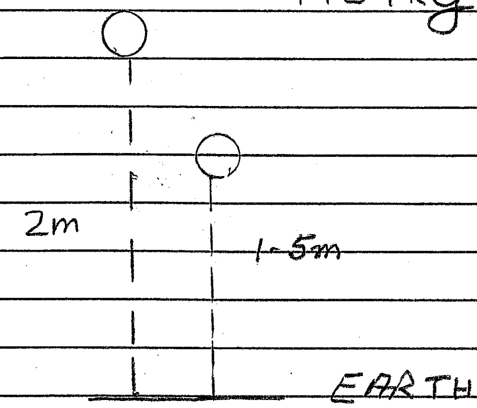
$$V_1' + V_1 = V_2 + V_2'$$

or $\quad V_1' - V_2' = V_2 - V_1 = -(V_1 - V_2)$

So Relative Velocity after the Collision is exactly negative of the relative velocity before the Collision.

8-6 No it is not totally Elastic, because the Ball loses kinetic energy but the Earth cannot pick up any kinetic Energy although it picks up linear momentum.

$M = 1\,kg$



Details before collision $\frac{1}{2}MV^2 = 2g \times 2.$

so $\underset{\rightarrow}{V_B} = -8.85\ m/s\ \hat{y}$

After collision $\underset{\rightarrow}{V_A} = +7.67\ m/s\ \hat{y}$ $\quad \frac{1}{2}MV^2 = 2 \times g \times 1.5$

So $\quad \Delta \underset{\rightarrow}{p}_{Ball} = 1 \times (7.67 + 8.85)\ \hat{y}$

$= 16.52\ kg\text{-}m/s\ \hat{y}$

Momentum Conservation requires

$$\Delta \underset{\rightarrow}{p}_{Ball} + \Delta \underset{\rightarrow}{p}_{Earth} = 0$$

So Earth picks up $\Delta \underset{\rightarrow}{p}_{Earth} = -16.52\ kg\text{-}m/s\ \hat{y}$

Change of K.E. $\Delta K_{Ball} = -9.8$ Joules

$\Delta K_{Earth} = 0 \quad b/c\ M_{Earth} = 6 \times 10^{24}\ Kg$

So collision is not totally Elastic as total (K.E.) after is less than total $(K.E.)_{Before}$

8-7

Given

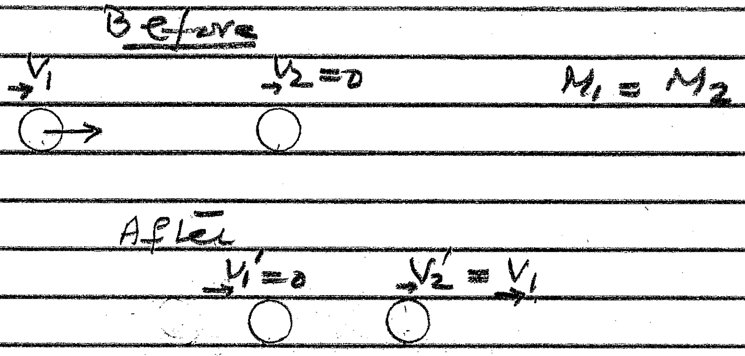
$$\underset{\rightarrow}{V_1'} = \frac{M_1 - M_2}{M_1 + M_2}\underset{\rightarrow}{V_1} + \frac{2M_2 \underset{\rightarrow}{V_2}}{M_1 + M_2}$$

PROVED IN CLASS.

