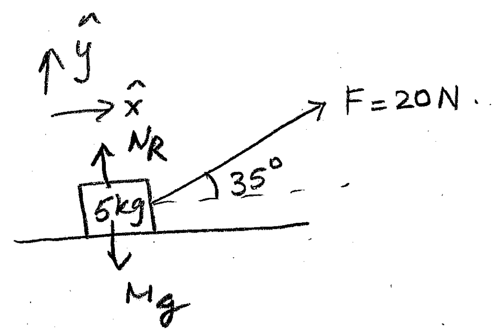
Week (7) Solutions



7-1 i, Work done $\Delta W = \vec{F}\cdot\vec{\Delta s} = F\Delta s \cos(F, \Delta s)$

$\Delta W = 20 \times 2 \times \cos 35^\circ$

$= 32.8$ Joules

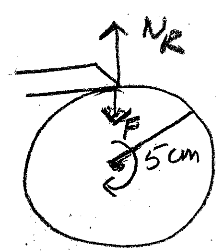
ii, Normal force N_R acts perpendicular to the displacement

$\therefore \cos(\vec{N}, \vec{\Delta s}) = \cos 90^\circ = 0$

$\therefore W = 0$

iii, Weight force 'mg' also acts perpendicular to the displacement

$\therefore W = 0$



7-2

Applied.

$\underset{\to}{F} = -20N\,\hat{r}$ $\quad [\underset{\to}{F} + \underset{\to}{N_R}] = 0$

$r = 5$ cm

$\mu = 0.3$

My applied force $= -$[Normal force on the tool]

$\therefore$ Frictional force on the tool $\underset{\to}{f} = \mu N_R \hat{\tau}$

$= 0.3 \times 20$

$= 6.67\,N\,\hat{\tau}$

This force does work as the displacement is in the direction of the frictional force.

In 10 revolutions; distance covered $= 2\pi r \times 10$

$= 2 \times 3.14 \times 0.05 \times 10$

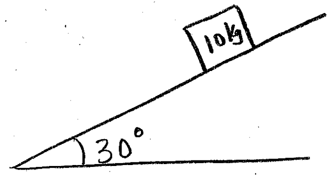
$= 3.14\,m.$

$\therefore$ Work $\Delta W = F \cdot \Delta s \cos(F, \Delta s)$

$= 6.67 \times 3.14 \times \cos(180^\circ)$

$= -20.944\,J$

<u>7-3</u> $\mu_k = 0.3$



Work $W = F \cdot \Delta s$

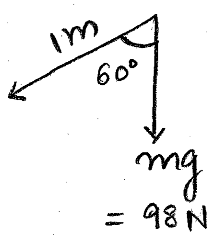
$= F \cdot \Delta s \cdot \cos(F, \Delta s)$

i, Normal force is perpendicular to displacement

$\therefore \cos(N, \Delta s) = 0$

$\Rightarrow \Delta W = 0\,J$.

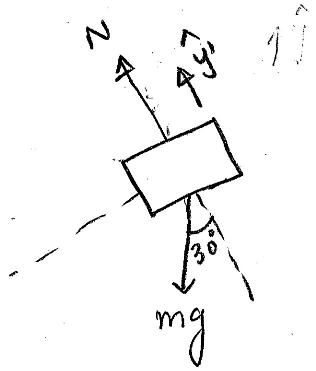
ii, Gravity



$\Delta W = 98 \times 1 \times \cos 60^\circ$

$= 49\,J$

iii, Friction : $f_k = \mu_k \times N$



on balancing force in $\hat{y}'$ direction

$N = mg\cos 30^\circ$

$= 98 \times \frac{\sqrt{3}}{2} = 84.868\,N$

$\Rightarrow f = 0.3 \times 84.868$

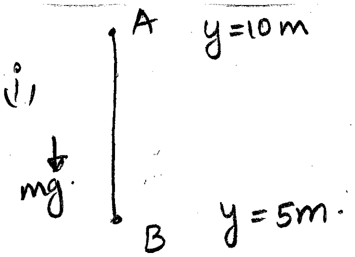
$= 25.46\,N$.

$W = 25.46 \times 1 \times \cos(180^\circ)$ ($\because$ friction is opposing motion)

$= -25.46\,J$.

