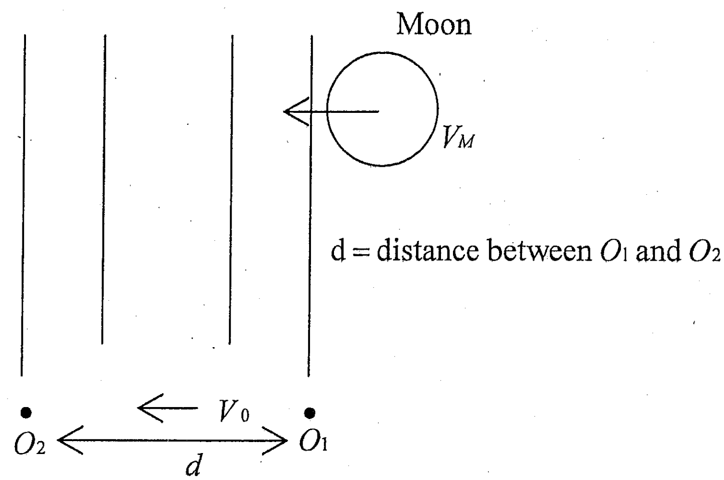
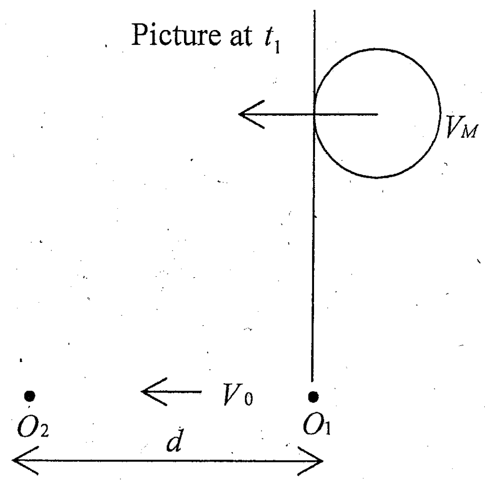
WEEK-4 SOLUTIONS

4-1 Speed and Size of Moon:
To access speed of moon we need two observers to go out at midnight on a full moon night and observe a star such that the moon intercepts the light from the star. Star is very far so light from it is a parallel beam. Both observers on same latitude so both have same velocity V_0 due to Earth's rotation.

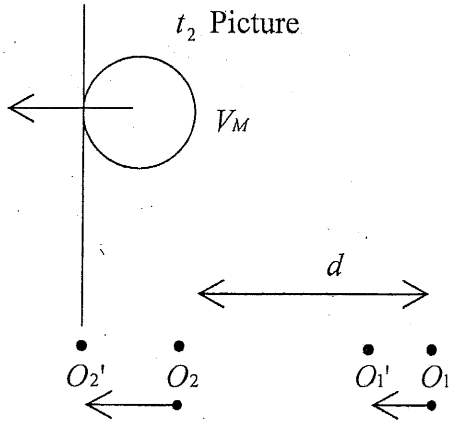
The picture is



At time t_1 moon intercepts light from star as seen by O_1



At time t_2 moon intercepts light from star as seen by O_2



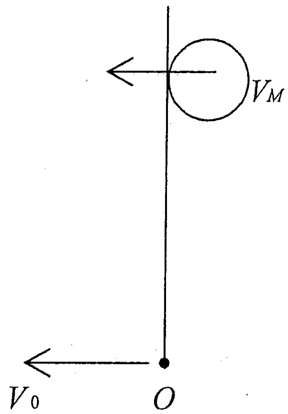
$O_2'O_2 = O_1'O_1$ = distance travelled by observer due to motion of Earth
Hence

$$V_M(t_2 - t_1) = d + V_0(t_2 - t_1)$$

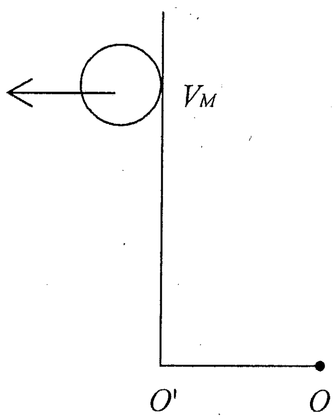
Speed of moon $\quad V_M = \dfrac{d}{(t_2 - t_1)} + V_0$

Once we know V_M a single observer can "measure" diameter of moon.

