Week 2 - Solutions

(2-1) Let's convert the speed 60 mph into ft per second as follows.

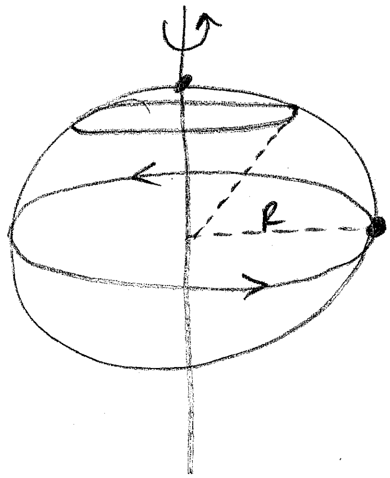
$$60 \text{ mph} = \frac{60 \text{ miles}}{1 \text{ hour}} = \frac{60 \times 5280 \text{ ft}}{3600 \text{ s}}$$

$$= \frac{5280}{60} \text{ ft/s}$$

$$= 88 \text{ ft/s}$$

So the speeds are the same.

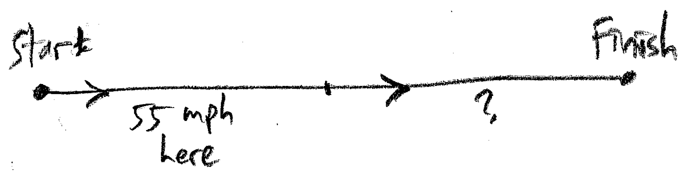
(2-2)



(i) The speed is the distance traveled divided by time. The maximum speed will be obtained if we stand on the Equator. In this case, $v = \frac{2\pi R}{t}$, where R is the radius of the Earth and $t = 24$ hours.

(ii) The speed is zero if we stand on the pole, since the distance traveled at that position as the Earth rotates is zero.

(2-3)



Since the automobile must finish the track in

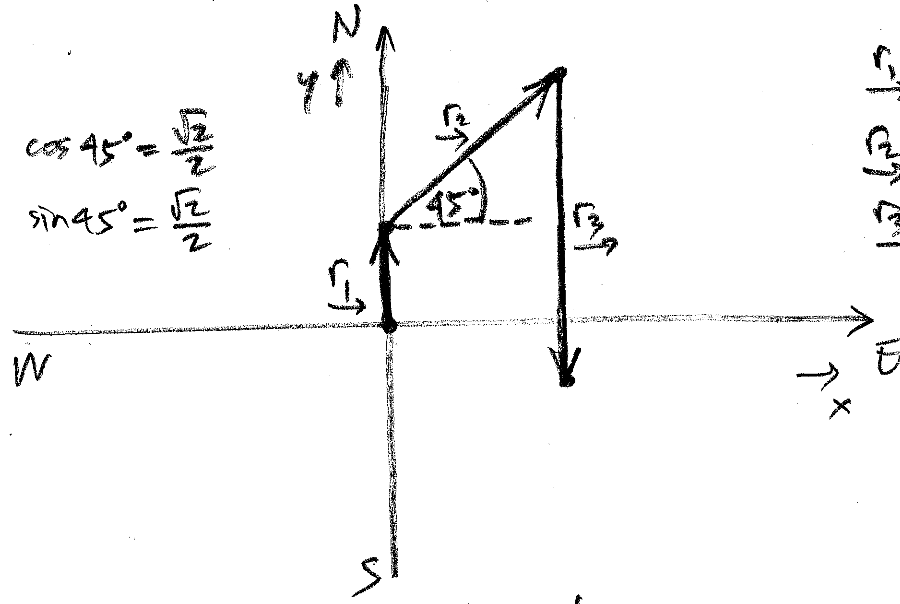
$$\frac{300 \text{ km}}{65 \text{ mph}} = \frac{300 \times 0.6215 \text{ miles}}{65 \text{ mph}} = 2.868 \text{ hours},$$

