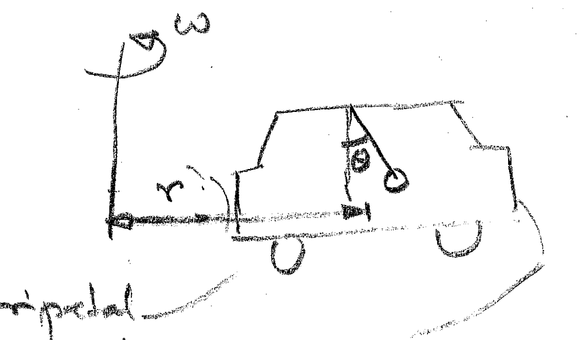
Week 6 **SOLUTIONS**

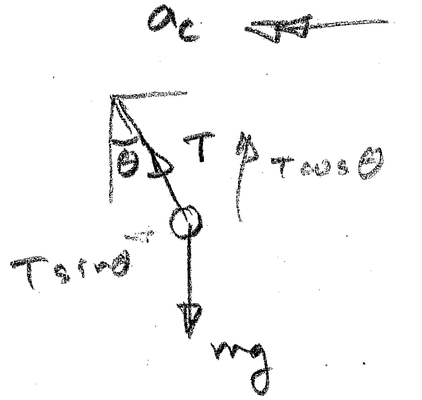
6-1 The pendulum cannot stay vertical. This is because the car along with the pendulum is describing a circular motion. The speed remains same, but due to constant change of the direction of velocity we have a non-zero acceleration acting towards the centre of the circular path.

Now if the string of the pendulum remains vertical, the horizontal component of Tension would be zero. So there would be no force to provide the necessary centripetal force to the pendulum. Hence the string has to make an angle with the vertical to provide the necessary centripetal force to the pendulum bob.

6-2



F.B.D



a_c = centripetal acceleration

$= \frac{v^2}{r}$

From F.B.D

$$T\cos\theta = mg$$

$$T\sin\theta = m a_c = \frac{mv^2}{r}$$

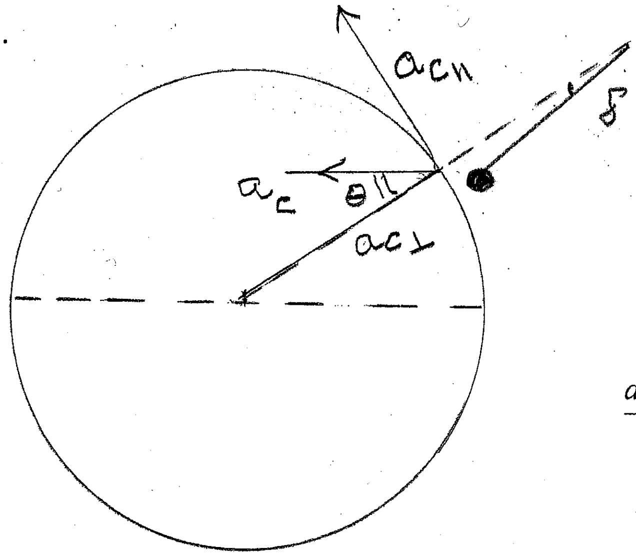
$$\therefore \tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{20 \times 30}{100 \times 9.8}\right)$$

$$= 0.387°$$

6-3 This is another effect of the rotation of Earth.

This Is the most subtle and happens for any Θ other than 0 and $\frac{\pi}{2}$ because $\underset{\rightarrow}{a_c}$ is NOT parallel to $\underset{\rightarrow}{g}$.



$$\underset{\rightarrow}{a_c} = R_E w_E^2 \cos\Theta$$

Indeed now $\underset{\rightarrow}{a}_c$ has a component parallel to surface of Earth

$$a_{c\parallel} = R_E w_E^2 \sin\Theta\cos\Theta$$

and a component along radius of Earth

$$a_{c\perp} = -R_E w_E^2 \cos^2\Theta\hat{r}$$

which is along r so it modifies "g" slightly.
Since we have an $a_{c\parallel}$, if you try to hang a pendulum, it cannot be vertical (parallel to $\hat{r}$).
It must tilt to yield a force to produce $a_{c\parallel}$

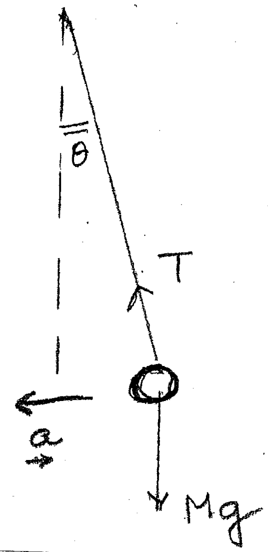
$$\tan\delta = \frac{a_{c\parallel}}{g}$$
$$\cong \frac{\sin\Theta\cos\Theta R_E w_E^2}{g}$$

