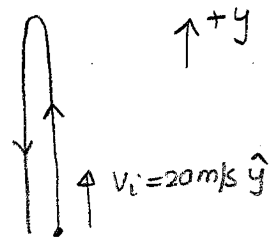
Week ③ - Solutions

3-1



i, $V_i = 20 \text{ m/s } \hat{y}$

$a = -9.8 \text{ m/s}^2 \ \hat{y}$

velocity and acceleration are in the opposite direction. Hence, the ball slows down, then comes to a halt and finally falls back towards the ground.

ii, $a = -9.8 \text{ m/s}^2 \ \hat{y}$

The acceleration of the ball is constant through out the journey.

iii, At the highest point ; velocity = 0

$V_i = 20 \text{ m/s } \hat{y}$

$V_f = 0$

$a = -9.8 \text{ m/s}^2 \ \hat{y}$

using $V_f^2 - V_i^2 = 2ay$; we get

$$0 - \left(20 \tfrac{m}{s}\right)^2 = 2 \times \left(-9.8 \tfrac{m}{s^2}\right) \times y_{max}$$

or $\boxed{y_{max} = 20.408 \text{ m}}$

(iv) Let total time of flight be 't' s.
Let time for upward journey be 't_1' s and time for downward journey be 't_2' s.

Then $t = t_1 + t_2$.

upward journey:

$$V_f = V_i + at_1$$

$$0\,\frac{m}{s} = 20\,\frac{m}{s} - 9.8\,\frac{m}{s^2}\cdot t_1$$

$$\Rightarrow t_1 \doteq 2.04\,s$$

downward journey:

$$y_{max} = V_i t_2 + \frac{1}{2}at_2^2$$

For downward journey ; $V_i = 0$

$$\therefore \quad y_{max} = \frac{1}{2}at_2^2$$

$$20.408\,m = \frac{1}{2}\times 9.8\,\frac{m}{s^2}\times t_2^2$$

$$\Rightarrow t_2 = 2.04\,s.$$

$$\therefore \quad \boxed{t = t_1 + t_2 = 4.08\,s}$$

Note that $t_1 = t_2$.

(V) Let's consider the downward journey.

$$V_i = 0\,\frac{m}{s}$$

$$a = 9.8\,\frac{m}{s^2}$$

$$y_{max} = 20.408\,m$$

using $V_f^2 - V_i^2 = 2a\,y_{max}$, we get

$$V_f^2 = 2\times 9.8\,\frac{m}{s^2}\times 20.408\,m$$

$$\Rightarrow \boxed{V_f = 20\,\frac{m}{s}}$$

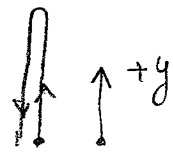
$$\vec{V}_f = -20\,m/s\,\hat{x}$$

Note that this is exactly opposite the velocity with which the ball was thrown up.

3-2

Let subscript 1 denote quantities from problem 1 and subscript 2 denote the new quantities.

$$\vec{a}_1 = \vec{a}_2 = -9.8 \frac{m}{s^2}\hat{y} = -g\hat{y} \quad \text{— (1)}$$



At the highest point ; $V_{f1} = V_{f2} = 0 \frac{m}{s}$ — (2)

Let the maximum heights be y_{max-1} and y_{max-2} — (3)

Let initial velocities be V_{i1} and V_{i2} — (4)

we need to find $\frac{V_{i2}}{V_{i1}}$ when we have $y_{max-2} = 3y_{max-1}$

Now $V_{f1}^2 - V_{i1}^2 = 2a_1 y_{max-1}$

$$0 - V_{i1}^2 = 2(-g)\cdot y_{max-1} \quad \text{— (5)}$$

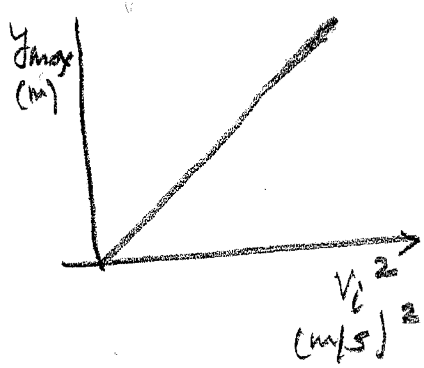
and $V_{f2}^2 - V_{i2}^2 = 2a_2 y_{max-2}$

$$0 - V_{i2}^2 = 2(-g)\, y_{max-2} \quad \text{— (6)}$$

we have used eqn-(1)