7-4

i,



$m = 1\,kg$

$\Delta s = -5m\ \hat{y}$

$F = 1 \times 9.8\,N\ -\hat{y}$

$= -9.8\,N\ \hat{y}$

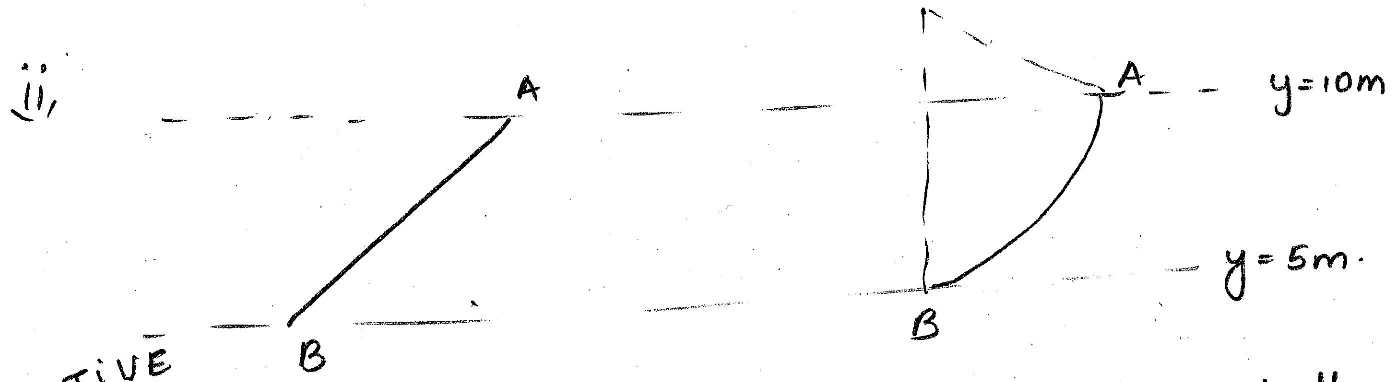
[$\Delta W = F_x \Delta x + F_y \Delta y + F_z \Delta z$

ONLY $F_y \neq 0$

So only Δy counts]

$\therefore\ W = 9.8 \times 5 = 49\,N.$

ii,



CONSERVATIVE FORCE → We Know that work is independent of the path chosen when work is done by gravitational force

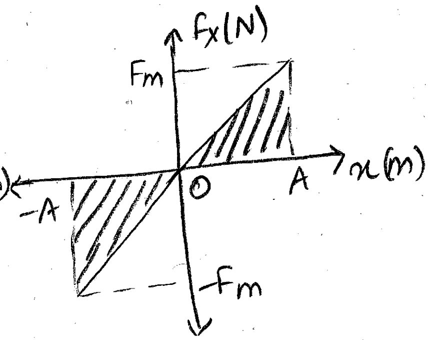
$\therefore$ in all the cases ; $W = 49\,N.$

Gravitational force is a Conservative force.

7-5 i, Work done = Area under the graph

= Area of the shaded region.

$= \frac{1}{2} \times$ Force $\times$ (Final - initial position)



$-A$ to 0 : area $= \frac{1}{2} \times (-F_m) \times (0-(-A))$

$= -\frac{F_m A}{2}\ J.$

ii, A to 0 : area $= \frac{1}{2} \times (F_m) \times (0-A)$

$= +\frac{F_m A}{2}\ J$

7-6 $KE = \frac{1}{2}mv^2$

Now earth completes one revolution in 365 days, 6 hours.

1 revolution $\longrightarrow T = \underset{\text{day}}{365} \times \underset{\text{hour}}{24} \times \underset{\text{min}}{60} \times \underset{\text{sec}}{60}$ sec $+ \underset{\text{hr}}{6} \times \underset{\text{min}}{60} \times \underset{\text{sec}}{60}$ sec

$T = 31557600$ sec.

This is the time period T.