$$\underset{\rightarrow}{V_2'} = \frac{M_2 - M_1}{M_1 + M_2}\underset{\rightarrow}{V_2} + \frac{2M_1 \underset{\rightarrow}{V_1}}{M_1 + M_2}.$$

i) $\underset{\rightarrow}{V_1} = 5\,m/s\,\hat{x}$

$\underset{\rightarrow}{V_2} = 0.$

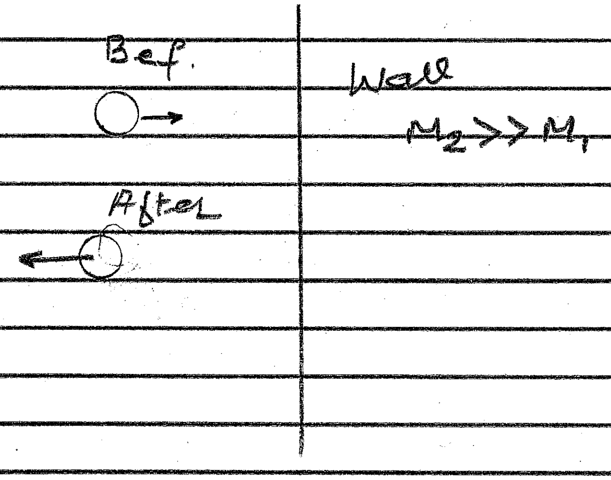


$\underset{\rightarrow}{V_1'} = 0$

$\underset{\rightarrow}{V_2'} = \frac{2 \times M \times 5}{2M} = 5\,m/s\,\hat{x}.$

(ii) $\underset{\rightarrow}{V_1} = 10\,m/s\,\hat{x}$

$\underset{\rightarrow}{V_2} = 0$



$\underset{\rightarrow}{V_1'} = -\underset{\rightarrow}{V_1} = -10\,m/s\,\hat{x}.$

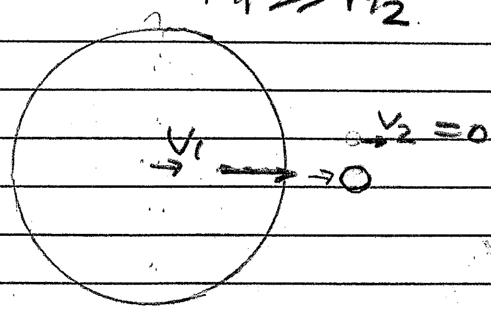
$\underset{\rightarrow}{V_2'} = 0 \quad [M_1 << M_2]$

Again, wall picks up momentum but no kinetic Energy because $M_2 >> M_1$

$\Delta \underset{\rightarrow}{P_1} = -20\,M_1\,kg\text{-}m/s\,\hat{x} \quad \Delta \underset{\rightarrow}{p_2} = +20\,M_1\,kg\text{-}m/s\,\hat{x}$

8-7 (iii) $\vec{V}_1 = 10\ m/s\ \hat{x}$

$M_1 >> M_2$



$\vec{V}_2 = 0$

$\vec{V}_1' = \vec{V}_1 = 10\ m/s\ \hat{x}$

$\vec{V}_2' = 2\vec{V}_1 = 20\ m/s\ \hat{x}$

However, M_2 carries away very little kinetic energy

(iv) $M_1 = 10\ kg \qquad M_2 = 5\ kg$

$\vec{V}_1 = 2\ m/s\ \hat{x} \qquad \vec{V}_2 = -4\ m/s\ \hat{x}$

$$\vec{V}_1' = \frac{10-5}{10+5} \times 2\ \hat{x} - \frac{2 \times 5 \times 4}{10+5}\ \hat{x}$$

$$= -2\ m/s\ \hat{x}$$

$$\vec{V}_2' = -\frac{5-10}{15} \times 4\ m/s\ \hat{x} + \frac{2 \times 10 \times 2}{15}\ m/s\ \hat{x}$$