Dividing $\frac{(5)}{(6)}$:- $\frac{-V_{i1}^2}{-V_{i2}^2} = \frac{2(-g)\,y_{max-1}}{2(-g)\,y_{max-2}}$

$$\left(\frac{V_{i1}}{V_{i2}}\right)^2 = \left(\frac{y_{max-1}}{y_{max-2}}\right) = \frac{1}{3}$$



$\Rightarrow$ $V_{i2}^2 = 3V_{i1}^2$

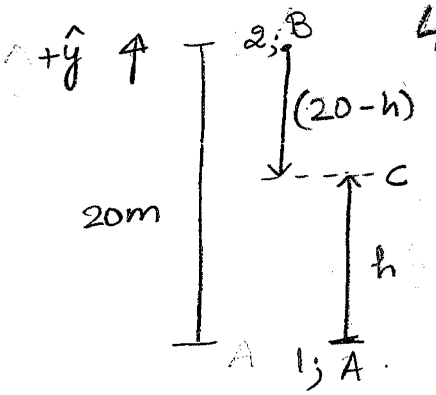
or $V_{i2} = \sqrt{3}\, V_{i1}$

$$\Rightarrow \boxed{\frac{V_{i2}}{V_{i1}} = \sqrt{3}}$$

Thus, I need to increase the initial velocity by factor of $\sqrt{3}$.

3-3

Let ball 1 be thrown upwards from point A with $V_{i1} = 15 \frac{m}{s} \hat{y}$.



Let ball 2 be thrown downwards from point B $(V_{i2} = 0 \frac{m}{s} \hat{y})$

Let the balls meet at point C.

Clearly; $AC + CB = 20m$

Let $AC =$ 'h' m. $\Rightarrow BC = (20-h)\,m$

and let the balls pass one another after 't' s.

Clearly $\vec{a_1} = \vec{a_2} = -9.8 \frac{m}{s^2} \hat{y}$

Using $y_f - y_i = v_i t + \frac{1}{2} a t^2$; we get

ball 1: $h = 15t - \frac{1}{2} \times 9.8 \times t^2$ — ①

ball 2: $h - 20 = \frac{1}{2} \times (-9.8) \times t^2$ — ②

substituting ② in ①:-

$$20 - \cancel{\frac{1}{2} \times 9.8 \times t^2} = 15t - \cancel{\frac{1}{2} \times 9.8 \times t^2}$$

$$\Rightarrow t = \frac{20}{15} = 1.33\,s$$

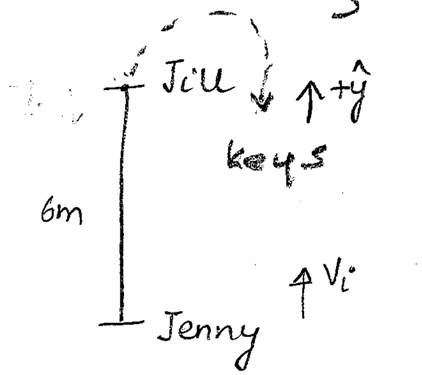
$$\boxed{t = 1.33s}$$

substituting this value in ②, we get

$$\boxed{h = 11.288\,m}$$

Hence, the balls meet after 1.33 s at a height of 11.288 m from the ground.

3-4 Let the keys have an initial velocity $= V_i \frac{m}{s}\hat{y}$



acceleration $\vec{a} = -9.8\, m/s^2\, \hat{y}$

Time of flight $= 3s$.

