Week 5 - Solutions

5-1

(i) 30 revolutions → 1 min = 60 sec

Let 1 revolution → \( x \) sec

\[ \frac{2}{x} = \frac{60}{30} \Rightarrow x = 2 \text{ sec} \]

\( \circ \) period = 2 sec

(ii) Since the motion is clockwise, \( \omega \) is negative.

\[ \omega = -\frac{2\pi}{1} = \frac{-2\pi \text{ rad}}{2 \text{ sec}} = -3.14 \text{ rad/sec} \]

(direction is obtained using right hand thumb rule)

(iii) At \( t = 1.5 \) sec:

Clearly \( \frac{1.5}{2} = \frac{3}{4} \) revolution and at \( t = 0 \), \( x = 2 \hat{m} \)

\[ x = 2 \hat{m} \]

\[ \hat{y} = \text{ tangential} \]

\[ \hat{x} = \text{ always towards centre and hence points in a direction opposite to } \hat{x} \]

\[ a_x = -1 \omega^2 \hat{y} = -(2\pi^2) \text{ m/s}^2 \hat{y} \]

5-2

(i) Time period = 2 sec

(ii) \[ \omega = \frac{2\pi}{1} = 3.14 \frac{\text{ rad}}{\text{ sec}} \]

(iii) Clearly \( x = -2 \hat{m} \), \( \hat{y} = 2\pi \text{ m/s} \) (tangential) \[ a_x = 2\pi^2 \text{ m/s}^2 \hat{y} \] (towards centre)

5-3

\( m = 1 \text{ kg, } r = 0.5 \text{ m} \)

15 revolutions in 1 min → one revolution in \( \frac{60 \text{ sec}}{15} = 4 \text{ sec} \)

\[ \omega = \frac{2\pi}{1} = \frac{\pi}{2} \text{ rad/sec} \]

\[ m.m.m. \]
As the mass is rotating, there should be a centripetal force which is clearly provided by the spring. It should be pointing towards centre:

\[ F_{cp} = -k \Delta r \hat{n} \]

For this force to be directed towards \(-\hat{n}\), \(k \Delta r > 0\)

\[ \Rightarrow \Delta r > 0 \]

\[ \Rightarrow \text{spring has to stretch} \]

\[ k = 10^3 \text{ N/m} \]

Now,

\[ F_c = F_{cp} \]

\[ -m r^2w^2 = -k \Delta r \]

\[ \Rightarrow 1 \times 0.5 \times \left( \frac{\pi}{2} \right)^2 = 10^3 \times \Delta r \]

\[ \Rightarrow \Delta r = 1.23 \times 10^{-3} \text{ m} \]

5-4

\[ r = 10 \text{ m} \]

\[ M = 0.3 \]

Clearly, to move on a curve, we need centripetal force which can only be provided by static friction otherwise car will skid!

\[ F_{fric} = F_c \]

\[ H(Mg) = \frac{MV^2}{r} \]

(where, \(M\) is mass of the car)

and \(H(Mg)\) is the maximum friction

\[ \Rightarrow V^2 = \frac{Mg r}{H} = 0.3 \times 10 \frac{m}{s^2} \times 10 \text{ m} \]

\[ = 30 \frac{m^2}{s^2} \]

\[ \Rightarrow V = 5.477 \frac{m}{s} \]

This is the maximum velocity with which we can drive without skidding.
5-5

\( r = 0.25 \, \text{m} \)

20 revolution in 1 min = 60 sec

\( \Rightarrow \) 1 revolution in 60 sec = 3 sec

\( \Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{3} = 2.093 \, \text{rad/sec} \)

\( \Rightarrow \) Centripetal force \( F_c = mr\omega^2 \) (where \( m \) is the mass of bob)

Let \( T \) be the tension

Clearly, using force balance

in \( \hat{y} \) direction: \( T \cos \theta = mg \)

\[ \Rightarrow T = \frac{mg}{\cos \theta} \]

in \( \hat{x} \) direction: the component of tension in this direction provides the centripetal force

\[ \Rightarrow T \sin \theta = mr\omega^2 \]

\[ \Rightarrow \tan \theta = \frac{r\omega^2}{g} \]

\[ = \frac{0.25 \times (2.093)^2}{10} = 0.11 \]

\( \Rightarrow \theta = \tan^{-1}(0.11) = 6.3^\circ \)

5-6

Forces on a person on equator are

\( N \hat{r} \): normal reaction radially outward

\(-Mg \hat{r} \): Force of attraction due to gravity radially inward.

