Week (14) Solutions

14-1 In a reversible process, the changes in the thermodynamic variables are carried out extremely slowly so that system is in equilibrium at all points. This means that at each instant, all the thermodynamic variables are well defined and can be represented on a graph. Hence, one can go both forward and backward.

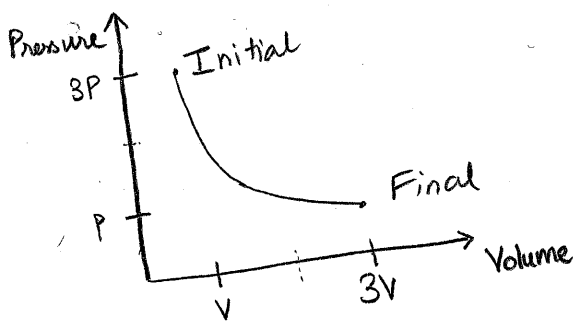
In an irreversible process, the system is in equilibrium only in the initial and final states. It is a one way process. There is no way to return to the original state once the process is carried out.

For an isotherm ; PV = constant.

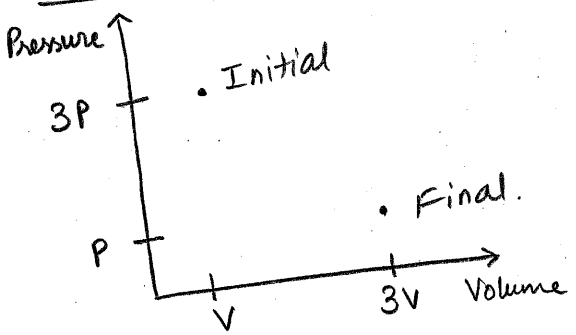
Let initial Pressure = $3P$, volume = V ; $P_i V_i = 3PV$

Then final Pressure = P, volume = $3V$; $P_f V_f = 3PV$

Reversible Isotherm



Irreversible Isotherm

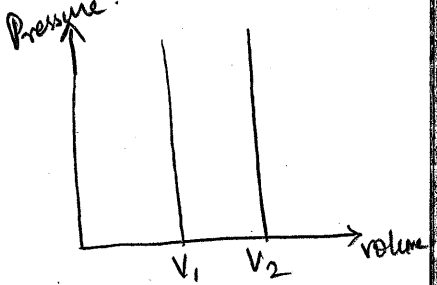


14-2 Isochores ⇒ volume is constant.

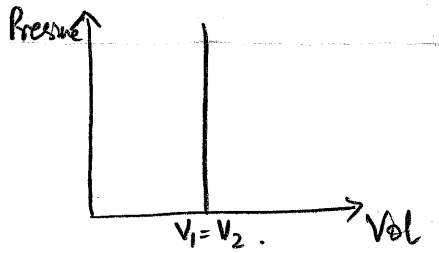
Let one isochore have a constant volume V_1 and other isochore have a constant volume V_2.

such that $V_1 \neq V_2$

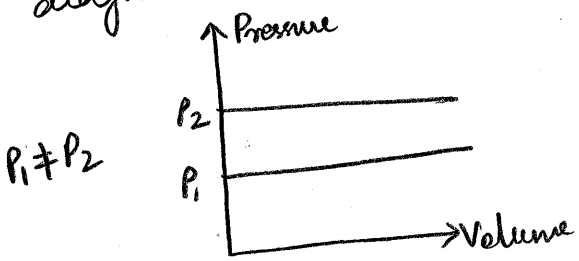
If these 2 isochores intersect at any point, then, at that point $V_1 = V_2$ which violates our assumption. If

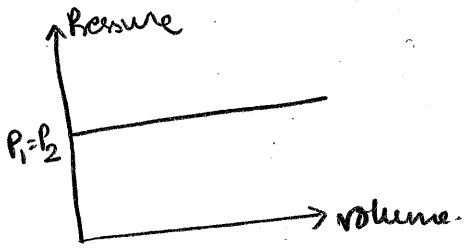


If in the begining we enforce $V_1 = V_2$, then $V_1 = V_2$ at all times and we have one isochore overlapping over the other.

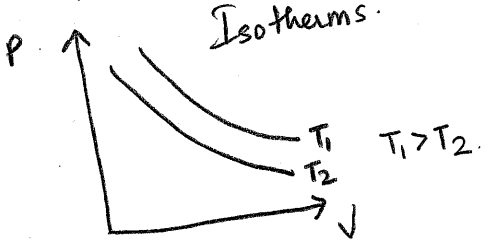


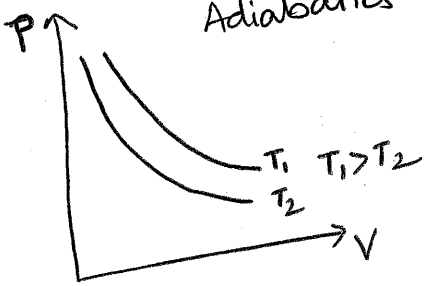
Similar arguement holds for isobars where instead of vertical lines, we get horizontal parallel lines in the P-V diagram.





