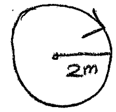
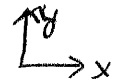
Week ⑤ - Solutions





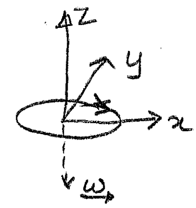
5-1 i) 30 revolutions $\rightarrow$ 1 min = 60 sec

Let 1 revolution $\rightarrow$ x sec

$$\Rightarrow \frac{x}{1} = \frac{60}{30} \Rightarrow x = 2\,\text{sec}$$

$\therefore$ period = 2sec

ii) since the motion is clockwise; $\vec{\omega}$ is negative.

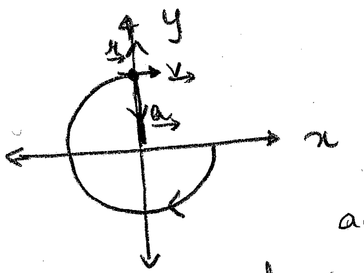


$$\vec{\omega} = -\frac{2\pi}{T} = \frac{-2\pi\,\text{rad}}{2\,\text{sec}} = -3.14\ \text{rad/sec}\ \hat{z}$$

(direction is obtained using right hand thumb rule)

iii) at $t = 1.5$ sec;

clearly $\frac{1.5}{T} = \frac{1.5}{2} = \frac{3}{4}$ revolution and at $t=0$; $\vec{r} = 2m\,\hat{x}$

$\Rightarrow$ 

$\vec{r} = 2m\,\hat{y}$

velocity is tangential

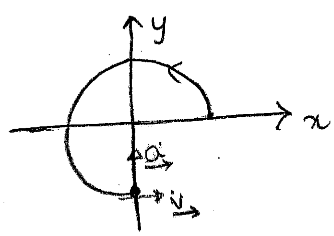
$\Rightarrow \vec{v} = |r\omega|\,\hat{x} = 2\pi\ m/s\ \hat{x}$

acceleration is always towards centre and hence points in a direction opposite to $\vec{r}$.

$$\vec{a} = -|r\omega^2|\,\hat{y} = -(2\pi^2)\ m/s^2\ \hat{y}$$

5-2 i) Time period = 2sec

ii) $\vec{\omega} = \frac{2\pi}{T} = 3.14\ \frac{\text{rad}}{s}\ \hat{z}$

iii) 

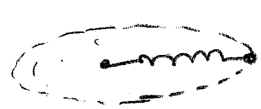
clearly $\vec{r} = -2m\,\hat{y}$; $\vec{v} = 2\pi\ m/s\ \hat{x}$ (tangential)

$\vec{a} = 2\pi^2\ m/s^2\ \hat{y}$ (towards centre)

5-3 $m = 1\,kg$, $r = 0.5\,m$

15 revolutions in 1 min $\Rightarrow$ one revolution in $\frac{60\,\text{sec}}{15} = 4$ sec

$$\Rightarrow \omega = \frac{2\pi}{T} = \frac{\pi}{2}\ \text{rad/sec}$$



As the mass is rotating ; there should be a centripetal force which is clearly provided by the spring. It should be pointing towards centre

$$\vec{F_{sp}} = -k \Delta r \, \hat{r}$$

for this force to be directed towards $-\hat{r}$; $k\Delta r > 0$

$\Rightarrow \quad \Delta r > 0$

$\Rightarrow$ spring has to strech

$K = 10^3 \frac{N}{m}$.

Now $\quad \vec{F_c} = \vec{F_{sp}}$

$$-mrw^2 = -k\Delta r$$

$$\Rightarrow \quad 1 \times 0.5 \times \left(\frac{\pi}{2}\right)^2 = 10^3 \times \Delta r$$

$$\Rightarrow \quad \Delta r = 1.23 \times 10^{-3} \, m$$

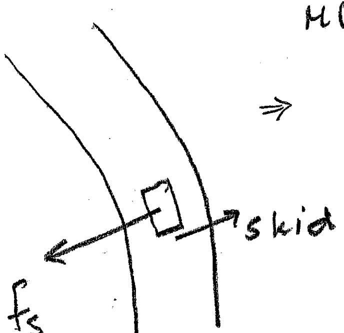
5-4 $\quad r = 10m$

$\mu = 0.3$

Clearly ; to move on a curve, we need centripetal force which can only be provided by static friction otherwise car will skid!

