\[ \begin{align*}
\text{i,} & \quad v_i = 20 \text{ m/s} \hat{y} \\
& \quad a = -9.8 \text{ m/s}^2 \hat{y} \\
\text{velocity and acceleration are in the opposite direction. Hence, the ball slows down, then comes to}
\text{a halt and finally falls back towards the ground.}
\end{align*} \]

\[ \begin{align*}
\text{ii,} & \quad a = -9.8 \text{ m/s}^2 \hat{y} \\
\text{The acceleration of the ball is constant throughout the journey.}
\end{align*} \]

\[ \begin{align*}
\text{iii,} & \quad \text{At the highest point, velocity} \quad v_f = 0 \\
& \quad v_i = 20 \text{ m/s} \hat{y} \\
& \quad a = -9.8 \text{ m/s}^2 \hat{y} \\
\text{using} \quad v_f^2 - v_i^2 = 2ay; \quad \text{we get}
\end{align*} \]

\[ 0 - (20)^2 = 2 \times (-9.8 \text{ m/s}^2) \times y_{\text{max}} \]

\[ y_{\text{max}} = 20.408 \text{ m} \]

\[ \begin{align*}
\text{iv,} & \quad \text{Let total time of flight be} \quad t \text{ s} \\
\text{let time for upward journey be} \quad t_1 \text{ s}; \quad \text{and time for}
\text{downward journey be} \quad t_2 \text{ s}. \\
\text{Then} \quad t = t_1 + t_2 \text{.}
\end{align*} \]
upward journey:

\[ v_f = v_i + at_1 \]
\[ 0 \text{ m/s} = 20 \text{ m/s} - 9.8 \frac{\text{m}}{\text{s}^2} \cdot t_1 \]
\[ \Rightarrow t_1 = 2.04 \text{ s} \]

downward journey:

\[ y_{\text{max}} = v_i t_2 + \frac{1}{2} a t_2^2 \]

For downward journey: \( v_i = 0 \)

\[ y_{\text{max}} = \frac{1}{2} a t_2^2 \]
\[ 20.408 \text{ m} = \frac{1}{2} \times 9.8 \frac{\text{m}}{\text{s}^2} \times t_2^2 \]
\[ \Rightarrow t_2 = 2.04 \text{ s} \]

\[ t = t_1 + t_2 = 4.08 \text{ s} \]

Note that \( t_1 = t_2 \).

\( \text{V} \) let's consider the downward journey.

\[ v_i = 0 \frac{\text{m}}{\text{s}} \]
\[ a = 9.8 \frac{\text{m}}{\text{s}^2} \]
\[ y_{\text{max}} = 20.408 \text{ m} \]

using \( v_f^2 - v_i^2 = 2ay_{\text{max}} \), we get

\[ v_f^2 = 2 \times 9.8 \frac{\text{m}}{\text{s}^2} \times 20.408 \text{ m} \]
\[ \Rightarrow v_f = 20 \frac{\text{m}}{\text{s}} \]

\[ v_f = -20 \text{ m/s} \] \( \text{oppo} \)

Note that this is exactly the velocity with which the ball was thrown up.
let subscript 1 denote quantities from problem 1 and subscript 2 denote the new quantities.

\[ a_1 = a_2 = -9.8 \, \text{m/s}^2 \, \hat{y} = -g \hat{y} \quad (1) \]

At the highest point: \( V_{f1} = V_{f2} = 0 \, \text{m/s} \quad (2) \)

let the maximum heights be \( y_{\text{max}-1} \) and \( y_{\text{max}-2} \) — (3)

let initial velocities be \( V_{i1} \) and \( V_{i2} \) — (4)

we need to find \( \frac{V_{i2}}{V_{i1}} \) when we have \( y_{\text{max}-2} = 3y_{\text{max}-1} \)