and since it finishes the first half track in

2-6

$\cos 45^\circ = \frac{\sqrt{2}}{2}$

$\sin 45^\circ = \frac{\sqrt{2}}{2}$



$\vec{r}_1 = 25\,m\,\hat{y}$

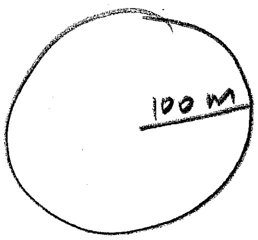
$\vec{r}_2 = 50\frac{\sqrt{2}}{2}\,m\,\hat{x} + 50\frac{\sqrt{2}}{2}\,m\,\hat{y}$

$\vec{r}_3 = -70\,m\,\hat{y}$

The total displacement vector is

$$\vec{S} = \vec{r}_1 + \vec{r}_2 + \vec{r}_3$$
$$= 25m\,\hat{y} + 50\frac{\sqrt{2}}{2}\,m\,\hat{x} + 50\frac{\sqrt{2}}{2}\,m\,\hat{y} - 70\,m\,\hat{y}$$
$$= 25\sqrt{2}\,m\,\hat{x} + (25\,m + 25\sqrt{2}\,m - 70\,m)\,\hat{y}$$
$$= 25\sqrt{2}\,m\,\hat{x} + (25\sqrt{2} - 45)\,m\,\hat{y}$$
$$= 35.35m\,\hat{x} - 9.65m\,\hat{y}$$

2-7



(i) The distance she traveled after one complete track is the circumference of the track, i.e.

$$2\pi R = 2\pi \times 100\ m = 628.3\ m$$

So her speed is

$$v = \frac{2\pi R}{t} = \frac{628.3\ m}{5\ \text{minutes}} = \frac{628.3\ m}{5 \times 60\ s}$$
$$= 2.1\ m/s$$

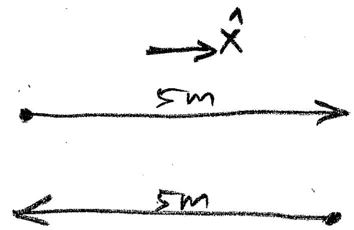
(ii) The displacement she has after one comple track is zero, so her velocity is zero.

$$\frac{150\text{ km}}{55\text{ mph}} = \frac{150 \times 0.6215\text{ miles}}{55\text{ mph}} = 1.695\text{ hours},$$

then it has to finish the second half track in $(2.868 - 1.695)$ hours, or 1.173 hours. The speed in the second half track then becomes

$$\frac{150\text{ km}}{1.173\text{ hours}} = \frac{150 \times 0.6215\text{ miles}}{1.173\text{ hours}} = 79.5\text{ mph}$$

(2-4) $\vec{r} = 5\text{m}\,\hat{x}$ has magnitude 5m.



$\vec{r} = -5\text{m}\,\hat{x}$ also has magnitude 5m, although the direction is different.

(2-5) $\vec{r}_1 = 3\text{m}\,\hat{x} + 4\text{m}\,\hat{y}$ has magnitude

$$\sqrt{(3\text{m})^2 + (4\text{m})^2} = \sqrt{9+16}\text{ m} = 5\text{ m}$$

$\vec{r}_2 = -3\text{m}\,\hat{x} - 4\text{m}\,\hat{y}$ has magnitude

$$\sqrt{(-3\text{m})^2 + (-4\text{m})^2} = \sqrt{9+16}\text{ m} = 5\text{ m}$$

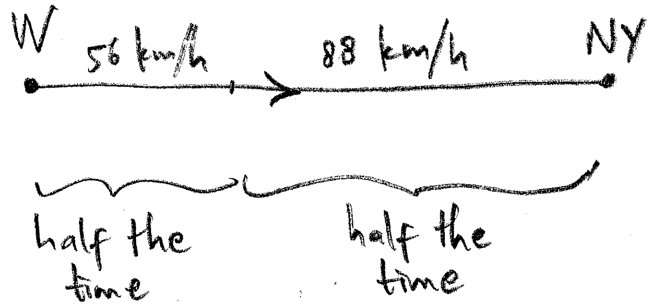
$\vec{r}_3 = 1\text{m}\,\hat{x} + 2\sqrt{2}\text{ m}\,\hat{y} + 4\text{m}\,\hat{z}$ has magnitude