$\delta \to 0 \qquad \Theta = 0$ and $\Theta = \pi/2$

The situation is exactly like the case of a pendulum hanging in a cart which has an acceleration $\underset{\rightarrow}{a} = -a\hat{x}$. (Prob. 4-16)

$$-T\sin\Theta = -Ma$$
$$T\cos\Theta - Mg = 0$$

$$\tan\Theta = \frac{a}{g}$$



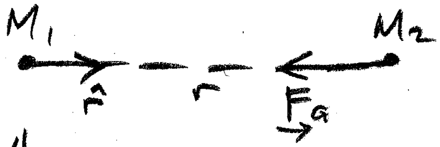
6-4 For a system to be inertial it cannot have an acceleration. However, the Earth goes around the sun once a year so even at the pole there is a centripetal acceleration. In this case $T = (365 \times 24 \times 3600)$ secs.

$$R = 1.5 \times 10^{11}\,m$$

so $$\underset{\rightarrow}{a_c} = -1.5\times10^{11} \times \left(\frac{2\pi}{365\times24\times3600}\right)^2 \hat{r} = -6\times10^{-3}\,m/s^2\,\hat{r}$$

Very small, but Not Zero.

(6-5) Because it is an attractive force, so the force on M_2 should point to M_1.



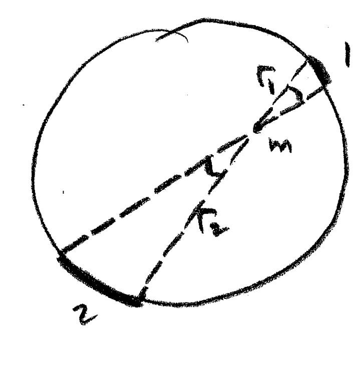
If we analyze the force on M_2, we see that $\vec{F}_G$ will point against $\hat{r}$, the unit vector from M_1 to M_2. So there should be a minus sign in the expression of $\vec{F}_G$.

Actually

$$\vec{F}_G = -\frac{G M_1 M_2}{r^2}\hat{r}$$

represents two forces ensuring that the force felt by M_2 due to M_1 is exactly equal but opposite to the force experienced by M_1 due to M_2.

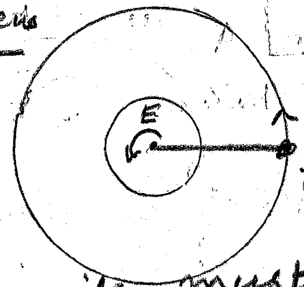
(6-6)



The force on m due to a piece of mass at position 1 is $\frac{GmM_1}{r_1^2}$. If the surface mass density is σ, then $M_1 = \sigma A_1$, where A_1 is the surface area of M_1. So the force on m due to M_1 is $Gm\sigma A_1/r_1^2$.

Similarly, for M_2, we have a force on m due to M_2, which is $Gm\sigma A_2/r_2^2$. Since the "solid angle" formed by M_1 and M_2 is the same, then we have $\frac{A_1}{r_1^2} = \frac{A_2}{r_2^2}$. Therefore, the force on m due to M_1 and M_2 will cancel each other. Since we choose M_1 (hence M_2) arbitrarily, we conclude that the net force on m is zero. A mass m does not feel gravitational force inside a hollow spherical surface.

T: 2nd View



The key concept is: the satelite has to be stationary with respect to a point on Earth. Hence it must move with the same angular speed as that of earth.

Now $T_{earth} = 1 \text{ day} = 24 \times 60 \times 60 \text{ s}$
$= 86400 \text{ s}$

$\therefore \omega = \frac{2\pi}{T} = 7.27 \times 10^{-5} \text{ rad/s}$

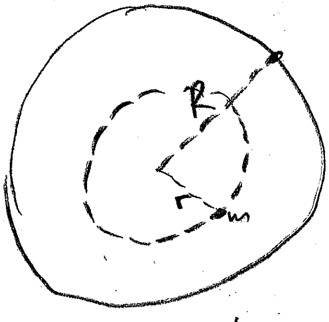
Now to keep the satelite in a circular orbit an external agent has to provide the necessary centripetal force. In this case, this is the gravitational force due to Earth. So,

$$\frac{GM\cancel{m}}{r^2} = \cancel{m}\omega^2 r \qquad \omega = \sqrt{\frac{GM}{r^3}} \quad \text{or} \quad r = \left(\frac{GM}{\omega^2}\right)^{1/3}$$

$G = 6.7 \times 10^{-11} \frac{N\text{-}m^2}{(kg)^2}$ $\qquad M_E = 6 \times 10^{24}$ kg

$$r = \left(\frac{6.7 \times 10^{-11} \times 6 \times 10^{24}}{(7.27 \times 10^{-5})^2}\right)^{1/3} = 4.16 \times 10^{7} \text{ m.}$$

(6-7)



m is inside all the shells between r and R.