Now

\[ V_{f1}^2 - V_{i1}^2 = 2a_1(y_{\text{max}-1}) \]

\[ 0 - V_{i1}^2 = 2(-9) \cdot y_{\text{max}-1} \quad (5) \]

and \( V_{f2}^2 - V_{i2}^2 = 2a_2(y_{\text{max}-2}) \)

\[ 0 - V_{i2}^2 = 2(-9) \cdot y_{\text{max}-2} \quad (6) \]

Dividing \( \frac{(5)}{(6)} \):

\[ \frac{-V_{i1}^2}{-V_{i2}^2} = \frac{2(-9)y_{\text{max}-1}}{2(-9)y_{\text{max}-2}} \]

\[ \left( \frac{V_{i1}}{V_{i2}} \right)^2 = \left( \frac{y_{\text{max}-1}}{y_{\text{max}-2}} \right) = \frac{1}{3} \]

\[ \left( \frac{V_{i1}}{V_{i2}} \right)^2 = \left( \frac{y_{\text{max}-1}}{y_{\text{max}-2}} \right) = \frac{1}{3} \]

\[ \Rightarrow \quad \frac{V_{i2}^2}{V_{i1}^2} = \frac{3}{1} \]

\[ \text{or} \quad V_{i1}^2 = \sqrt{3} \cdot V_{i2}^2 \]

\[ \Rightarrow \quad \frac{V_{i2}}{V_{i1}} = \sqrt{3} \]

Thus, I need to increase the initial velocity by a factor of \( \sqrt{3} \).
Let ball 1 be thrown upwards from point A with $V_{i1} = 15 \, \text{m/s} \, \hat{y}$.

Let ball 2 be thrown downwards from point B ($V_{i2} = 0 \, \text{m/s} \, \hat{y}$).

Let the balls meet at point C.

Clearly, $AC + CB = 20 \, \text{m}$

Let $AC = 't' \, \text{m}$. $\Rightarrow EC = (20 - 't') \, \text{m}$

and let the balls pass one another after 't' s.

Clearly, $a_1 = a_2 = -9.8 \, \text{m/s}^2 \, \hat{y}$

Using $y_f - y_i = V_i t + \frac{1}{2}at^2$; we get

ball 1: $h = 15t - \frac{1}{2} \times 9.8 \times t^2$ — (1)

ball 2: $h - 20 = \frac{1}{2} \times (-9.8) \times t^2$ — (2)

Substituting (2) in (1):—

$20 - \frac{1}{2} \times 9.8 \times t^2 = 15t - \frac{1}{2} \times 9.8 \times t^2$

$\Rightarrow t = \frac{20}{15} = 1.33 \, \text{s}$

$t = 1.33 \, \text{s}$

Substituting this value in (2), we get

$h = 11.288 \, \text{m}$

Hence, the balls meet after $1.33 \, \text{s}$ at a height of 11.288 m from the ground.
3-4 Let the keys have an initial velocity

\[ \text{acceleration } a = -9.8 \text{ m/s}^2 \]

\[ y_f = v_i t + \frac{1}{2} at^2 \]

\[ y_f = 0 \text{ m \hat{y}} \quad y_f = 6 \text{ m \hat{y}} \]

Using \( y_f - y_i = v_i t + \frac{1}{2} at^2 \)

\[ (6-0) = (v_i \text{ m/s})(3) + \frac{1}{2} \times (-9.8 \text{ m/s}^2) \times (3)^2 \]

\[ \Rightarrow v_i = -16.7 \text{ m/s \hat{y}} \rightarrow \text{This is the velocity with which Jenny throws} \]

Using \( v_f^2 - v_i^2 = 2a(y_f - y_i) \)

\[ v_f^2 = (-16.7)^2 + 2 \times (-9.8) \times (6-0) \]

\[ \Rightarrow v_f = -12.7 \text{ m/s \hat{y}} \rightarrow \text{This is the velocity with which Jill catches} \]

Be careful: \( v_f = (16.7 - 3 \times 9.8) \text{ m/s \hat{y}} \) so \( v_f \) is downward.