Apparent weight \( \Rightarrow N_{\hat{r}} \) [Weighting machine measures \( N_{\hat{r}} \)]

Now \( -(N_{\hat{r}} - Mg) \hat{r} \) is the net inwards radial force. This provides the centripetal force.
\[ \Rightarrow Mg - N_R = MR_e \omega_e^2 \]

\[ N_R = M(g - R_e \omega_e^2) \]

Now for earth's rotation; \( R_e \omega_e^2 = 0.03 \, \frac{m}{s^2} \) at equator

(Check Text on 'Some Consequences of Earth's Rotation')

If we want apparent weight to be zero (i.e. feel weightlessness), then \( N_R = 0 \Rightarrow g = R_e \omega_f^2 \) (where \( \omega_f \) is the velocity with which earth should rotate for feeling weightlessness)

\[ \Rightarrow R_e \omega_f^2 = 9.8 \, \frac{m}{s^2} \]

\[ \frac{R_e \omega_f^2}{R_e \omega_e^2} = \frac{9.8}{0.03} \Rightarrow \omega_f = 18.07 \, \omega_e \]

\[ \therefore \text{earth should rotate } 18.07 \text{ times faster} \]

5-7

I'll lose contact if my apparent weight becomes zero.

Forces acting on me:

\[ N_R \hat{a} \rightarrow \text{normal force radially outwards} \]
\[-Mg \hat{a} \rightarrow \text{gravitational force radially inwards} \]

The resultant force provides centripetal force

\[ (N_R - Mg) \hat{a} = -\frac{MV^2}{R} \hat{a} \]

\[ N_R = 0 \Rightarrow -Mg = -\frac{MV_{\max}^2}{R} \]
\[ \Rightarrow V_{\max} = \sqrt{gR} = \sqrt{9.8 \times 20} = 14 \, \text{m/s} \]

5-8

In this case, centripetal force is \( -\frac{MV_{\max}^2}{R} \hat{a} \)

\[ N_R - Mg = +\frac{MV_{\max}^2}{R} \]

\[ N_R = Mg + \frac{MV_{\max}^2}{R} \]
\[ = 2Mg \]
5-9 Let man of mass \( M \) kg. \( V = 60 \) mph = \( 60 \times 0.447 \) m/s = \( 26.82 \) m/s.
\( r = 300 \) m.

Let's balance the forces acting on the car.

**y-direction:** \( \text{Ncos} \theta \hat{y} \) and \(-\text{Mg} \hat{y}\) as there is no acceleration in \( \hat{y} \)-direction

\[(\text{Ncos} \theta - \text{Mg}) \hat{y} = 0\]

\[\Rightarrow \text{Ncos} \theta = \text{Mg}. \quad (\text{I})\]

**x-direction:** The component of \( N \) provides centripetal acceleration.

\[\text{Nsin} \theta = \frac{MV^2}{r}\]

Substituting \( N \) from eqn (I)

\[\frac{\text{Mg}}{\cos \theta} \cdot \sin \theta = \frac{MV^2}{r}\]

\[\Rightarrow \tan \theta = \frac{V^2}{mg}\]

\[= \frac{(26.82)^2}{300 \times 9.8} = 0.2446\]

\[\Rightarrow \theta = 13.8^\circ\]

5-10 \( M = 0.5 \) kg.
\( T = 1.2 \) sec.

Both strings are of 1 m. Clearly, the figure is an isosceles triangle and hence both strings make equal angles with the vertical.

\( \Rightarrow \) Vertical force is zero on the mass

\[\Rightarrow T_1 \cos \theta = T_2 \cos \theta\]

\[\Rightarrow T_1 = T_2\]
Horizontal force \( = 2T \sin \theta \)

This provides the centripetal force.

\[
2T \sin \theta = \frac{MV^2}{R}.
\]

From the figure, clearly \( R = \frac{1}{\sin \theta} \)

\[
= (\sin \theta) m
\]

\[
\therefore \quad 2T \sin \theta = \frac{MV^2}{R} = \frac{MV^2}{\sin \theta}
\]

\[
\Rightarrow \quad T = \frac{MV^2}{2 \sin^2 \theta} \quad - (1)
\]

Now, the time period of rotation \( = T = 1.2 \) sec.

\[
T = \frac{2 \pi}{\omega} = \frac{2 \pi}{V} = \frac{2 \pi R}{V} = \frac{2 \pi \sin \theta}{V}
\]

\[
\Rightarrow \quad V = \frac{2 \pi \sin \theta}{T} \quad - (2)
\]

Substituting (2) in (1)

\[
T = \frac{M \cdot 4 \pi^2 \sin^2 \theta}{2 \sin^2 \theta \cdot T^2} = \frac{M \cdot 2 \pi^2}{T^2}
\]

\[
= \frac{0.5 \times 2 \times (3.14)^2}{(1.2)^2}
\]

\[= 6.84 \text{ N} \]

This is the tension in each string.

\[
F = 10^{-10} \text{ N}
\]

\[
m = 1 \text{ kg m}
\]

\[
R = 0.5 \text{ m}
\]

Viscous force provides centripetal force:

\[
F = \frac{MV^2}{R} = m \omega^2
\]

\[
10^{-10} = 10^6 \times 0.5 \times \omega^2
\]

\[
\Rightarrow \quad \omega = 0.0141 \text{ rad/sec}
\]

\[
T = \frac{2 \pi}{\omega} = 445.7 \text{ sec}
\]
At any point of time, the centripetal force is always perpendicular to the displacement. Hence, it does not do any work.