Again, concentrate on light from a star being intercepted by moon.



at t_d light just disappears



at t_A light just appears

$OO' = V_0(t_A - t_d)$

Distance moved by moon = $d_M + V_0(t_A - t_d)$
Where d_M = diameter of moon

$$V_M(t_A - t_d) = d_M + V_0(t_A - t_d)$$
$$d_M = (V_M - V_0)(t_A - t_d)$$

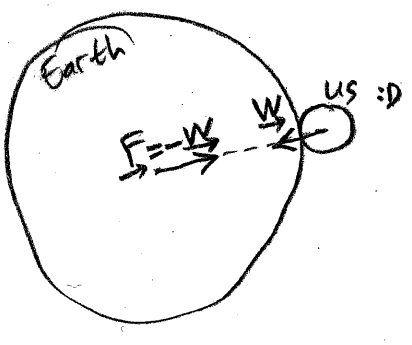
Which will allow us to measure d_M.

② A) No, because the mass does not depend on the gravitational acceleration or other external quantity; it is an intrinsic quantity.

B) My weight on the moon is

$$\underrightarrow{W} = m\, g_{moon}$$
$$= (50\ kg) \times \left(-1.65\ \frac{m}{s^2}\right) \hat{r}$$
$$= -82.5\ N\ \hat{r}$$

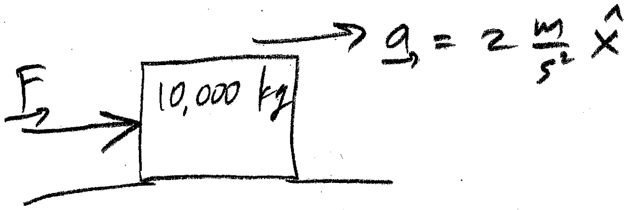
③



We pull on Earth with the force $\underrightarrow{F} = -\underrightarrow{W} = Mg\vec{r}$, due to the third Newton's law. Since the Earth pulls us with force $\underrightarrow{W}$, then it means that we pull on Earth with force $-\underrightarrow{W}$.

The force $\underrightarrow{F}$ $(=-\underrightarrow{W})$ acts on the center of Earth's mass.

④

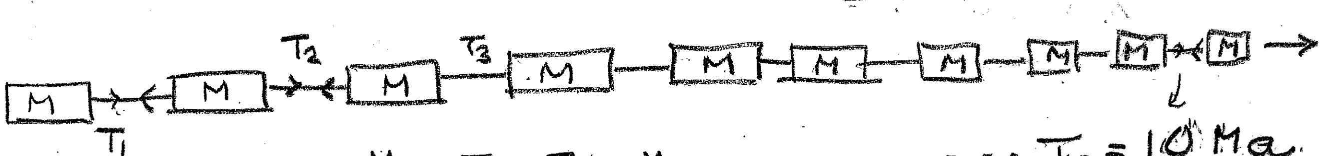


$M\underrightarrow{a} = \sum \underrightarrow{F_i}$ at that pt. at that time!

Let's apply Newton's law: $\underrightarrow{F} = m\underrightarrow{a}$

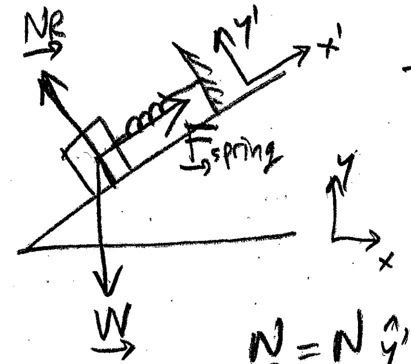
$$= (10^4\ kg)\left(2\ \frac{m}{s^2}\hat{x}\right) = 2\times10^4 N\hat{x}$$

Explain



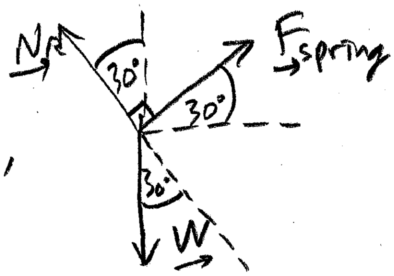
$T_1 = Ma \quad T_2 - T_1 = Ma, \quad T_3 - T_2 = Ma \ldots \ldots \quad T_{10} = 10\, Ma$