$y_i = 0m\,\hat{y}$; $y_f = 6m\,\hat{y}$

using $\quad y_f - y_i = v_i t + \frac{1}{2}at^2$

$$(6-0)m = \left(V_i \tfrac{m}{s}\right)(3s) + \tfrac{1}{2}\times\left(-9.8\,\tfrac{m}{s^2}\right)\times(3s)^2$$

$\Rightarrow \boxed{V_i = -16.7\ \frac{m}{s}\hat{y}}$ → This is the velocity with which Jenny throws

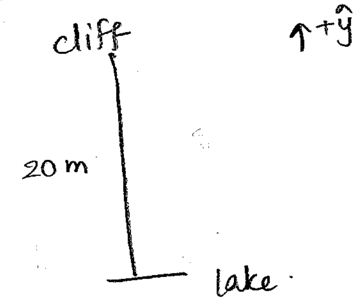
using $\quad V_f^2 - V_i^2 = 2a(y_f - y_i)$

$$V_f^2 = (16.7)^2 + 2\times(-9.8)\times(6-0)$$

$\Rightarrow \boxed{V_f = -12.7\ \frac{m}{s}\hat{y}}$ → This is the velocity with which Jill catches.

Be Careful: $\vec{V_f} = (16.7 - 3\times9.8)\,m/s\ \hat{y}$ so $\vec{V_f}$ is Downward.

3-5 Both the balls hit the water at the same time.



Ball 1 travels for 't' s

Ball 2 travels for '(t-1)' s.

Both balls travel a distance of 20 m.

$V_{i1} = 0$; $V_{i2} = -V\frac{m}{s}\cdot\hat{y}$

acceleration $= -9.8\,\frac{m}{s^2}$

Ball 1 : $\quad y_f - y_i = v_{i1}t + \frac{1}{2}at^2$

$$0m - 20m = 0 - \tfrac{1}{2}\times 9.8\,\tfrac{m}{s^2}\times t^2$$

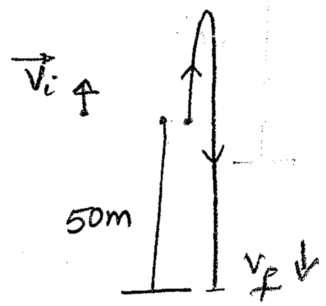
$\Rightarrow \boxed{t = 2s}$

Ball 2:- $\quad -20m = -V(t-1) - \frac{1}{2}\times9.8\times(t-1)^2$

$\Rightarrow \boxed{\vec{V} = -15.1\ \frac{m}{s}}\hat{y}$

<u>3-6</u> $y_i = 50\text{m } \hat{y}$

$\uparrow +\hat{y}$



i) Same reason.

ii) $\underset{\rightarrow}{a} = -9.8 \frac{m}{s^2} \hat{y}$

iii) $V_f = 0 \frac{m}{s}$ at highest point ; $V_i = 20 \text{ m/s } \hat{y}$

$$V_f^2 - V_i^2 = 2a(y_f - y_i)$$

$$0^2 - (20)^2 = 2 \times (-9.8) \times (y_f - 50)$$

$$\Rightarrow \quad y_f - 50 = 20.408 \text{ m}$$

$$\Rightarrow \boxed{y_f = 70.408 \text{ m } \hat{y}} \rightarrow \text{maximum height}$$

iv) and (v) We'll divide the journey into 2 parts

1- Ball leaving for upward journey and returning back to top of the Tower

2- Ball continuing downward journey from top of tower to ground.

We've already calculated time taken for part 1 and final velocity at the end of part 1. This will be our initial velocity for part 2. (See 3-1)

For part 2 : $V_f^2 - V_i^2 = 2a(y_f - y_i)$

$$V_f^2 - (20)^2 = 2 \times (-9.8)(0 - 50)$$

$$\Rightarrow \boxed{\underset{\rightarrow}{V_f} = -37.148 \frac{m}{s}} \hat{y}$$

This is the velocity with which ball hits the ground.

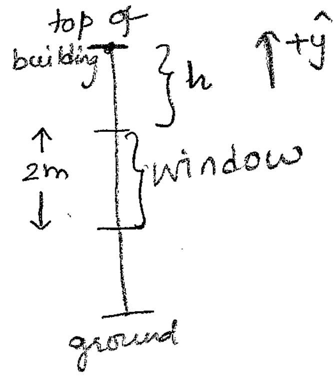
Time for part 2 :-

$$V_f = V_i + at$$

$$-37.148 = -20 + (-9.8)t$$

$$\Rightarrow t = 1.75\,s$$

$\therefore$ total time for journey $= 4.08 + 1.75 = \underline{\underline{5.83\,s}}$.