3-5 Both the balls hit the water at the same time.

Ball 1 travels for \( t \) s.

Ball 2 travels for \( (t-1) \) s.

Both balls travel a distance of 20 m.

\[ v_i = 0 \quad v_i = -v \text{ m/s \hat{y}} \]

acceleration \( = -9.8 \text{ m/s}^2 \)

Ball 1:

\[ y_f - y_i = v_i t + \frac{1}{2} at^2 \]

\[ 0 - 20 \text{ m} = 0 - \frac{1}{2} \times 9.8 \text{ m/s}^2 \times t^2 \]

\[ \Rightarrow t = 2 \text{ s} \]

Ball 2:

\[ -20 \text{ m} = -v(t-1) - \frac{1}{2} \times 9.8 \text{ m/s}^2 \times (t-1)^2 \]

\[ \Rightarrow v = -15.1 \text{ m/s \hat{y}} \]
\[ y_i = 50 \text{ m} \hat{y} \]

\[ v_i = \text{some reason} \]

\[ a_t = -9.8 \frac{\text{m}}{\text{s}^2} \hat{y} \]

\[ v_f = 0 \frac{\text{m}}{\text{s}} \text{ at highest point} ; \quad v_i = 20 \frac{\text{m}}{\text{s}} \hat{y} \]

\[ v_f^2 - v_i^2 = 2a(y_f - y_i) \]

\[ 0^2 - (20)^2 = 2 \times (-9.8) \times (y_f - 50) \]

\[ y_f - 50 = 20.408 \text{ m} \]

\[ y_f = 70.408 \text{ m} \hat{y} \rightarrow \text{maximum height} \]

\[ v_f^2 - v_i^2 = 2a(y_f - y_i) \]

\[ v_f^2 - (20)^2 = 2 \times (-9.8) \times (0 - 50) \]

\[ v_f = -37.148 \frac{\text{m}}{\text{s}} \hat{y} \]

This is the velocity with which ball hits the ground.

iv) We'll divide the journey into 2 parts

1- Ball leaving for upward journey and returning back to top of the Tower
2- Ball continuing downward journey from top of tower to ground.

We've already calculated time taken for part 1 and final velocity at the end of part 1. (See 3-i)

This will be our initial velocity for part 2. (See 3-i)

For part 2:

\[ v_f^2 - v_i^2 = 2a(y_f - y_i) \]

\[ v_f^2 - (20)^2 = 2 \times (-9.8) \times (0 - 50) \]

\[ v_f = -37.148 \frac{\text{m}}{\text{s}} \hat{y} \]

This is the velocity with which ball hits the ground.
Time for part 2:

\[ V_f = V_i + at \]
\[-37.148 = -20 + (-9.8)t \]
\[ \Rightarrow t = 1.75 s \]

\[ \text{total time for journey} = 4.08 + 1.75 = 5.83 s \]

3-7 Let the height of the house above the window be \( h \) m.

Let the velocity of flower pot at the top of the window be \((-V \frac{m}{s}) \hat{y}\). It of course starts falling with a zero initial velocity.

For the length of the window

\[ t = 0.55 \]
\[ V_f - V_i = -2 \text{ m} \]
\[ a = -9.8 \frac{m}{s^2} \hat{y} \]

Using \[ V_f - V_i = V_i t + \frac{1}{2} at^2 \] we get

\[ -2 = -V \times 0.5 + \frac{1}{2} (-9.8)(0.5)^2 \]
\[ \Rightarrow V = -1.55 \frac{m}{s} \hat{y} \]

\[ \therefore \text{velocity of flower pot at top of the window} \]

\[ V = -1.55 \frac{m}{s} \hat{y} \]