We can also extend this arguement to isotherms and adiabatics but instead of parallel lines we have non-intersecting curves in these cases.





14-3 Ideal gas $\Rightarrow$ $PV = nRT$

Adiabatic ideal gas $\rightarrow$ $PV^\gamma = \text{const.}$

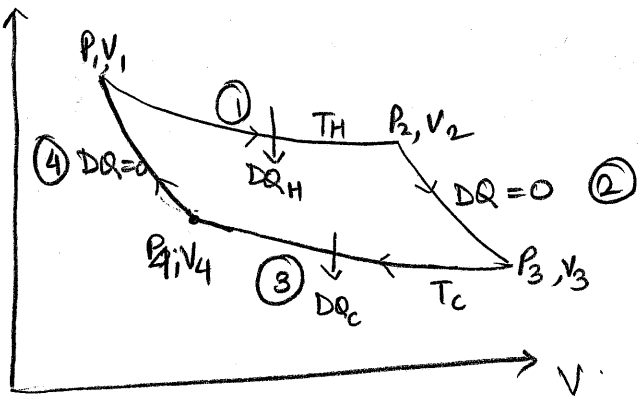
writing P in terms of T from the 1st equation

$$PV^\gamma = \frac{nRT}{V} \cdot V^\gamma = nR \cdot TV^{\gamma-1} = \text{constant}$$

$$\Rightarrow TV^{\gamma-1} = \text{const.}$$

14-4 In nature, there are many processes which are not prohibited by the First law of Thermodynamics, but yet they never occur (for example a ship drawing energy from water around it, a cup of water heating up or cooling down spontaneously at room temperature etc) This suggests that there must be another law which enforces 'direction' to thermodynamic process. Hence 'entropy' was introduced and the 2nd law of Thermodynamics was established which allows only those processes where the final entropy is either equal or greater than the initial entropy.

14-5



Curve ① : Isothermal Expansion

T is constant = T_H

$U \propto T$

$\Rightarrow dU = 0$ since $\Delta T = 0$

$DQ_H = dU + DW$

$= DW = nRT_H \ln\left(\frac{V_2}{V_1}\right)$ ——— A

curve ② : Adiabatic expansion

Temperature changes from T_H to T_C

$DQ = 0$ $\quad T_C V_3^{\gamma-1} = T_H V_2^{\gamma-1}$ B (using result from 14-3)

curve ③ Isothermal Contraction

Temperature = T_C is a constant in this process

$dU = 0$; $DQ_C = DW = nRT_C \ln\left(\frac{V_4}{V_3}\right)$ ——— C

Curve (4) : Adiabatic contraction

$DQ = 0$. Temperature increases from T_C to T_H.

$$T_H V_1^{\gamma-1} = T_C V_4^{\gamma-1} \quad \text{——} \quad \boxed{D}$$

combining [A] and [C] :-

$$\frac{DQ_H}{T_H} + \frac{DQ_C}{T_C} = nR\left[\ln\left(\frac{V_2}{V_1}\right) + \ln\left(\frac{V_4}{V_3}\right)\right]$$

$$= nR \ln\left(\frac{V_2 V_4}{V_1 V_3}\right) \quad \text{——} \quad \boxed{E}$$

Now From [B]

$$T_C V_3^{\gamma-1} = T_H V_2^{\gamma-1}$$

$$\frac{T_C}{T_H} = \left(\frac{V_2}{V_3}\right)^{\gamma-1}$$

From [D] $\quad \frac{T_C}{T_H} = \left(\frac{V_1}{V_4}\right)^{\gamma-1}$

$$\therefore \left(\frac{V_2}{V_3}\right)^{\gamma-1} = \frac{T_C}{T_H} = \left(\frac{V_1}{V_4}\right)^{\gamma-1}$$

$$\Rightarrow V_2 V_4 = V_1 V_3$$

$$\Rightarrow \ln\left(\frac{V_2 V_4}{V_1 V_3}\right) = \ln 1 = 0$$

Substituting this in [E] ;

$$\frac{DQ_H}{T_H} + \frac{DQ_C}{T_C} = 0.$$

14-6

$T_H = 100^\circ C = 373K$.

$\eta = 0.4$

$T_c = ?$

$$\eta = 1 - \frac{T_c}{T_H}$$

$$0.4 = 1 - \frac{T_c}{373} = \frac{373 - T_c}{373}$$

$$\Rightarrow T_c = 373(1 - 0.4) = 224K$$

14-7

$$COP = \frac{Q_H}{Q_H - Q_C} = \frac{T_H}{T_H - T_C}$$

$T_c = 0^\circ C$; $T_H = 25^\circ C$

and input $= Q_H - Q_C = 1kJ$

$$COP = \frac{T_H}{T_H - T_C} = \frac{25+273}{(25+273)-(0+273)} = \frac{298}{25} = 11.92$$

Note that temperatures have been converted to Kelvin.

$$\frac{Q_H}{Q_H - Q_C} = \frac{T_H}{T_H - T_C}$$

$$Q_H = (Q_H - Q_C) \cdot COP$$

$$= 1kJ \cdot 11.92$$

$$= 11.92\ kJ$$