3-7 Let the height of the house above the window be 'h' m.

Let the velocity of flower pot at the top of the window be $\left(-V \frac{m}{s}\right)\hat{y}$. It ofcourse starts falling with a zero initial velocity

For the length of the window

$$t = 0.5\,s$$

$$y_f - y_i = -2\,m$$

$$\vec{a} = -9.8 \frac{m}{s^2}\hat{y}.$$

using $y_f - y_i = V_i t + \frac{1}{2}at^2$; we get

$$-2 = -V \times 0.5 + \frac{1}{2}(-9.8)(0.5)^2$$

$$\Rightarrow \vec{V} = -1.55 \frac{m}{s}\hat{y}$$

$\therefore$ velocity of flower pot at top of the window $= -1.55 \frac{m}{s}\hat{y}$

Now for the length from top of building to top of the window:-

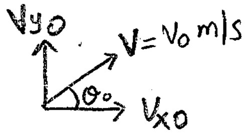
using
$$V_f^2 - V_i^2 = 2a(y_f - y_i)$$

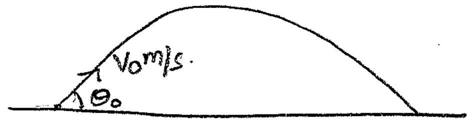
$$(1.55)^2 - 0^2 = 2 \times (-9.8) \times (-h)$$

$$\Rightarrow \boxed{h = 0.1226\,m}$$

This is the required height.

3-8





Breaking the initial velocity vector into components; we have

$$V_{xo} = V_o \cos\theta_o \ \frac{m}{s} \ \hat{x}$$

$$V_{yo} = V_o \sin\theta_o \ \frac{m}{s} \ \hat{y}$$

$$a = -9.8 \ \frac{m}{s^2} \ \hat{y}.$$

Using $y_f - y_i = v_{yi} t + \frac{1}{2} a_y t^2$
and from your text on 'kinematics in 2-dimension' :—
– Projectile motion

Eqn (3) $\rightarrow$ $y = (V_i \sin\theta_i)t - 4.9t^2$

$$= V_o \sin\theta_o t - 4.9t^2 \quad \text{— (A)}$$

Now, let the x-component of position be x

Then $x = v_o \cos\theta_o t$ (again from eqn (3)) $\because a_x = 0$

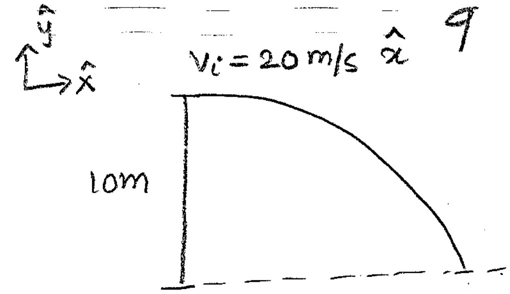
$$\Rightarrow t = \frac{x}{V_o \cos\theta_o}$$

substituting this in eqn (A); we obtain

$$y = V_o \sin\theta_o \left(\frac{x}{V_o \cos\theta_o}\right) - 4.9\left(\frac{x}{V_o \cos\theta_o}\right)^2$$

or $$y = x \tan\theta_o - 4.9\left(\frac{x}{V_o \cos\theta_o}\right)^2$$

3-9 at end



3-10 $\quad V_i = 20\ \frac{m}{s}\hat{x} = V_{ix}\hat{x} + V_{iy}\hat{y}$

$y = 10\ m$

Time taken for downward journey :-

$$y_f - y_i = V_{iy}t + \frac{1}{2}a_y t^2$$

$$-10\ m = 0 + \frac{1}{2}\left(-9.8\ \frac{m}{s^2}\right)t^2$$

$$\Rightarrow t = 1.43\ s$$

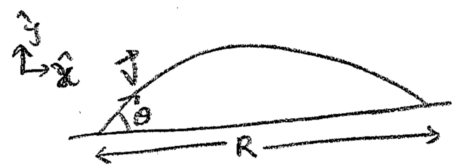
In this time ; horizontal distance travelled is