Now for the length from top of building to top of the window:

\[ V_f^2 - V_i^2 = 2a(y_f - y_i) \]
\[ (1.55)^2 - 0^2 = 2 \times (-9.8) \times (-h) \]
\[ \Rightarrow h = 0.1226 \text{ m} \]

This is the required height.
Breaking the initial velocity vector into components, we have

\[ V_{x0} = V_0 \cos \theta_0 \text{ m/s } \hat{x} \]
\[ V_{y0} = V_0 \sin \theta_0 \text{ m/s } \hat{y} \]

\[ a = -9.8 \text{ m/s}^2 \hat{y} \]

Using \[ y_f - y_i = v_{yi}t + \frac{1}{2}at^2 \]
and from your textbook on 'Kinematics in 2-dimensions':

Eqn 3 \[ y = (v_i \sin \theta_i) t - 4.9t^2 \]
\[ = v_0 \sin \theta_0 t - 4.9t^2 \] — (A)

Now, let the \( x \)-component of position be \( x \)

Then \[ x = v_0 \cos \theta_0 t \] (again from eqn 3)

\[ \Rightarrow t = \frac{x}{v_0 \cos \theta_0} \]

Substituting this in eqn (A), we obtain

\[ y = \frac{v_0 \sin \theta_0}{v_0 \cos \theta_0} \left( \frac{x}{v_0 \cos \theta_0} \right) - 4.9 \left( \frac{x}{v_0 \cos \theta_0} \right)^2 \]

or \[ y = x \tan \theta_0 - 4.9 \left( \frac{x}{v_0 \cos \theta_0} \right)^2 \]

3-9 at end
$v_i = 20 \text{ m/s} \hat{x} = v_{ix} \hat{x} + v_{iy} \hat{y}$

$y = 10 \text{ m}$

Time taken for downward journey:

$y_f - y_i = v_{iy} t + \frac{1}{2} g t^2$

$-10 \text{ m} = 0 + \frac{1}{2} (-9.8 \text{ m/s}^2) t^2$

$\Rightarrow t = 1.48 \text{ s}$

In this time, horizontal distance travelled is

$x_f - x_i = v_{ix} t + \frac{1}{2} a x t^2$

$x_f = 20 \times 1.48 + 0 \quad \left(\because a_x = 0 \text{ m/s}^2\right)$

$= 28.6 \text{ m}$

Clearly he misses his target by $(40 - 28.6) \text{ m} = 11.4 \text{ m}$

3-11

$\vec{V} = v_{i \cos \theta} \hat{x} + v_{i \sin \theta} \hat{y}$

To calculate range, we need time of flight. Using $y_f - y_i = v_{iy} t + \frac{1}{2} a t^2$

$0 = v_{i \sin \theta} t - \frac{1}{2} g t^2 \quad \left(\because a = -g\right)$

$\Rightarrow t = \frac{2v_{i \sin \theta}}{g}$

In this time; Range $R = \frac{v_{i \cos \theta} \cdot 2v_{i \sin \theta}}{g} = \frac{v_{i}^2 \sin 2\theta}{g}$

Now $\sin[2(\frac{\pi}{2} - \theta)] = 2 \sin(\frac{\pi}{2} - \theta) \cos(\frac{\pi}{2} - \theta) = 2 \cos \theta \sin \theta = \sin 2\theta$

$\Rightarrow$ for 2 angles $\theta$ and $(\frac{\pi}{2} - \theta)$; we get same value of sine.