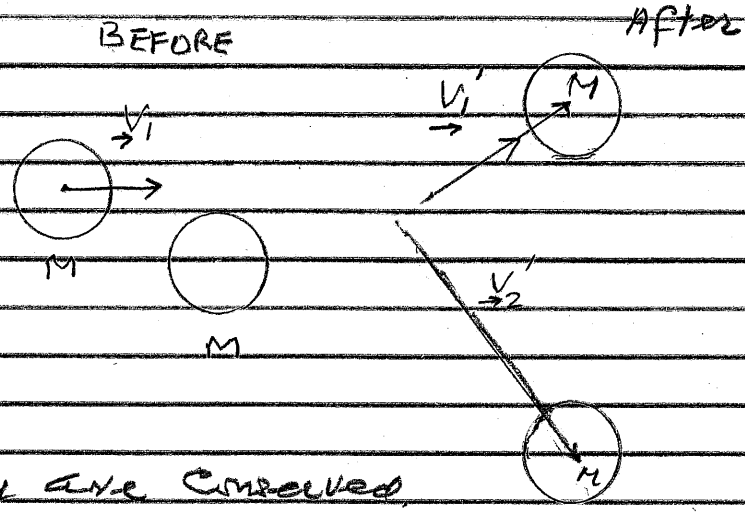
$$= 4\ m/s\ \hat{x}$$

Note Rel. Vel After $\vec{V}_2' - \vec{V}_1' = 6\ m/s\ \hat{x}$ is

−ive of Rel. Vel. Before $\vec{V}_2 - \vec{V}_1' = -6\ m/s\ \hat{x}$

as shown in Prob 8-5

8-8 GLANCING, TOTALLY ELASTIC COLLISION:



Totally Elastic means that both Linear momentum and Kinetic Energy are Conserved

Linear mom$^{m.}$ $\quad M\vec{V}_1' + M\vec{V}_2' = M\vec{V}_1$ $\quad$ —(1)

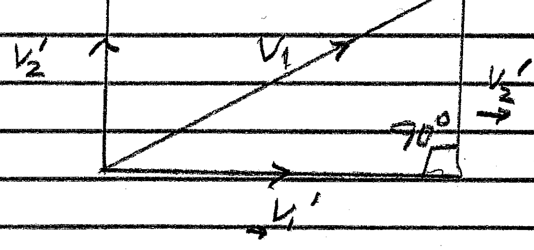
Kinetic Energy $\quad \frac{1}{2}MV_1'^2 + \frac{1}{2}MV_2'^2 = \frac{1}{2}MV_1^2$ $\quad \rightarrow$(2).

Eq (1) implies $\quad \vec{V}_1' + \vec{V}_2' = \vec{V}_1$ $\quad$ —(3)

and (2) gives $\quad V_1'^2 + V_2'^2 = V_1^2$ $\quad$ —(4).

Eqn. (3) says that $\vec{V}_1$ is the vector sum of $\vec{V}_1'$ and $\vec{V}_2'$

The only way this will lead to Eq (4) is if $\vec{V}_1'$ and $\vec{V}_2'$ are at right angles to one another because then V_1 will be the hypotenuse.



8-9 This is a totally inelastic Collision

so after Collision $\underset{\to}{V_1}' = \underset{\to}{V_2}' = \underset{\to}{V}'$

Momentum Conservation requires

$$(M_1 + M_2)\,\underset{\to}{V}' = M_1 \underset{\to}{V_1} + M_2 \underset{\to}{V_2}$$

$$9\left[\,V_x'\,\hat{x} + V_y'\,\hat{y}\,\right] = 20\,\text{kg m/s}\,\hat{x} - 20\,\text{kg-m/s}\,\hat{y}$$

$$V_x' = \frac{20}{9} = 2.2\,\text{m/s}$$

$$V_y' = -\frac{20}{9} = -2.2\,\text{m/s}$$

so $\underset{\to}{V}' = 2.2\,\text{m/s}\,\hat{x} - 2.2\,\text{m/s}\,\hat{y}$

8-10 In 8-7 (iv) momentum before Collision

$$\underset{\to}{p}_B = (20 - 20)\,\text{kg-m/s}\,\hat{x}$$

So $\underset{\to}{p}_A = 0 = (M_1 + M_2)\,\underset{\to}{V}'$

Hence $\underset{\to}{V}' = 0$ The two objects came together and stop moving.