$$\sqrt{(1\text{ m})^2 + (2\sqrt{2}\text{ m})^2 + (4\text{ m})^2} = 5\text{ m}$$

$\vec{r}_4 = 1\text{m}\,\hat{x} + 4\text{m}\,\hat{y} - 2\sqrt{2}\text{ m}\,\hat{z}$ has magnitude

$$\sqrt{(1\text{ m})^2 + (4\text{ m})^2 + (-2\sqrt{2}\text{ m})^2} = 5\text{ m}.$$

(2-8) (i)



Suppose the total time from W to NY is T. So, at the first half time, we traveled a distance

$$56 \text{ km/h} \times \frac{T}{2}.$$

Also, at the second half time, we traveled a distance

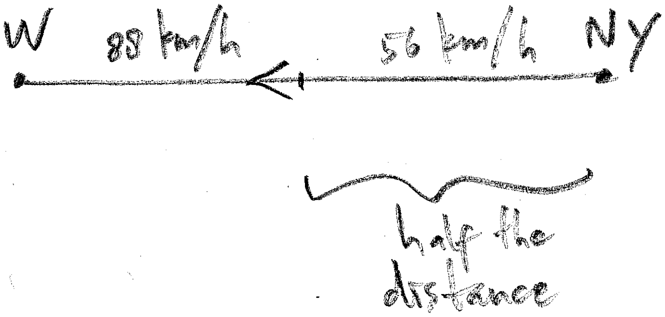
$$88 \text{ km/h} \times \frac{T}{2}.$$

The total distance from W to NY then becomes

$$(56 \text{ km/h} + 88 \text{ km/h}) \times \frac{T}{2}.$$

$$\text{Average speed} = \frac{\text{total distance}}{T} = \frac{56+88}{2} \text{ km/h} = 72 \text{ km/h}.$$

(ii)



Suppose the total distance from NY to W is L. So, at the first half distance, we traveled for a period

$$\frac{L/2}{56 \text{ km/h}}.$$

Also, at the second half distance, we traveled for a period

$$\frac{L/2}{88 \text{ km/h}}.$$

The total time from NY to W then becomes

$$\frac{L/2}{56\ \text{km/h}} + \frac{L/2}{88\ \text{km/h}} = \frac{L}{2}\left(\frac{1}{56\ \text{km/h}} + \frac{1}{88\ \text{km/h}}\right).$$

$$\text{Average speed} = \frac{L}{\text{total time}}$$

$$= \frac{L}{\frac{L}{2}\left(\frac{1}{56\ \text{km/h}} + \frac{1}{88\ \text{km/h}}\right)}$$

$$= \frac{2}{\frac{1}{56} + \frac{1}{88}}\ \frac{\text{km}}{\text{h}}$$

$$= 68.4\ \text{km/h}.$$

(iii) The distance from W to NY is $L = 225.8$ miles $= 363.3$ km.

For the journey in part (i), we traveled in a period

$$t_1 = \frac{L}{72\ \text{km/h}} = \frac{363.3\ \text{km}}{72\ \text{km/h}} = 5.04\ \text{hours}.$$

For the journey in part (ii), we traveled in a period

$$t_2 = \frac{L}{68.4\ \text{km/h}} = \frac{363.3\ \text{km}}{68.4\ \text{km/h}} = 5.31\ \text{hours}$$