⑤



There are 3 forces that work on the block. Let's redraw it.

$$\vec{N} = N\,\hat{y}'$$
$$\vec{W} = -W_{x'}\,\hat{x}' - W_{y'}\,\hat{y}'$$



(spring is stretched along $-\hat{x}'$) so

$$\vec{F}_{spring} = +k\Delta x\,\hat{x}'$$

(i) By projecting the forces into the y'-direction (see picture), we have

$$\sum \vec{F}_{(\text{in } y'\text{-direction})} = \vec{0} \quad \text{(since the block is still)}$$

$$N_R\hat{y}' - W_{y'}\,\hat{y}' = \vec{0}$$

$$N_R\hat{y}' = W_{y'}\,\hat{y}'$$

$$N_R = mg\cos 30°$$

$$= (100\text{ kg})(9.8\ \tfrac{m}{s^2})\,\tfrac{1}{2}\sqrt{3}$$

$$= 490\sqrt{3}\text{ Newton}$$

So $\vec{N}_R = 490\sqrt{3}\text{ Newton}\ \hat{y}'$.

(ii) Let's project the forces into the x'-direction.

$$\sum \vec{F}_{(\text{in } x'\text{-direction})} = \vec{0} \qquad k = 10^4\,N/m$$

$$-W_{x'}\,\hat{x} + \vec{F}_{spring} = \vec{0}$$

$$\vec{F}_{spring} = W_{x'}\,\hat{x}'$$

$$k\Delta x\,\hat{x} = mg\sin 30°\,\hat{x}'$$

$$(10^4\ N/m)\,\Delta x = (100\text{ kg})(9.8\ \tfrac{m}{s^2})\,\tfrac{1}{2}$$

$$\Delta x = 4.9 \times 10^{-2}\text{ m}$$

$$= 4.9\text{ cm}$$

(iii) If we cut the spring, then $\underset{\rightarrow}{F}_{spring}$ would be zero. So the only force along the plane (x'-direction) is the projection of $\underset{\rightarrow}{W}$, i.e. $-W_{x'}\,\hat{x}'$. So, by using Newton's law,

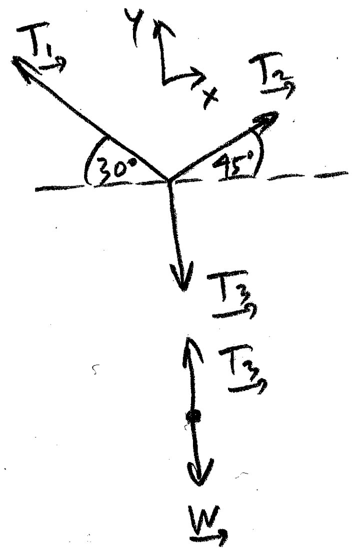
$$\Sigma \underset{\rightarrow}{F} = m\underset{\rightarrow}{a}$$

$$-W_{x'}\,\hat{x}' = m\underset{\rightarrow}{a}$$

$$\underset{\rightarrow}{a} = -\frac{mg\sin 30^\circ}{m}\,\hat{x}'$$

$$= -4.9\ \frac{m}{s^2}\,\hat{x}'.$$

(6)



For the first drawing, we have

$$\Sigma \underset{\rightarrow}{F_i} = \underset{\rightarrow}{0}$$

In x-direction, $T_1\cos 30^\circ = T_2 \cos 45^\circ$

$$T_1 \tfrac{1}{2}\sqrt{3} = T_2 \tfrac{1}{2}\sqrt{2}$$

$$T_1\sqrt{3} = T_2\sqrt{2}$$

In y-direction, $T_1\sin 30^\circ + T_2 \sin 45^\circ = T_3$.