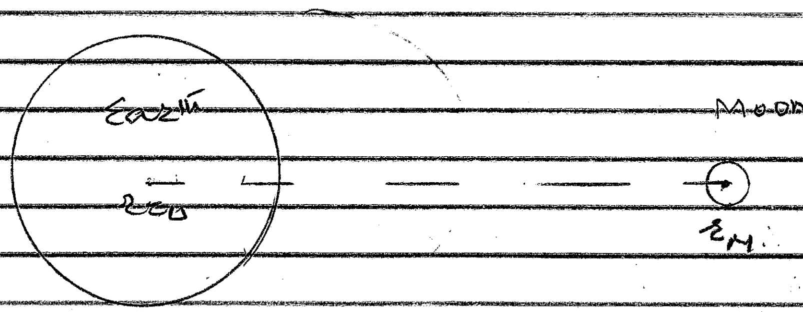
8-11 The position of the Center of mass is

$$\underset{\to}{r}_{CM} = \frac{\sum M_i \underset{\to}{r_i}}{\sum M_i}$$

Let us put earth at $r = 0$.

Then $r_{Moon} = 4 \times 10^5$ km.

$$M_E = 81\,M_M$$



$$r_{CM} = \frac{M_E \times 0 + M_M \times 4 \times 10^5}{M_E + M_M}$$

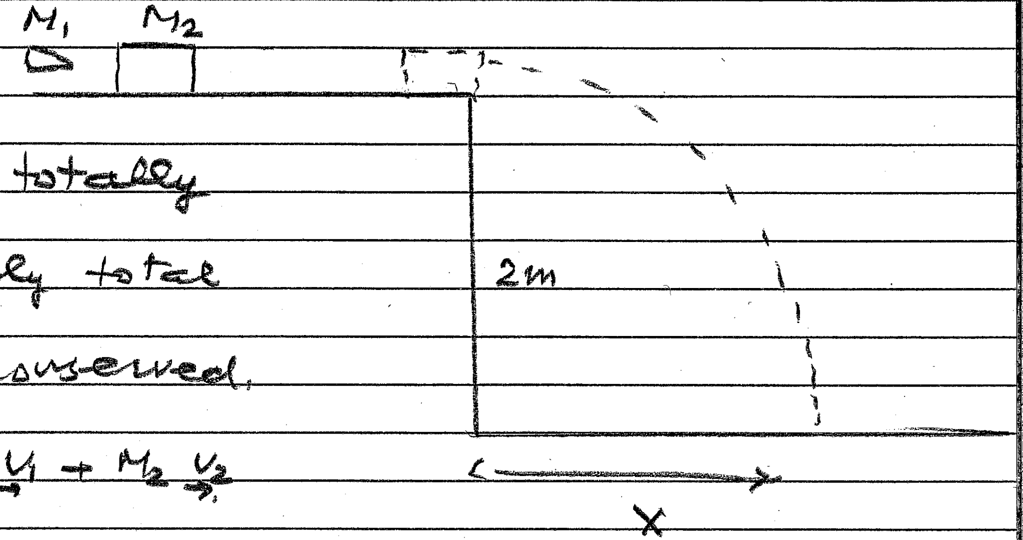
$$= \frac{M_M \times 4 \times 10^5}{81 M_M + M_M} = 4878 \text{ km.}$$

Since radius of Earth is 6000 km the Center of Mass of the Earth - Moon System is "INSIDE" THE EARTH!

8-12 This could be called a "TRICK" question. It has been set up to show you that there is not enough information to calculate the velocities after the collision. The only case where the velocities after collision could be assessed would be if the two objects stuck together (Totally inelastic collision). If the collision is totally elastic we still have only three equations* and there are four unknowns V_{1x}', V_{1y}', V_{2x}', V_{2y}' so one cannot calculate them.

* 3 EQS: Two for x and y components of linear momentum and one for total kinetic Energy.

8-13



The collision is totally Inelastic so only total momentum is conserved.