The gravitational force on m at radius $\vec{r}$ is $\vec{F}_G = -\frac{GM'm}{r^2}\hat{r}$, where M' is the mass of the sphere inside the radius r. Why do we use M', not M (the total mass)? Because the mass m that sits at radius r does not feel gravitational force from the mass of solid sphere that sits outside r.

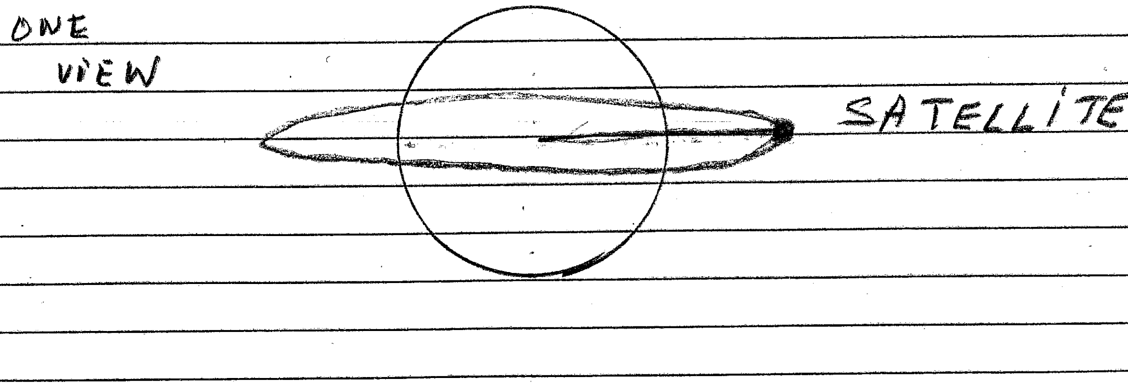
Now, since $M_1 = \rho \times$ volume $= \rho \times \frac{4}{3}\pi r^3$, we will have

$$\vec{F}_G = -\frac{G\rho\frac{4}{3}\pi r^3 m}{r^2}\hat{r} = -\frac{4\pi}{3}G\rho m r\hat{r} \qquad (\text{for } r<R).$$

If, instead, the mass m sits on the radius r outside R, then this mass m will feel a force from the whole mass M, so the force on m is

$$\vec{F}_G = -\frac{GMm}{r^2}\hat{r} \qquad (\text{for } r>R).$$

6-8 IF THE SATELLITE IS TO REMAIN STATIONARY WITH RESPECT TO A POINT ON EARTH, THE PLANE OF ITS ORBIT MUST BE THE SAME AS THE PLANE OF THE CIRCLE TRAVELLED BY A POINT ON EARTH, SO ITS ORBIT MUST BE IN THE EQUATORIAL PLANE



(6-9) Apply Newton's law on the moon of Earth,

$$\frac{G M_E m_m}{r^2} = \frac{m_m v^2}{r} = \frac{m_m \omega^2 r^2}{r} = m_m \left(\frac{2\pi}{T}\right)^2 r$$

[NOTE THAT FOR KEPLERIAN ORBITS

$$T_{sat}^2 = \frac{4\pi^2}{G M_E} R_{sat}^3 \quad \text{Earth}$$

and

$$T_{sat}^2 = \frac{4\pi^2}{G M_J} R_{sat}^3 \quad \text{Jupiter}]$$

So, $\frac{G M_E}{r^3} = \frac{4\pi^2}{T^2}$, or $r^3 = \frac{G M_E}{4\pi^2} T^2$

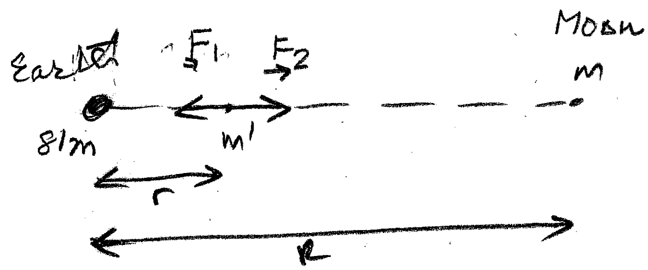
The equation for the moon of Jupiter is similar with the equation above. If we divide the two equations, we will have

$$\frac{r_1^3}{r_2^3} = \frac{\frac{G M_E}{4\pi}}{\frac{G M_J}{4\pi}} \frac{T_1^2}{T_2^2}$$

$$\frac{(4 \times 10^5\ km)^3}{(7 \times 10^5\ km)^3} = \frac{M_E}{M_J} \frac{(27\ days)^2}{(3.5\ days)^2}$$

so $\frac{M_E}{M_J} = 3.14 \times 10^{-3}$

(6-10)



There are two forces on m'. The total will be zero if they are equal and opposite.