$$T_1 \tfrac{1}{2} + T_2 \tfrac{1}{2}\sqrt{2} = T_3.$$

$$T_1 + T_2\sqrt{2} = 2T_3.$$

For the second drawing, we have

$$\Sigma \underset{\rightarrow}{F_i} = \underset{\rightarrow}{0}$$

In y-direction, $T_3 = W$

$$= mg$$

$$= (10\ kg)(9.8\ \tfrac{m}{s^2})$$

$$= 98 \text{ Newton}.$$

So, we have these three equations:

$$T_1\sqrt{3} = T_2\sqrt{2}$$

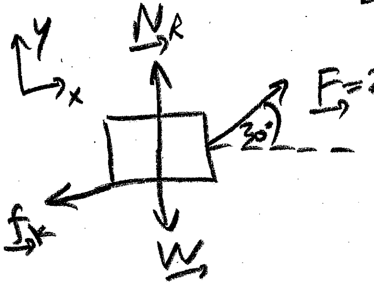
$$T_1 + T_2\sqrt{2} = 2T_3$$

$$T_3 = 98 \text{ Newton}$$

So we can write

$$T_1 + T_1\sqrt{3} = 2T_3$$
$$T_1(1+\sqrt{3}) = 2T_3$$
$$T_1 = \frac{2}{1+\sqrt{3}}\,(98 \text{ Newton})$$
$$= 71.74 \text{ Newton}$$

$$T_2 = T_1\sqrt{\frac{3}{2}} = 87.86 \text{ Newton.}$$

(7)


Use Newton's law. speed is const. Mass m ≡ m!

$$\sum \underset{\rightarrow}{F_i} = \underset{\rightarrow}{0}$$

In x-direction, $F\cos 30° - f_k = 0$

In y-direction, $F\sin 30° + N_R = W$

$$f_k = F\cos 30° = (20\ N)\,\tfrac{1}{2}\sqrt{3} = 10\sqrt{3} \text{ Newton.}$$

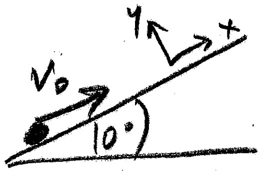
So $\underset{\rightarrow}{f_k} = -\dfrac{20\times\sqrt{3}}{2}\,\hat{x} = -17.32N\,\hat{x}$

In y dir. $N_R = Mg - F\sin 30 = 5\times 9.8 - 10 = 39N$

$$\mu_k = \frac{17.32}{39} = 0.444$$

If object is stationary $f_s \leq \mu_s N_R \leq (f_s)_{max} = 19.5N$

$(F\cos 30) \leq \mu_s N_R$ then $\underset{\rightarrow}{F}$ will not be able to move the mass.

(8)


The acceleration along the plane is

$$\underset{\rightarrow}{a} = -g\sin 10°\,\hat{x}.$$

Or, $a = g\sin 10°$. (the magnitude of $\underset{\rightarrow}{a}$).

$$15\,km/h = \frac{15000}{3600} = 4.17\,m/s$$

At the maximum point,

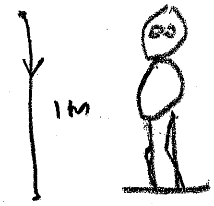
$$v^2 = V_0^2 - 2as$$
$$0 = V_0^2 - 2as$$
$$s = \frac{V_0^2}{2a} = \frac{(15\ km/h)^2}{2\times(9.8\ \frac{m}{s^2})\times\sin 10°} = 5.1\ m.$$

(9) Her speed before touching the ground can be calculated as follows.

$$v^2 = v_0^2 + 2gh \; ; \qquad h = 1 \text{ m}$$

$$= 0 + 2(9.8 \text{ m/s}^2)(1 \text{ m})$$

$$v = 4.43 \text{ m/s}$$



(i) Then, for the next 20 cm, the acceleration on her torso is assumed to be constant with value calculated as below.

$$0 = v^2 + 2as \; ; \qquad s = 20 \text{ cm}$$

$$19.6 \text{ m}^2/\text{s}^2 = 2a\left(\tfrac{20}{100} \text{ m}\right)$$

$$\underset{\rightarrow}{a} = -49 \text{ m/s}^2 \, \hat{y}$$

So the upward force on her torso is $F = ma = (50 \text{ kg})(49 \text{ m/s}^2)\hat{y}$