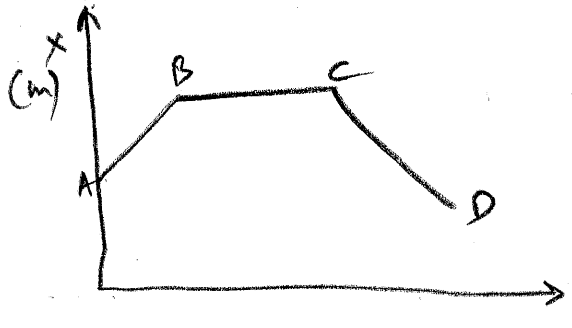
So the total time is $t = t_1 + t_2 = (5.04 + 5.31)$ hours

$$= 10.35\ \text{hours}.$$

The average speed of complete trip is

$$v = \frac{2L}{t} = \frac{2 \times 363.3\ \text{km}}{10.35\ \text{hours}} = 70.2\ \text{km/h}.$$

(2-9)

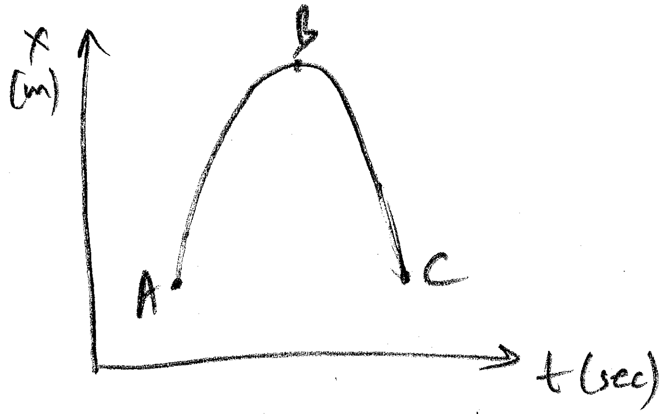


Interval AB: the slope is (+), so the velocity is (+).

Interval BC: the slope is zero, so the velocity is zero.

Interval CD: the slope is (−), so the velocity is (−).

(2-10)



(i) (a) A→B: the slope is (+), so the velocity is (+).

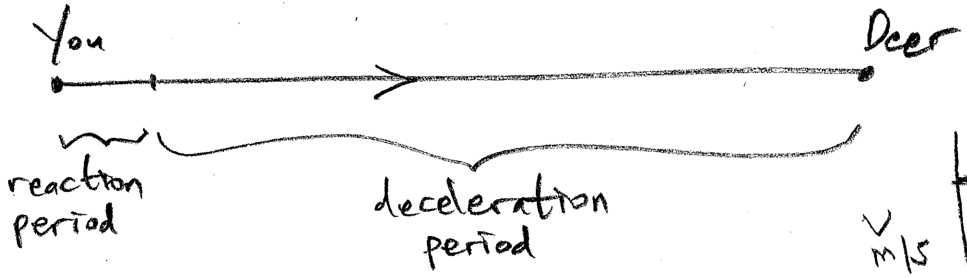
(b) B→C: the slope is (−), so the velocity is (−).

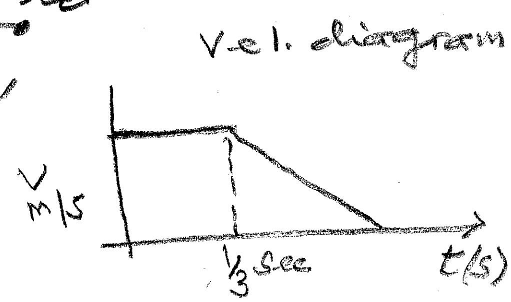
(ii) Since the velocity changes from positive to negative, then the acceleration is negative.

(iii) Since A and C have the same x-coordinate, then the average velocity between them is zero.

(iv) The instantaneous velocity is zero when the slope is zero, so it is at B.