$(M_2 + M_1)\underset{\rightarrow}{V}' = M_1 \underset{\rightarrow}{V_1} + M_2 \underset{\rightarrow}{V_2}$

$\underset{\rightarrow}{V_1} = 100\,m/s\,\hat{x}$ $\quad V_2 = 0$

$M_1 = 0.01\,kg$ $\quad M_2 = 2\,kg$

$$\underset{\rightarrow}{V}' = \frac{0.01 \times 100}{2.01}\,m/s\,\hat{x} = 0.50\,m/s\,\hat{x}$$

Time to drop by 2m $\quad y = y_0 + v_{oy}t - 4.9t^2$

$$0 = 2 - 4.9t^2$$

$$t = \sqrt{\frac{2}{4.9}} = 0.64\,sec$$

Distance X travelled

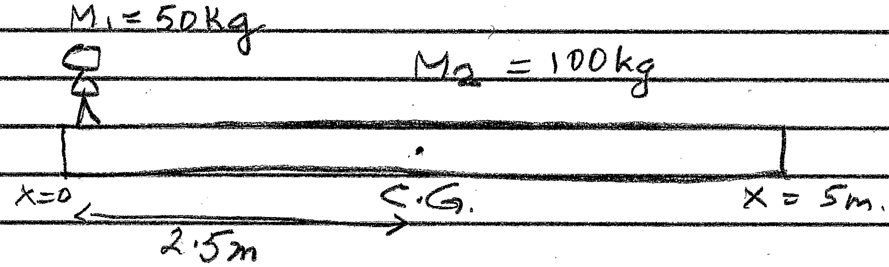
$$X = v't = 0.32\,m$$

8-14 Since external force is zero. Total linear momentum is conserved. The velocity of the center of mass must be constant. Since center of mass is at rest before the person starts walking it must stay at rest throughout so we calculate X_{CM} in the beginning and X_{CM} at the end to calculate how much the plank moved.

<u>Beginning</u>

$M_1 = 50\,kg$

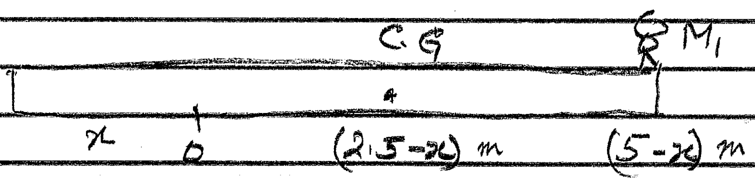
$M_2 = 100\,kg$



$x_1 = 0 \quad , \quad X_2 = 2.5m$ (center of Gravity)

$$X_{CM} = \frac{50 \times 0 + 100 \times 2.5}{150} = 1.67\,m. \rightarrow (1)$$

<u>End</u> Let plank move to the left by x meters



Now $$X_{CM} = \frac{100 \times (2.5 - x) + 50(5-x)}{150.}$$

$$= \frac{250 - 100x + 250 - 50x}{150} = \frac{500 - 150x}{150} \quad (2)$$

FROM Eqs. (1) and (2)

$$500 - 150x = 250$$

$$x = \frac{250}{150} = 1.67\,m.$$

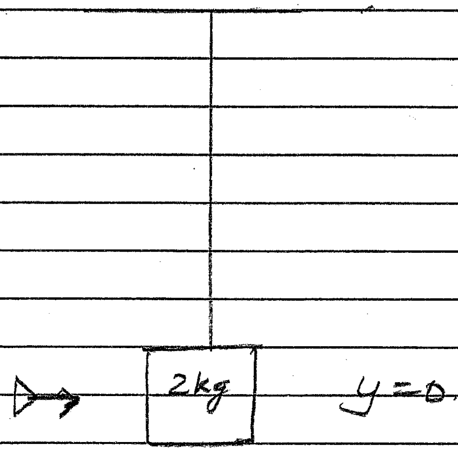
8–15 BALLISTIC PENDULUM.

First, as in Prob 8–13

this is a totally

inelastic Collision.

$\underset{\rightarrow}{v}' = 0.50\ m/s\ \hat{x}$



Kinetic Energy of Bullet + Block

$$= \frac{1}{2} \times 2.01 \times (0.5)^2 = 0.25\ J$$

Next, we use Conservation of Energy (No friction).

$$P_f + K_f = P_i + K_i$$

$$Mgh + 0 = 0 + 0.25\ J$$

$$h = \frac{0.25}{2.01 \times 9.8} = 0.013\,m.$$