$\therefore \quad \vec{F_{fric}} = \vec{F_c}$

$\mu(Mg) = \frac{Mv^2}{r}$ (where M is mass of the car) and $\mu(Mg)$ is the maximum friction



$\Rightarrow \quad v^2 = \mu g r = 0.3 \times 10 \frac{m}{s^2} \times 10m$

$= 30 \frac{m^2}{s^2}$

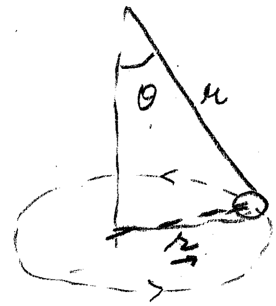
$\Rightarrow \quad v = 5.477 \frac{m}{s}$.

This is the maximum velocity with which we can drive without skidding

5-5 $r = 0.25\ m$

20 revolution in 1 min (= 60 sec)

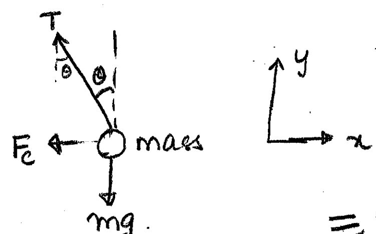
$\Rightarrow$ 1 revolution in $\frac{60\ sec}{20\ rev} = 3\ sec$



$\Rightarrow \underset{\rightarrow}{\omega} = \frac{2\pi}{T} = \frac{2\pi}{3} = 2.093\ \frac{rad}{sec}$

$\Rightarrow$ centripetal force $F_c = mr\omega^2$ (where m is the mass of bob)

Let T be the tension



clearly, using force balance

$\equiv$ m along Y

$\underset{\rightarrow}{a}_c$ along $-\hat{r}$

in $\hat{y}$ direction : $T\cos\theta = mg$

$\Rightarrow T = \frac{mg}{\cos\theta}$

in $\hat{r}$ direction : the component of tension in this direction provides the centripetal force

$\Rightarrow T\sin\theta = mr\omega^2$

$\frac{mg}{\cos\theta}\sin\theta = mr\omega^2$

$\Rightarrow \tan\theta = \frac{r\omega^2}{g}$

$= \frac{0.25 \times (2.093)^2}{10} = 0.11$

$\Rightarrow \theta = \tan^{-1}(0.11)$

$= 6.3^\circ$

5-6 Forces on a person on equator are

$N\hat{r}$: normal reaction radially outward

$-Mg\hat{r}$: Force of attraction due to gravity radially inward.

Apparent weight $= N_R$ [Weighing machine measures N_R]

Now $-(N_R - Mg)\hat{r}$ is the net inwards radial force. This provides the centripetal force.

$$\Rightarrow \quad Mg - N_R = M R_E \omega_E^2$$

$$N_R = M(g - R_E\omega_E^2)$$

Now for earth's rotation; $R_E\omega_E^2 = 0.03 \ \frac{m}{s^2}$ at equator

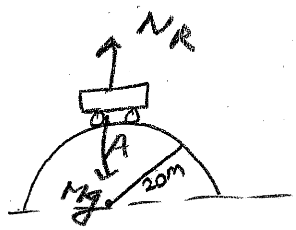
(Check Text on 'Some Consequences of Earth's Rotation')

If we want apparent weight to be zero (ie feel weightlessness), then $N_R = 0 \Rightarrow g = R_E\omega_F^2$ (where ω_F is the velocity with which earth should rotate for feeling weightlessness)

$$\Rightarrow R_E\omega_F^2 = 9.8 \ \frac{m}{s^2}$$

$$\frac{R_E\omega_F^2}{R_E\omega_E^2} = \frac{9.8}{0.03} \Rightarrow \omega_F = 18.07\,\omega_E$$

$\therefore$ earth should rotate 18.07 times faster



5-7 I'll lose contact if my apparent weight becomes zero.

Forces acting on me:

$N_R\hat{r}$ → normal force radially outwards

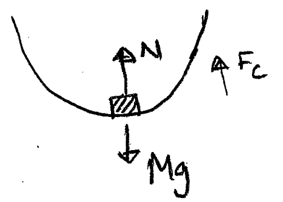
$-Mg\hat{r}$ → gravitational force radially inwards.