$|\vec{\omega}| = \frac{2\pi}{T}$ and $|\vec{v}| = |r\vec{\omega}|$

$$\therefore \quad KE = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times m \times (r\omega)^2$$
$$= \frac{1}{2} mr^2\omega^2$$
$$= \frac{1}{2} \times mr^2 \left(\frac{2\pi}{T}\right)^2$$
$$= \frac{1}{2} \times \underbrace{6 \times 10^{24}}_{m} \times \underbrace{(1.5 \times 10^8 \times 10^3)^2}_{r} \times \left(\underbrace{\frac{2\pi}{31557600}}_{\frac{2\pi}{T}}\right)^2$$
$$= \frac{10^{24} \times 10^{22} \times 266.209}{(31557600)^2}$$
$$= 10^{46} \times 2.673 \times 10^{-13}$$
$$= 2.673 \times 10^{33} \text{ J}$$

7-7: A conservative force is that force for which the work done in moving a body from an initial to a final point is independent of the path taken by the body.

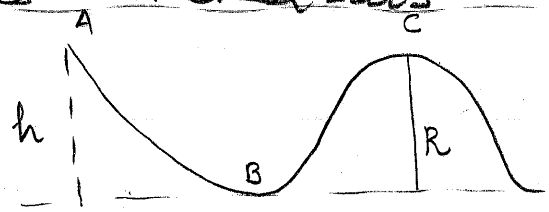
7-8: [For our cases] Frictional force always opposes motion. Hence the force and the displacement are always in the opposite direction. $\cos(f, \Delta s) = \cos(180^\circ) = -1$. $\therefore$ Work done by frictional forces is always negative.

7-9: Potential energy of a body is by virtue of its position and arrangement in space. in the presence of a

Conservative force [see Notes handed out]

7-10

(iii) For a body to go over C, without losing contact; the criterion is smallest value of $|v_c|$:



partiii is solved after partii

$$v_c = \sqrt{gR} \quad \text{(Check problem 5-7)}$$

Conservation of energy yields.

$$mgh = \frac{1}{2}mv_c^2 + mgR.$$
$$= \frac{1}{2}mgR + mgR.$$
$$\Rightarrow \boxed{h = \frac{3R}{2}}$$

(ii) The criterion to reach C is $|\vec{v_c}| = 0$

$\Rightarrow$ by energy conservation

$$mgh = mgR + \frac{1}{2}mv_c^2$$
$$mgh = mgR$$
$$\Rightarrow \boxed{h = R}$$

7-11 Satellite is moving in a circular orbit

The centripetal force is provided by the earth's gravitational force

$$-\frac{Mv^2}{R}\hat{r} = -\frac{GM_EM}{R^2}\hat{r}$$

$$\therefore v^2 = \frac{GM_E}{R}$$

Kinetic energy $k = \frac{1}{2}Mv^2 = \frac{1}{2}\cdot M\frac{GM_E}{R}$

Potential energy $P = -\frac{GM_EM}{R}$ $\quad\therefore\quad \boxed{P = -2K}$

7-12 If a body escapes earth's gravitational field, then final Gravitational potential energy = 0. To obtain minimum escape velocity, we should also enforce final kinetic energy = 0

Conserving energy:

Initial energy = Final energy

$$-\frac{GM_EM}{R_E} + \frac{1}{2}Mv_{esc}^2 = 0$$

$$\frac{1}{2}Mv_{esc}^2 = \frac{GM_EM}{R_E}$$

$$\Rightarrow \quad v_{esc} = \sqrt{\frac{2GM_E}{R_E^2}R_E}$$

$$= \sqrt{2gR_E} = \sqrt{2 \times 9.8 \times 6400 \times 10^3}$$

$$= 11.2 \times 10^3 \, m/s$$

7-13 $m = 10^3 kg$.

$F = 200N$.

i, $20 \frac{m}{s}$ for 10 sec

distance covered:

$$x = v_i t$$

$$= 20 \frac{m}{s} \times 10 \, sec = 200 m$$

Work $\Delta W = F.\Delta S$

$$= 200N \times 200m$$

$$= 40000 J.$$

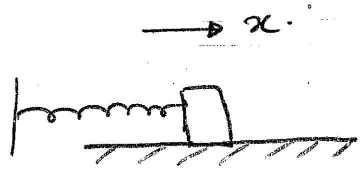
ii, $v_f = 20 m/s$, $v_i = 0 m/s$, $t = 10s$

$$\Rightarrow \quad a = \frac{v_f - v_i}{t} = \frac{20}{10} = 2 m/s^2$$

$$x = v_i t + \frac{1}{2}at^2$$

$$= \frac{1}{2} \times 2 \times 10 \times 10 = 100 m$$

$$\Rightarrow \Delta W = 200N \times 100m = 20000 J.$$



7-14 $m = 0.2\,kg$.