(2-11)





In the reaction period, we have traveled

$$55 \text{ mph} \times \frac{1}{3} s = \frac{55 \times 1609 \text{ meters}}{3600 \quad s} \times \frac{1}{3} s = 8.19 \text{ m}$$

so the distance to the deer now is $(170 - 8.19)\ m = 161.8\ m$.
After that we apply the brake, hoping we can avoid hitting the deer which is 161.8 m in front of us.. Our speed at that time is 55 mph, and the brake deceleration is $2\ m/s^2$. Let's calculate the distance needed to stop and hope that this distance is smaller than 161.8 m.

$$0 = V_0^2 + 2aL$$

$$= (55\ mph)^2 + 2(-2\ m/s^2)L$$

$$0 = \left(\frac{55 \times 1609}{3600}\ \frac{m}{s}\right)^2 + 2(-2\ m/s^2)L$$

or, $L = 151.1\ m$, so the deer is safe.

(2-12) $\vec{V} = \vec{V_0} + \vec{a}t$, where $\vec{a} = 5\ m/s^2\ \hat{x}$.

Its present velocity is $-10\ m/s\ \hat{x}$.

(i) At 2.5 s earlier, we want to find the velocity $\vec{V_0}$ at that time such that the present velocity is $\vec{V} = -10\ \frac{m}{s}\hat{x}$.

So, $\vec{V} = \vec{V_0} + \vec{a}t$

$$-10\ \frac{m}{s}\hat{x} = \vec{V_0} + \left(5\ \frac{m}{s^2}\hat{x}\right)(2.5\ s)$$

$$\vec{V_0} = -10\ \frac{m}{s}\hat{x} - 12.5\ \frac{m}{s}\hat{x}$$

$$= -22.5\ \frac{m}{s}\hat{x}$$

(ii) At 2.5 s later, we want to find the velocity $\vec{V}$ at that time such that the present velocity is $\vec{V_0} = -10\ \frac{m}{s}\hat{x}$.

So, $\underset{\rightarrow}{v} = \underset{\rightarrow}{v_0} + \underset{\rightarrow}{a} t$

$$= -10 \frac{m}{s} \hat{x} + \left(5 \frac{m}{s^2} \hat{x}\right)(2.5\ s)$$

$$= -10 \frac{m}{s} \hat{x} + 12.5 \frac{m}{s} \hat{x}$$

$$= 2.5 \frac{m}{s} \hat{x}$$

(2-13) We need a velocity 200 mph $\hat{x}$ and we have a track of 1.1 km long to accelerate.

$$v^2 = v_0^2 + 2a(x - x_i)$$

$$(200\ \text{mph})^2 = 0 + 2a\,(1.1\ \text{km})$$

$$= 2a\,(1.1 \times 0.6215\ \text{miles})$$

$$= 2a\,(0.684\ \text{miles})$$

So, $a = \dfrac{(200\ \text{mph})^2}{2 \times 0.684\ \text{miles}} = 29255\ \text{mph}$

(2-14) $\underset{\rightarrow}{x} = (5 + 3t - 4t^2)\,\hat{x}$

(i) At $t = 5$ s, we have

$$\underset{\rightarrow}{x} = (5 + 3 \times 5 - 4 \times 5^2)\ m\ \hat{x} = -80\ m\ \hat{x}$$

$$\underset{\rightarrow}{v} = (3 - 8t)\,\hat{x}$$

$$= (3 - 8 \times 5) \frac{m}{s} \hat{x} = -37 \frac{m}{s} \hat{x}$$

$$\underset{\rightarrow}{a} = -8 \frac{m}{s^2} \hat{x}$$

(ii) $\underset{\rightarrow}{x}_1 = 0$

$(5+3t-4t^2)\hat{x} = 0$

So, $5+3t-4t^2=0$.

Rearrange the terms, $4t^2-3t-5=0$.

So we have $t = \frac{3 \pm \sqrt{9-4(4)(-5)}}{2 \cdot 4}$ s

$= \frac{3 \pm \sqrt{89}}{8}$ s

Therefore, $\underset{\rightarrow}{x}_1 = 0$ at $t_1 = \frac{3+\sqrt{89}}{\cancel{-}8}$ s and $t_2 = \frac{3-\sqrt{89}}{\cancel{-}8}$ s.

$= 1.55$ s and $t_2 = -0.8$ s

t_2 refers to time before clock was turned on.