The resultant force provides centripetal force

$$(N_R - Mg)\hat{r} = -\frac{MV^2}{R}\hat{r}$$

$$N_R = 0 \Rightarrow -Mg = -\frac{MV_{max}^2}{R}$$

$$\Rightarrow V_{max} = \sqrt{gR} = \sqrt{9.8 \times 20} = 14 \ m/s$$



5-8 In this case, centripetal force is $-\frac{MV_{max}^2}{R}\hat{r}$

$$N_R - Mg = +\frac{MV_{max}^2}{R}$$

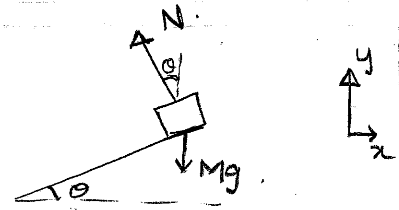
$$N_R = Mg + \frac{MV_{max}^2}{R}$$

$$= 2Mg$$

5-9 Let mass of my car be M kg.

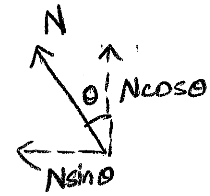
$v = 60\ mph = 60 \times 0.447\ m/s = 26.82\ m/s$.

$r = 300m$



Let's balance the forces acting on the car.

y-direction:– $N\cos\theta\,\hat{y}$ and $-Mg\,\hat{y}$



as there is no acceleration in $\hat{y}$-direction

$$(N\cos\theta - Mg)\hat{y} = 0$$

$$\Rightarrow N\cos\theta = Mg. \quad — (1)$$

x-direction :– The component of N provides centripetal acceleration

$$N\sin\theta = \frac{Mv^2}{r}$$

substituting N from eqn (1)

$$\frac{Mg}{\cos\theta} \cdot \sin\theta = \frac{Mv^2}{r}$$

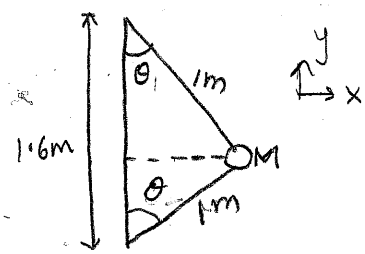
$$\Rightarrow \tan\theta = \frac{v^2}{rg}.$$

$$= \frac{(26.82)^2}{300 \times 9.8} = 0.2446$$

$$\Rightarrow \theta = 13.8^\circ$$

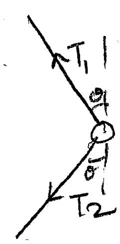
5-10 $M = 0.5\ kg$.

$T = 1.2\ sec$.



Both strings are of 1m.

clearly, the figure is an isosceles triangle and hence both strings make equal angle with the vertical.



∴ vertical force is zero on the mass

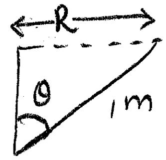
$$\Rightarrow T_1\cos\theta = T_2\cos\theta$$

$$\Rightarrow T_1 = T_2.$$

Horizontal force $= 2T_1 \sin\theta$

This provides the centripetal force.

$$\Rightarrow 2T_1 \sin\theta = \frac{Mv^2}{R}$$



From the figure; clearly $R = 1 \times \sin\theta$ m $= (\sin\theta)$ m.

$$\therefore \quad 2T_1 \sin\theta = \frac{Mv^2}{R} = \frac{Mv^2}{\sin\theta}$$

$$\Rightarrow \quad T_1 = \frac{Mv^2}{2\sin^2\theta} \quad \text{— (1)}$$

Now as time period of rotation $= T = 1.2$ sec.

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\left(\frac{V}{R}\right)} = \frac{2\pi R}{V} = \frac{2\pi \sin\theta}{V}$$

$$\Rightarrow \quad V = \frac{2\pi \sin\theta}{T} \quad \text{— (2)}$$

substituting (2) in (1)

$$T_1 = \frac{M \cdot 4\pi^2 \sin^2\theta}{2\sin^2\theta \cdot T^2} = \frac{M \cdot 2\pi^2}{T^2}$$

$$= \frac{0.5 \times 2 \times (3.14)^2}{(1.2)^2}$$

$$= 6.84 \text{N}$$

This is the tension in each string.

5-11 $\quad F = 10^{-10}$ N

$m = 1$ Mgm

$r = 0.5$ m.

viscous force provides centripetal force.

$$F = \frac{mv^2}{r} = mr\omega^2$$

$$10^{-10} = 10^{-6} \times 0.5 \times \omega^2$$

$$\Rightarrow \omega = 0.0141 \text{ rad/sec}$$

$$T = \frac{2\pi}{\omega} = 445.7 \text{ sec}$$

rps $n_s = \frac{1}{T} = 2.24 \times 10^{-3}$ per sec.

5-12 At any point of time; the centripetal force is always perpendicular to the displacement. Hence, it does not do any work.