$$x_f - x_i = V_{ix}t + \frac{1}{2}a_x t^2$$

$$x_f = 20 \times 1.43 + 0 \qquad \left(\because a_x = 0\ \frac{m}{s^2}\right)$$

$$= 28.6\ m$$

Clearly he misses his target by $(40 - 28.6)m = \underline{11.4\ m}$



3-11 $\quad \vec{V} = v_i\cos\theta\,\hat{x} + v_i\sin\theta\,\hat{y}$

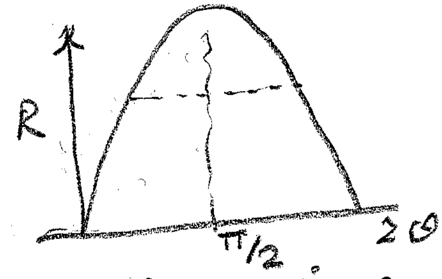
To calculate range ; we need time of flight. Using $y_f - y_i = v_{yi}t + \frac{1}{2}at^2$ $\qquad (\because a = -g)$

$$0 = v_i\sin\theta\, t - \frac{1}{2}gt^2$$

$$\Rightarrow t = \frac{2v_i\sin\theta}{g}$$

In this time ; Range $R = v_i\cos\theta \cdot \frac{2v_i\sin\theta}{g}$

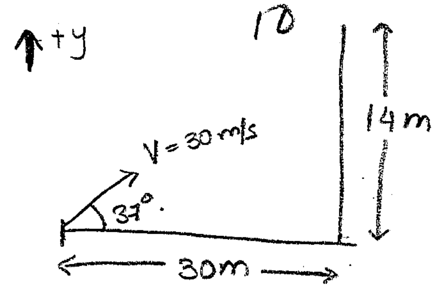
$$= \frac{v_i^2\sin 2\theta}{g}$$



Now $\sin\left[2\left(\frac{\pi}{2} - \theta\right)\right] = 2\sin\left(\frac{\pi}{2} - \theta\right)\cos\left(\frac{\pi}{2} - \theta\right) = 2\cdot\cos\theta\cdot\sin\theta = \sin 2\theta$

$\Rightarrow$ for 2 angles θ & $\left(\frac{\pi}{2} - \theta\right)$; we get same value of sine.

$\therefore$ There are two launch angles with same range.



3-12 The initial velocity of the ball is

$$\vec{V_i} = V_i \cos\theta_i \hat{x} + V_i \sin\theta_i \hat{y}$$
$$= 30 \cos 37^\circ \hat{x} + 30 \sin 37^\circ \hat{y}$$
$$= 30 \times \frac{4}{5} \hat{x} + 30 \times \frac{3}{5} \hat{y}$$
$$= 24 \frac{m}{s} \hat{x} + 18 \frac{m}{s} \hat{y}$$

The time after which the ball is 14 m high :-

$$y_f - y_i = V_{yi} t + \frac{1}{2} a t^2$$
$$14 = 18t - \frac{1}{2} \times 10 t^2 \qquad (\text{for simplicity } g = 10 \tfrac{m}{s^2} \text{ is assumed})$$
$$5t^2 - 36t + 28 = 0.$$
$$t = \frac{36 \pm \sqrt{36^2 - 4 \times 28 \times 5}}{2 \times 5} = \frac{36 \pm 27.13}{10} = 0.887s, 6.31s$$

We obtain 2 times for the same displacement.

$t_1 = 0.887s$ is the time when displacement is 14m and ball continues moving upward

$t_2 = 6.31s$ is the time when displacement is 14m and ball is on its return journey.

In between t_1 and t_2 ; ball stays higher than 14m.

Now, the time taken for the ball to cover 30m horizontally is

$$t_3 = \frac{d}{v} = \frac{30}{24} = 1.25s$$

$\because$ $t_1 < t_3 < t_2$ $\Rightarrow$ the ball clears the wall.