$$\underset{\rightarrow}{F} = 2450 \text{ Newton } \hat{y}$$

(ii) If $s = 4$ cm, then $0 = v^2 + 2as$

$$19.6 \text{ m}^2/\text{s}^2 = 2a\left(\tfrac{4}{100} \text{ m}\right)$$

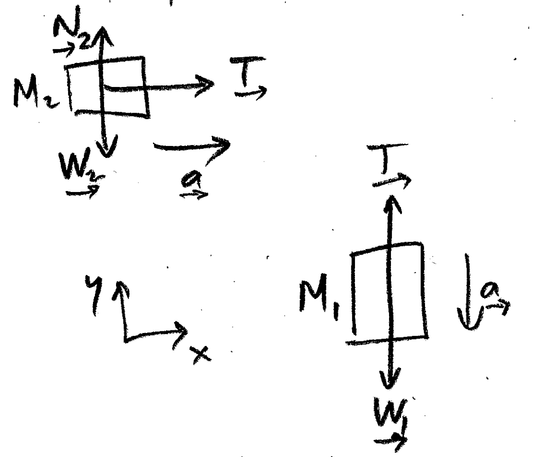
$$\underset{\rightarrow}{a} = -245 \text{ m/s}^2 \, \hat{y}$$

So the upward force on her torso is $\underset{\rightarrow}{F} = ma = (50 \text{ kg})(245 \text{ m/s}^2)\hat{y}$

$$= 12250 \text{ Newton } \hat{y}$$

(Cracked Ribs!)

(10) (i) If the table is frictionless,



$$\Sigma \underset{\rightarrow}{F} = m\underset{\rightarrow}{a}$$

For M_2, in x-direction,

$$T = M_2 a$$

For M_1, in y-direction,

$$T - W_1 = M_1(-a)$$

Substitute for T, we have $M_2 a - W_1 = -M_1 a$, or

$$(M_1+M_2)a = W_1 = M_1 g$$

$$a = \frac{M_1}{M_1+M_2} g$$

$$= \frac{2\,kg}{2\,kg + 5\,kg} 9.8\ m/s^2$$

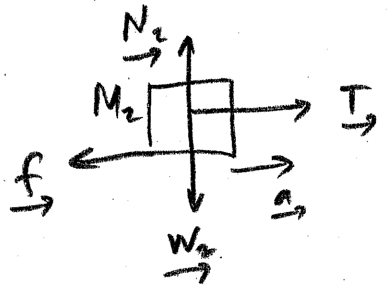
$$= 2.8\ m/s^2$$

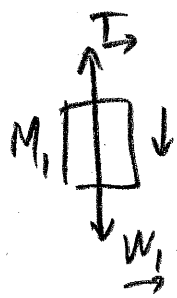
$$T = M_2 a$$

$$= (5 \times 2.8) N$$

$$= 14 N$$

(ii) If the table has $\mu_k = 0.15$,





$$\Sigma \vec{F} = M\vec{a}$$

For M_2, in x-direction,

$$T - f = M_2 a$$

For M_2, in y-direction,

$$N_2 - W_2 = 0$$

For M_1, in y-direction,

$$T - W_1 = M_1(-a)$$

Since $f = \mu_k N_2$, we will have

$$T - \mu_k N_2 = M_2 a$$

$$(W_1 - M_1 a) - \mu_k W_2 = M_2 a$$

$$M_1 g - \mu_k M_2 g = (M_1 + M_2) a$$

$$a = \frac{M_1 - \mu_k M_2}{M_1 + M_2} g$$

$$= \frac{2\,kg - 0.15 \times 5\,kg}{2\,kg + 5\,kg} 9.8\ m/s^2$$

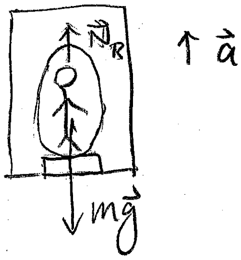
$$= 1.75\ m/s^2$$

$$T = M_2 a + \mu_k M_2 g = 5(1.75 + 0.15 \times 9.8)$$

$$= 16.1 N$$

4-11

The weight of the person is equal to the magnitude of the normal force N_R due to the weighing machine.



Forces on Person

$\uparrow N_R$

$\downarrow Mg$

Newton's law

$M\vec{a} = (N_R - Mg)\hat{y}$

at that pt. at that time.