Let's say I put a mass m' on the distance r from Earth.
Apply Newton's law, $\frac{G M m'}{r^2} - \frac{G m m'}{(R-r)^2} = 0$.

$$\frac{GMm'}{r^2} = \frac{Gmm'}{(R-r)^2}$$

$$\frac{M}{r^2} = \frac{m}{(R-r)^2}$$

$$\left(\frac{R-r}{r}\right)^2 = \frac{m}{M}$$

$$\frac{R-r}{r} = \sqrt{\frac{m}{M}} = \sqrt{\frac{m}{81\,m}}$$

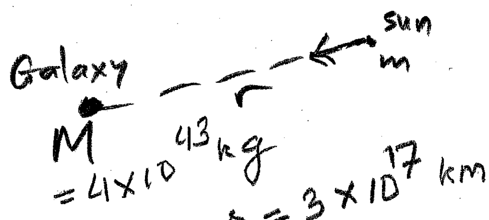
$$\frac{R-r}{r} = \frac{1}{9}$$

$$9(R-r) = r$$

$$9R = 10r$$

$$r = \frac{9}{10}R$$

(6-11) $$\frac{GMm}{r^2} = \frac{mv^2}{r}$$



So, $v^2 = \frac{GM}{r}$

$$v = 9.42\times10^7\ \mathrm{m/s}$$

and the period of the sun is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{v/r} = \frac{2\pi r}{v}$$

$$= \frac{2\pi\times3\times10^{20}}{9.42\times10^{7}} = 2\times10^{13}\ \mathrm{sec.}$$

(6-12) The orbital speed is $v_p = \frac{2\pi R_p}{T_p}$. Since $T_p^2 \propto R_p^3$, then $T_p \propto R_p^{3/2}$. So, $v_p = \frac{2\pi R_p}{T_p} \propto \frac{2\pi R_p}{R_p^{3/2}} = \frac{2\pi}{R_p^{1/2}}$.

If R_p is increased, then v_p would decrease.

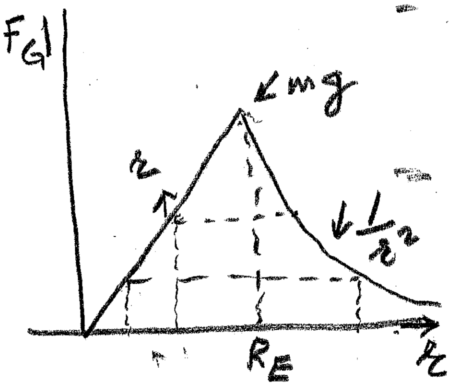
6-13

Referring to prob 6-7 we found that

INSIDE

$$\vec{F}_G = -\frac{4\pi}{3} G \rho m r \hat{r} \quad \text{for } r < R$$

$$= -\cancel{\frac{4\pi}{3}} G \frac{M}{\cancel{\frac{4\pi}{3}} R^3} m r \hat{r}$$



$$= -\left(\frac{GMm}{R^2}\right)\left(\frac{r}{R}\right)\hat{r}$$

At surface $r = R_E$ $\quad \vec{F}_G = -\frac{GMm}{R_E^2}\hat{r}$

$$= -mg$$

So $\frac{GMm}{R_E^2} = mg$

or $\vec{F}_G = -mg\left(\frac{r}{R_E}\right)\hat{r}$ for $r < R$

So if $F_G = \frac{1}{4} mg$ then $\frac{r}{R_E} = \frac{1}{4}$ $\therefore r = \frac{1}{4} R_E$

Similarly $r = \frac{1}{2} R_E$ for $F_G = \frac{1}{2} mg$

Outside $\vec{F}_G = -\frac{G M_E m}{r^2}\hat{r}$

So you get $\frac{Mg}{2}$ at $\sqrt{2}\, R_E$

$\frac{Mg}{4}$ at $2 R_E$

6-19 $\quad \frac{GMm}{r^2} = m\omega^2 r$ $\quad$ m = mass of moon

$\omega = \frac{2\pi}{T}$ $\quad \frac{GMm}{r^2} = \frac{4\pi^2 m}{T^2} r$ $\quad$ M = mass of earth

r = radius of orbit

$\therefore \quad T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$

$\therefore \quad \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

We have

$T_1 = 27$ days

$r_1 = 400,000$ km

$T_2 = 1$ day

$r_2 = ?$

$\therefore \quad r_2 = r_1 \left(\frac{T_2}{T_1}\right)^{2/3} = r_1 \left(\frac{1}{27}\right)^{2/3} = \frac{r_1}{9}$ km

$r_2 = \frac{400,000}{9}$ km $= 4.44 \times 10^7$ m.

(slight over estimate, compare 6-8)