The y-component of velocity at this time is :

$$V_{yf} = V_{yi} + at$$
$$= 18 - 9.8 \times 1.25$$
$$= 5.75 \frac{m}{s}$$

$\therefore$ velocity is $\vec{V} = 24 \frac{m}{s} \hat{x} + 5.75 \frac{m}{s} \hat{y}$

3-13 (i) down stream: my motion is in the same direction as river flow

down $\longrightarrow$ $600 \frac{m}{hr}$

river $\longrightarrow$ $500 \frac{m}{hr}$

up $\longleftarrow$ $600 \frac{m}{hr}$

$\longrightarrow +x$

$$\therefore \vec{V_d} = 600 + 500$$
$$= 1100 \frac{m}{hr} \hat{x}$$

(ii) upstream: my motion is opposite to the flow of the river

$$\therefore \vec{V_u} = -600 + 500$$
$$= -100 \frac{m}{hr} \hat{x}$$

A $\xrightarrow{100\text{ m}}$ B down

A $\longleftarrow$ B up

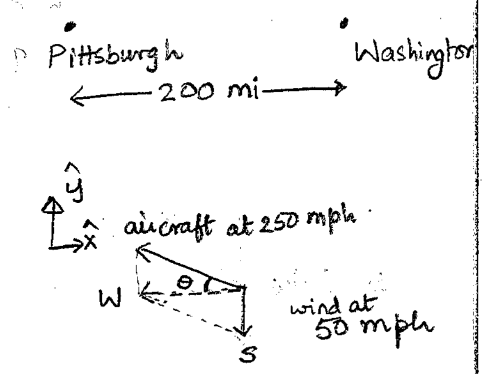
(iii) time for the trip.

$$t_d = \frac{d}{V_d} = \frac{100 \text{ m} \hat{x}}{1100 \frac{m}{hr} \hat{x}} = 0.09 \text{ hr}$$

$$t_u = \frac{d}{V_d} = \frac{-100 \text{ m} \hat{x}}{-100 \frac{m}{hr} \hat{x}} = 1 \text{ hr}$$

$$\therefore \text{total time} = t_u + t_d = 1.09 \text{ hr} \quad \text{or } 65.4 \text{ min}$$

3-14 I will fly in a direction as shown in the figure such that my resultant velocity points towards west.



my velocity $\vec{V}_{ac} = -250\cos\theta\,\hat{x} + 250\sin\theta\,\hat{y}$

wind velocity $\vec{V}_w = -50\,\hat{y}$

I want ground velocity only along $-\hat{x}$ (west) $\Rightarrow$ y-component of velocity $=0$

$250\sin\theta = 50$

$\Rightarrow \boxed{\theta = 11.54^\circ}$

$\Rightarrow$ Net velocity $\vec{V}_{net} = \vec{V}_{ac} + \vec{V}_w$

$= -250\times\cos(11.54^\circ)\,\hat{x}$

$= -250\times 0.9798\,\hat{x}$

$= -244.95$ mph $\hat{x}$

$\therefore$ time taken for travel $t = \frac{d}{v} = \frac{200}{244.95} = 0.8165$ hr

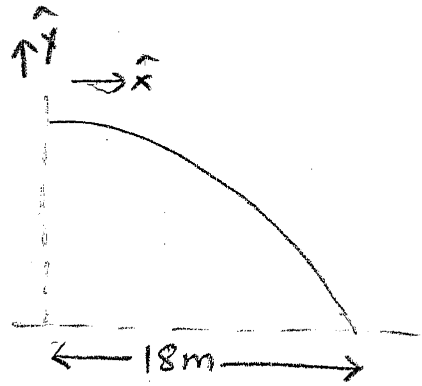
$= \underline{49 \text{ min}}$

3-15 Inertial observer is one whose co-ordinate system moves at a constant velocity. (i.e. both magnitude and direction of velocity are constant).

3-9 $V_i = 90\ \text{miph}\ \hat{x}$

$= 90 \times 0.447\ \frac{m}{s}\hat{x}$

$= 40.23\ \frac{m}{s}\hat{x}$



Time taken to travel 18m is

$$t = \frac{18\,m}{40.23\,\frac{m}{s}} = 0.447\ \text{sec}$$

In this time; vertical drop is :-

$$y_f - y_i = V_{iy}t + \frac{1}{2}a_y t^2$$

$$= 0 + \frac{1}{2}\times(-9.8)\times(0.447)^2$$

$$= -2.19\,m$$

$\therefore$ ball drops by 2.19 m