Let $x=0$ be the point where is mass 'm' is present when spring is relaxed.

$\Delta x = -10\,cm = -0.1\,m$

$\mu_K = 0.2$

$K = 100\,N/m$

i, $WD = \frac{1}{2}K\Delta x^2$

$= \frac{1}{2} \times 100 \times (-0.1)^2 = 0.5\,J$

ii, $f = \mu N$ (check part (iv) before part (ii) to obtain distance travelled before halt)

$= \mu mg$

$= 0.2 \times 0.2 \times 9.8$

$= 0.392\,N$

$WD = f \cdot \Delta x \cos(f, \Delta x) = 0.392 \times 0.1 \times \cos(180^\circ) = -0.0392\,J$

iii, Conservation of energy gives

$\frac{1}{2}k\Delta x^2 = \frac{1}{2}mv^2$

$\frac{1}{2} \times 100 \times (0.1)^2 = \frac{1}{2} \times 0.2 \times v^2$

$\Rightarrow v = 2.236\,m/s$

[when the mass leaves spring it has kinetic Energy

$k = (0.5 - 0.0392)\,J$

$= 0.4608\,J$]

iv, $f = \mu mg = ma$

$\Rightarrow a = \mu g = 0.2 \times 9.8 = 1.96\,m/s^2$

For the mass;

$a = -1.96\,m/s^2$ ($\because$ friction opposes motion causing deceleration)

$v_i = 2.236\,m/s$

$v_f = 0$

$\therefore v_f^2 - v_i^2 = 2a\Delta x \Rightarrow -2.236^2 = 2 \times -1.96 \times \Delta x$

$\Rightarrow \Delta x = 1.2755\,m$

$[k - f_k \cdot \Delta S = 0]$ $f_S = \frac{k}{f_k} = \frac{0.4608}{0.392} = 1.18\,m$

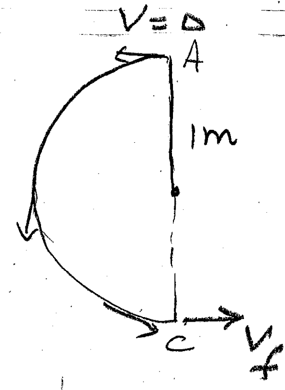
7-15

$R = 1m$

$F_T = 10N\,\hat{i}$

$m = 0.2\,kg$

$K_f + P_f = K_i + P_i + W_{NCF}$



Statement of Work-energy theorem:

Work done = change in energy of the body.

$WD = (KE_f - KE_i) + (PE_f - PE_i)$

$W_{NCF} = WD = F_T \cdot \Delta s \cos(F_T, \Delta s)$

$= 10 \times (\pi \times 1) \times \cos(0^\circ)$

$= 31.4J$

$K_f = 0 + (P_i - P_f) + W_{NCF}$

$= 0 + Mg\,2R + W_{NCF}$

$= (0.2 \times 9.8 \times 2 + 31.4)\,J = \frac{1}{2} m V_f^2$

using $V_f = \sqrt{\dfrac{2 \times 35.32}{0.2}} = 18.8\,m/s$

7-16

$M = 50\,kg$

$h = 15\,m$.

$\therefore$ change in PE $= mgh = 50 \times 15 \times 9.8 = 7350\,J$

This is the work I have done in 3 sec

$\therefore$ Power $= \dfrac{WD}{time} = \dfrac{7350}{3} = 2450\,W$

7-17

3.2×10^6 people consume $6 \times 10^{22}\,J$ in 1 year

$\Rightarrow$ per capita consumption $= \dfrac{\text{Total Power}}{\text{population}} = \left(\dfrac{1}{3.2 \times 10^6}\right) \times \left(\dfrac{6 \times 10^{22}}{365 \times 24 \times 60 \times 60}\right)$

$= 5.94 \times 10^8\,W$

useful power of sun $= \frac{20}{100} \times 1000\,W/m^2 = 200\,W/m^2 \Rightarrow$ Area needed $= \dfrac{5.94 \times 10^8}{200} = 2970000\,m^2$