$\therefore$ There are two launch angles with same range.
The initial velocity of the ball is
\[ \vec{v}_i = v_i \cos \theta \hat{i} + v_i \sin \theta \hat{j} \]
\[ = 30 \cos 37^\circ \hat{i} + 30 \sin 37^\circ \hat{j} \]
\[ = 24 \text{ m/s} \hat{i} + 18 \text{ m/s} \hat{j} \]

The time after which the ball is 14 m high is:
\[ y_f - y_i = v_{yi} t + \frac{1}{2} a t^2 \]
\[ 14 = 18 t - \frac{1}{2} \times 10 t^2 \]
\[ 5t^2 - 36t + 28 = 0 \]
\[ t = \frac{-36 \pm \sqrt{36^2 - 4 \times 28 \times 5}}{2 \times 5} = \frac{36 \pm 27.13}{10} = 0.8878 \text{ s, 6.31 s} \]

We obtain 2 times for the same displacement:
- \( t = 0.8878 \text{ s} \) is the time when displacement is 14 m and ball continues moving upward.
- \( t = 6.31 \text{ s} \) is the time when displacement is 14 m and ball is on its return journey.

In between \( t_1 \) and \( t_2 \), ball stays higher than 14 m.

Now, the time taken for the ball to cover 30 m horizontally is
\[ t_3 = \frac{d}{v} = \frac{30}{24} = 1.25 \text{ s} \]

Thus, \( t_1 < t_3 < t_2 \Rightarrow \) the ball clears the wall.

The y-component of velocity at this time is:
\[ v_{yf} = v_{yi} + at \]
\[ = 18 - 9.8 \times 1.25 \]
\[ = 5.75 \text{ m/s} \]

\[ \therefore \text{velocity is } \overrightarrow{v} = 24 \text{ m/s} \hat{i} + 5.75 \text{ m/s} \hat{j} \]
\[ 3-13 \text{ in downstream: my motion is in the same direction as river flow} \]

\[ \vec{V}_d = 600 + 500 \]
\[ = 1100 \text{ m/hr} \]

\[ \text{ii, upstream: my motion is opposite to the flow of the river} \]
\[ \vec{V}_u = -600 + 500 \]
\[ = -100 \text{ m/hr} \]

\[ \text{iii, time for the trip} \]
\[ t_d = \frac{d}{V_d} = \frac{100 \text{ m}}{1100 \text{ m/hr}} = 0.09 \text{ hr} \]

\[ t_u = \frac{d}{V_u} = \frac{-100 \text{ m}}{-100 \text{ m/hr}} = 1 \text{ hr} \]

\[ \therefore \text{ total time} = t_u + t_d = 1.09 \text{ hr or 65.4 min} \]
I will fly in a direction as shown in the figure such that my resultant velocity points towards west.

my velocity $\vec{V}_a = -250 \cos \theta \hat{x} + 250 \sin \theta \hat{y}$

wind velocity $\vec{V}_w = -50 \hat{y}$

I want ground velocity only along $-\hat{x}$ $\Rightarrow$ y-component of velocity $= 0$

$250 \sin \theta = 50$

$\Rightarrow \quad \theta = 11.54^\circ$

$\Rightarrow$ Net velocity $\vec{V}_{net} = \vec{V}_a + \vec{V}_w$

$= -250 \cos (11.54^\circ) \hat{x}$

$= -250 \times 0.9798 \hat{x}$

$= -244.95 \text{ mph} \hat{x}$

$\Rightarrow$ Time taken for travel $t = \frac{d}{u} = \frac{200}{244.95} = 0.8165 \text{ hr} = 49 \text{ min}$
Inertial observer is one whose co-ordinate system moves at a constant velocity (i.e., both magnitude and direction of velocity are constant).

\[ V_i = 90 \text{ m/s} \]

\[ = 90 \times 0.447 \text{ m/s} \]

\[ = 40.23 \text{ m/s} \]

Time taken to travel 18 m is

\[ t = \frac{18 \text{ m}}{40.23 \text{ m/s}} = 0.447 \text{ sec} \]

In this time, vertical drop is:

\[ y_f - y_i = V_{iy}t + \frac{1}{2}ayt^2 \]

\[ = 0 + \frac{1}{2} \times (-9.8) \times (0.447)^2 \]

\[ = -2.19 \text{ m} \]

\[ \therefore \text{ ball drops by } 2.19 \text{ m} \]