(i) If the elevator is at rest, we have

$\vec{F} = \vec{N}\hat{y} - mg\hat{y} = 0$

$|\vec{N}| = mg = 588\ N$ #

(ii) constant velocity of $20\hat{y}$ m/s $\Rightarrow \vec{a} = 0$

$\vec{N} = 588\ N$

(iii) const. acceleration $\vec{a} = 0.2\ m/s^2\ \hat{y}$

$\vec{F} = \vec{N}\hat{y} - mg\hat{y} = m\vec{a} \Rightarrow \vec{N} = 60 \times (9.8 + 0.2)\hat{y} = 600\,N\ \hat{y}$

$|\vec{N}| = 600\,N$ #

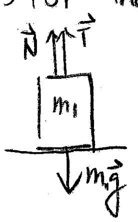
(iv) $\vec{a} = -0.2\ m/s^2\ \hat{y}$

$\vec{F} = \vec{N}\hat{y} - mg\hat{y} = m(-0.2)\hat{y}$

$|\vec{N}| = (60) \times (9.8 - 0.2) = 576\ N$ #

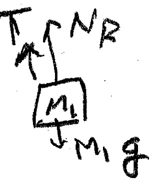
4-12

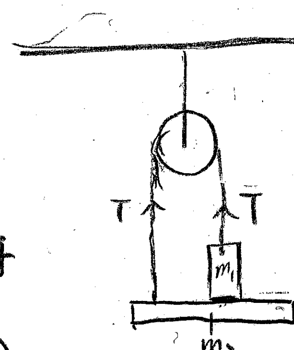
(i) For the painter (m_1), the force diagram is



$\because \vec{a} = 0$, $\vec{F}_{net} = 0$

$\Rightarrow \hat{y}: N_R + T - m_1 g = 0$ —①





No acc.

$\uparrow 2T$

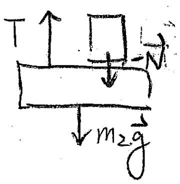
$\downarrow\downarrow (m_1+m_2)g$

w/ the acc

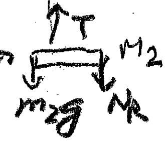
$\uparrow 2T$ $\uparrow \vec{a}$

$\downarrow\downarrow$

For the platform (m_2), the force diagram is $(\vec{a} = 0)$



$\Rightarrow \hat{y}: T = m_2 g + N_R = m_2 g + m_1 g - T$ —②



$T = \frac{1}{2}(m_1 + m_2)g = 441\ N$ #

(ii) If the painter has $\vec{a} = 0.4\ m/s^2$, eqs. ① ② can be written as

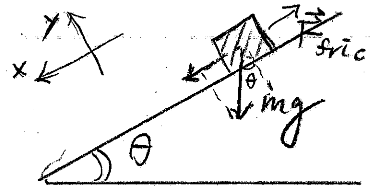
Eq ① $\Rightarrow$ $\begin{cases} N + T - m_1 g = m_1 a \\ T - N - m_2 g = m_2 a \end{cases}$ $\Rightarrow$ ① + ② $\Rightarrow 2T - (m_1 + m_2)g = (m_1 + m_2)a$

Eq ② $\Rightarrow$

$T = \frac{1}{2}(m_1 + m_2)(g + a)$

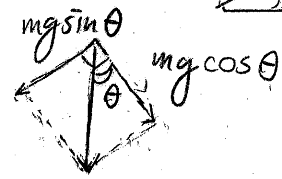
$= \frac{1}{2}(75 + 15)(9.8 + 0.4) = 459\ N$ #

4-13



$M_s = 0.5$ (coefficient of max. static friction)

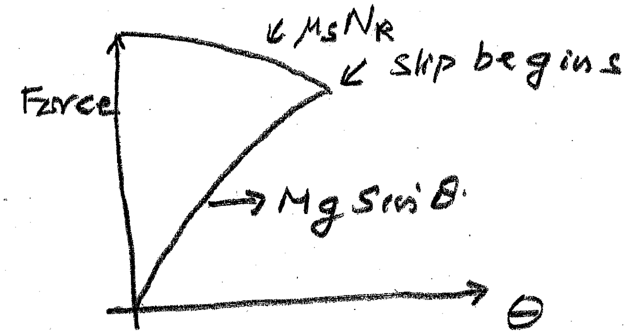
We can separate the forces in to. x, y components



$$\begin{cases} \text{x-direction}: \ mg\sin\theta\hat{x} - f_s\hat{x} = 0 \\ \text{y-direction}: \ N\hat{y} - mg\cos\theta\hat{y} = 0 \end{cases}$$

Before the block slides, the friction will reach its maximum value

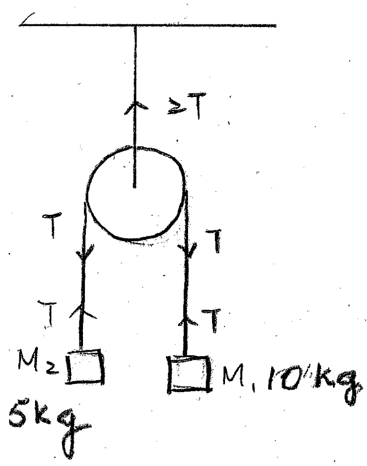
$\vec{F} = -M_s N_R \hat{x}$



$$\Rightarrow \begin{cases} N_R = mg\cos\theta \\ mg\sin\theta = M_s N_R = M_s mg\cos\theta \end{cases}$$

$\Rightarrow \tan\theta = M_s = 0.5 \Rightarrow \underline{\theta = 26.6^\circ}$ #

4-14



(i) For M_1, M_2, we have the Newton's eqs.

$$\begin{cases} T\hat{y} - M_1 g\hat{y} = -M_1 a\hat{y} \quad \text{—①} & , \ \vec{a}_1 = -a\hat{y} \\ T\hat{y} - M_2 g\hat{y} = M_2 a\hat{y} \quad \text{—②} & , \ \vec{a}_2 = a\hat{y} \end{cases}$$

eq② − eq①

$\Rightarrow (M_1 - M_2)g = (M_1 + M_2)a$

$\Rightarrow \vec{a} = \dfrac{(M_1 - M_2)}{(M_1 + M_2)} g = \underline{3.267 \ m/s^2} \Rightarrow \begin{cases} \vec{a}_1 = -3.267 \ m/s^2 \ \hat{y} \\ \vec{a}_2 = 3.267 \ m/s^2 \ \hat{y} \end{cases}$

(ii) From ①, we have

$T = M_1(g - a) = 10[9.8 - 3.27] = 65.3 N$

(iii) Since the net force on the pulley is zero, we can obtain

$\vec{T}_{rope} = 2T\hat{y} = \underline{130.6 N}$ #

4-15

For 1-dim. motion, we have the equation for displacement Δy

$$\Delta y = v_0 t + \frac{1}{2} a t^2$$

In this case, $v_0 = 0$, $t = 2s$, $\vec{a}_2 = 3.267\ m/s^2\ \hat{y}$

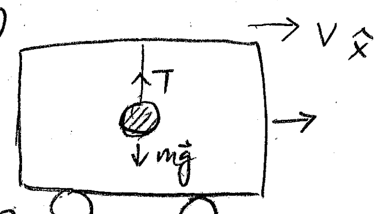
$$\Delta y = \frac{1}{2} \times (3.267\ m/s^2)(2s)^2 = \underline{6.534\ m}$$

M_2 will move $6.534\ m$ upwards.

4-16

(1) If the car moves at a uniform velocity $V\hat{x}$, there's no acceleration for the pendulum in the x direction, so it will also move with the velocity $V\hat{x}$ as shown in Fig. ①.

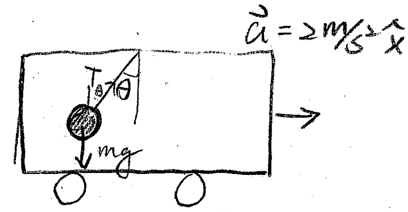
Fig ①



And, $\vec{F} = T\hat{y} - mg\hat{y} = 0$

(2) If the car has a uniform acceleration $\vec{a} = 2\ m/s^2\ \hat{x}$, the ~~mass~~ will also have the same acceleration in the x-direction.

Fig ②



$$\vec{a} = 2\ m/s^2\ \hat{x}$$

The Newton's eqs would be

$M\vec{a} = \sum \vec{F}_i$ at that pt. that time

$$\begin{cases} \text{x-direction}: \vec{F}_x = T\sin\theta\ \hat{x} = m \times (2\ m/s^2)\ \hat{x} \quad \text{— ①} \\ \text{y-direction}: \vec{F}_y = T\cos\theta\ \hat{y} - mg\ \hat{y} = 0 \end{cases}$$

$$\Rightarrow T\cos\theta = mg \quad \text{— ②}$$

Thus, we have $\frac{①}{②}$

$$\Rightarrow \tan\theta = \frac{2}{g} = 0.2041$$

$$\underline{\theta = 11.54^\circ}$$

Note In case 2 this is no longer an inertial system. As soon as you see that the pendulum is not vertical you know that system must be accelerating.